Course Name: Combustion of Solid Fuels and Propellants
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Lecture: 27 Choice of Pressure Index (n); Tutorial Problems

Hello everyone. So, let us continue our discussion on the choice of pressure index N for stable operation of solid propellant rocket. We had begin our discussion / considering one in burning rocket and there we discussed that how the mass generation and mass leaving the nozzle is going to be you know related, because we have seen that mass generation is going to be coming from the regression of the propellant. So, we can relate the mass generation with the linear regression rate R and then we can use the burn rate law

$$r = a*P^n$$

and we have incorporated that in the equation. And for steady state operation we have said that mass generate mass generation rate is going to be equal to mass leaving the nozzle mass leaving the nozzle and doing so, we have obtained a equation for equilibrium pressure and we have said that if the value of N is close to unity there is a you know little change in the parameters for example, like throat area, burning surface area, propellant density, the value of A is going to make a huge change in the equilibrium pressure. On the other hand if there is a small you know value of N the change in these parameters will not make any significant change in the equilibrium pressure.

So, let us see what we have written earlier like we have written that mass generation rate can be you know written as S b which is the burning surface area * R * ρ p and that we have written in terms of burn rate law which is like in place of R we have written A p^n * ρp and for mass leaving the nozzle we have written P c A t / so, here P c A t / C* and equating these two we get the steady state or equilibrium pressure as ρ p * A * C* * S b / A t whole to the power 1 / 1 - N and here we have said if N is near to unity little change in these parameters will make a huge change in the equilibrium pressure whereas, if N is very very small than 1 then there is no significant change in the equilibrium pressure ok. Now we have also said that while considering these we did not consider the change in the volume of the gas because of the regression of the propellant. Now if we say that if the due to the regression of the propellant the volume of the gas is going to change. So, let us say the using the ideal gas equation mass of the gas we can relate with the volume as P v / R T where v is the volume of the gas in the chamber, R is the specific gas constant, R means here R = universal gas constant / molecular weight, T is the gas temperature.

Now if we want to write this with the equations what we earlier had that we had the dm dt = m dot g - m dot n or we had written in terms of like S b * ah A p to the power n * ρ p - P

A t / C* . Now we can write this / replacing m as P v / R T that is = S b * A * p to the power n * ρ p - P A t / C* we may use P c here P c or we can leave it like this let us not use it because it will be easier for us to write. So, we just simply said P here, P is actually the chamber pressure. Now let us look at the this derivative here d d t of P v / R T what we can write here is since the temperature of the gas does not vary much we can actually take the temperature outside of the bracket R is also constant. So, we can have this derivative in the form of like dp dt + P / R T d v d t.

Now what can we say about the d v d t? We can say d v d t is the the change in the rate of change of volume d v d t which we can easily relate with the change in this burning surface area * linear regression because the volume rate the change of volume rate is depending on how the regression is taking place. So, this is my regressing surface the volume is changing because of the regression so it is coming to another this place this place we can relate that simply with the S B * R because length is changing so we can just simply say that S B * R and what about this P / R T you can simply say it is the gas density so P / R T will be the ρ which is like the density of the gas this is basically the product gas.

So, now if you write the equation here we want this equation to be written in the form of like the pressure change so let us say we just write this part or we can just replace the whole thing there in this equation let us say this is equation 1 this is equation 2 this is equation 3 so let us try to write in the equation 2. In terms of this so what we can write V / R T dP dt + this is becoming ρ now ρ we can say the gas density as ρ g so ρ g * S B * R S B R means R is basically A P ^ n that is going to be = this part which is S B * A * P to the power n ρ P - P AT / C* now for steady state condition we can say we can say dP dt = 0 so this equation 4 from here we can write that we can keep this density of propellant and density of gas one side so if we do that it will come as S B * A P to the power n * ρ P - ρ g that is going to be = P AT / C* so the equilibrium pressure equation will become

$$(A * C^* * \rho P - \rho g * S B / AT)^{1 / 1 - n}$$

so the difference between the previous equation and this equation is that earlier we since we did not consider the change in the gas volume it was only the propellant density but now we have seen that this part is modified to ρ P - ρ g but one thing we can notice here is that ρ P is actually very very you know larger than ρ g because ρ g is simply the gas density ρ P is like the propellant density so it is like a huge value compared to the ρ g so therefore even if you neglect this one it is not going to make any much difference in that sense the equations we had earlier it is equally you know can be useful so what is earlier got P Q was = A C * ρ P * S B / AT ^ 1 / 1 - n that was our equation / simply equating m dot g and m dot n here there is a small change that ρ P is becoming now ρ P - ρ g so for a more accurate analysis we can use that now why we are doing this exercise we want to see that what does what is the typical values of n supposed to be in order to have like stable operation of the rocket.

So what we can do is we can actually try to plot the m dot g which is in terms of like S B A P to the power n * ρ P and m dot n which is mass leaving the nozzle C* we want to just plot this as a function of you know pressure and we want to see that for what values of n, n is greater than 1 or n is less than 1 we can get this stable operation so if we just plot them let us say we plot this for n greater than 1 so the mass leaving the nozzle is straight way is going to relate to the pressure uniform I mean linearly so this is m dot n which is simply PcAt / C* so this plot is for m dot versus pressure for n greater than 1 the mass generation curve will look like this so this is m dot g which is you know S B * A P ^ n * ρ P.

So, the intersecting point will tell us the equilibrium pressure as per our equation also that once we are equating the m dot g and m dot n we are going to get the equilibrium pressure so this is for n greater than 1 now same thing we can plot it for n less than 1 this is straight way m dot n for m dot g it will come like this so this is the intersecting point which will give us the equilibrium pressure now what exactly happening here is that if we try to understand that once n is greater than 1 and what n less than 1 what is exactly happening in case of n greater than 1 if there is a small change in the you know small increase the pressure beyond the equilibrium pressure let us say the pressure is slightly increased to some extent and it is beyond the equilibrium pressure the rate of mass generation is exceeding the rate of mass leaving the nozzle.

So, the pressure will tries to you know increase further because your mass generation is less compared to the mass leaving the nozzle so this will increases the with pressure and the pressure will continues to continues to you know increase until the rocket burst so there is a situation is going to arise that pressure will keep on increasing until the rocket burst now if you look at the other side of it in case that there is a due to some momentary disturbances there is a result in you know decrease in pressure below the equilibrium pressure what is going to happen here you see the mass leaving the nozzle is higher than the mass generation rate so since the mass generation rate is lower the pressure tries to fall since mass leaving the nozzle is higher than the mass generation so this will keep on you know decreasing the pressure until the you know stable operation is not possible.

So, it is going to extinguish finally so eventually for the value of n greater than 1 stable operation is not possible now if you look at the cases for n less than 1 for n less than 1 if p is beyond you know p q or p equilibrium that will cause the m dot generation so let us say if we due to some momentary disturbances your our p is beyond p equilibrium pressure so it will bring to a situation where m dot g is less than m dot n this will cause the pressure to decrease and get back to the equilibrium pressure similarly when there is a you know drop in pressure due to some momentary effect and here our generation will be higher than the mass leaving the nozzle that will cause the increase the pressure and it will come towards the equilibrium condition.

So, what we just said in both the cases either if it goes beyond the equilibrium pressure or if it is going below the equilibrium pressure in both the cases it will try to move to the equilibrium pressure so here the for n less than 1 stable operation of the rocket is possible so because of that the choice of pressure index is such that it it is supposed to be less than n and we have already discussed also that for double base propellant the value of n is in the range of like 0.5 to 0.6 and few cases even higher than that in case of you know composite propellant we have seen that it is in the order of like 0.3 to 0.5 or so but of course they are less than 1 for stable operation of rocket.

So, I think with that we pretty much understood the concept of you know mechanism of different type of propellants burning we have related the the choice of you know pressure index n how it is decided for stable operation of the rocket. Now we want to close this module / solving some example problems related to the burning rate that we want to you know calculate some of the performance parameters when the burn rate law is given so that we can tyπcally you know understand that the concept of burn rate how we can actually calculate the values of mass flow rate through the you know mass mass generation rate and how the mass leaving the nozzle.

So, we can relate to that burn rate equation and we can actually get the data from there. One more thing I should try to tell here is that sometime you know the burn rate law is expressed in terms of some reference pressure that R = A P / P reference to the power n because you say one thing we should remember that the unit of R or the burn rate it is basically the length per time unit and in most occasion it is expressed in terms of like millimeter per second or centimeter per second. Now if you look at the unit of A it is like R divided / P to the power n. So it is kind of a very arbitrary I mean awkward unit. So in order to you know relate this unit of A with the unit of R P is non-dimensionalized / dividing it with reference pressure and the reference pressure value is taken as 7 MPa or 70 atmosphere and in that case the value of R at 7 MPa is denoted as A 70 and the unit of A is of course is going to be unit of R.

So, I mean this is like you know this 7 MPa pressure or the 70 atmosphere pressure is kind of near to the you know operating pressure of the high pressure solid rocket. So that is why this is you know adopted in practice. Now in some problem may be if the value of A 70 is given that inherently mean that we are talking about that the reference pressure is given as 7 MPa. So I will try to show you one example problems relating to A 70 as well. So let us try to solve one or two example problems so that we can close this module with you know greater confidence.

Let us look at the tutorial problems. So in this tutorial problem what is saying is a double base solid propellant rocket is employed in a missile application following data are given. It is an n burning solid rocket so n burning means we can immediately we should have the πcture in our mind that it is like the burning surface area is not changing it is neutral type of burning. So it is n burning rocket means it is burning like this surface area regressing like this so the surface area is not changing so it is a neutral type of burning. There is saying it is need to provide it is given the throat diameter as 30 mm so immediately if we use you can actually calculate the throat area we should calculate the throat area it needs to provide a constant thrust of 5000 Newton or 5 kilo Newton.

So, 5000 Newton. The propellant follows a burn rate law R = 0.004 P to the power 0.5 where P is given in Pascal and R is given in millimeter per second you see this units are important. So, now you know that the value of P is in Pascal and R is in millimeter per second.

So, this is tyπcally R = A P to the power n so the value of A is given here and n is given here n is 0.5 and A is given as 0.004 of course, it is millimeter per second means we have to carefully use the unit because pressure is in Pascal and R is in millimeter per second. So, we have to carefully use this so this is going to be very much you know we have to make it consistent with the unit. Now, other values are given like I s p is given as 1900 Newton second per kg C* or the characteristic velocity is given as 1600 meter per second ρ P is given as 1540 kg per meter cube.

Now, it is asking to find out the diameter of the propellant grain. So, if we know the S b we can find out the diameter of the grain if the rocket has to operate for 200 second determine the length of the solid propellant grain. So, these are the two questions it is asking. So, in order to proceed that we will immediately as I said we will calculate the throat area. So, you can do it yourself, but the values I got is like 0.000707 meter square. You are requested to check these values now m dot we can immediately write m dot as S b * r * ρ P. How do you find the m dot? m dot we can relate with the thrust and the I s p data. So, thrust is given as 5000 Newton I s p is given as 1900. So, if you do this we can find the value of m dot which is coming around 2.63 kg per second. Now, we can simply write our equation you know we had m dot = P c A t / C* is given here A t we have calculated. So, from here we can easily find out the pressure and it is coming something around you know 5951909 Pascal. So, you can use this equation of burn rate r = A P to the power n here it is 0.004 P to the power 0.5.

So, you put the values here you will end up getting you know the value of burn rate as 9.759 millimeter per second. Now, you bring this m dot equation because it is asking about the diameter of the propellant grain. So, in order to get the diameter of the propellant grain we have to find out the value of S b or the burning surface area. So, if you relate this m dot with the burning surface area S b * r * ρ p we can actually get the value of S b / dividing the m dot / you know r * ρ p r we just got.

So, we put the value of 2.63. Now, 2.63 millimeter per second. So, we multiplied this 10^-3 which is going to be the going to be in the meter per second r is sorry this is this is wrong this was our mass flow rate.

Here the burn rate was given as 9.9759 here we have to multiply with 10^- 3 and the density of the propellant is given as 1540 kg per meter cube. If you do that you will get the value of S b which is coming something around 0.175 meter square. Now, the diameter of the grain you can find out because you know the surface area.

So, πd^2 / 4 is going to be equal to your S b. So, this is your grain diameter if you solve for it I think it is coming as diameter of the grain is coming as 0.472 meter or 472 millimeter ok. So, this is the answer for question number 1. I would still suggest you please check the calculation once.

Now, the second question was it was asking that if the rocket has to operate for 200 second determine the length of the solid propellant grain. So, if it is operating for 200 second we know that for 200 second how much mass is required. We can say the mass will be like m dot * the time of firing. So, we can simply multiply that with the m dot * 200 second that will come as 526 kg. You can get the volume of the propellant which is like you know mass divided / the density, density is 1540 that much meter cube.

So, it will come as 0.341558 meter cube. So, the length of the grain how do we get the length of the grain? Length of the grain will be simply volume divided / the surface area of the grain. So, you can put the value of this 341558 / 0.175. So, this will come as here we have already calculated the surface area of the grain.

So, we can get the value of the length of the grain is coming as 1.952 meter. So, that is for the answer of question number 2. So, this way if you just know how to you know relate the burning rate with pressure with the burn rate law and the other parameters and you know the relationship of our known you know performance parameters like thrust, the specific impulse, we can easily find out the values from the given values. So, this is one type of one type of you know problems one can easily solve in this case.

Now, second problem I just wanted to bring you here the A 70 thing what we have just discussed that A 70 is the given at a reference pressure. So, in this question it is you know asking us to let us see what is saying. The problem statement goes / this that an an end burning rocket employs dB propellant grain, propellant density of this with a diameter of 125 millimeter and generous 205 Newton over 250 second with the characteristic velocity of 1350 meter per second and thrust coefficient of 1.2. Consider the following values for this propellant n = 0.5, A 70 is 3.2 millimeter per second, determine the length of the grain, chamber pressure and throat diameter of the nozzle. So, I will just tell you how to do it then I think you can you can try yourself and do this thing. So, immediately once it is given the you know diameter of the grain and it says the n burning again it is a n burning rocket. So, we can immediately write the surface area as π / 4 the diameter is given 0.125 that much you know meter square since it was given as 125 meter 125 millimeter.

So, I am converting that to meter. So, you can easily find out the value of this what I got is like 0.0123 meter square. Then recall the burn rate law we have r = A p to the power n here it is given as A 70 is given. So, we should recall what we discussed already.

So, you can write A is 3.2 millimeter per second. So, 3.2 * 10 to the power - 3 * p as in Pascal. So, let us say p we can convert that p reference is 70 MPa which is 70 * 10 to the power 6 to the power 0.5. So, now this r is going to be meter per second since we have converted that to meter per second. So, the unit of r will become meter per second here and remember the value of A 70 is given means immediately we should use the equation / employing the reference pressure p r f p r e f which is 70 atmosphere or 7 MPa which is 70 * 10 to the power 6 Pascal. Specific impulse is given no specific impulse is not given. So, we can simply find out the specific impulse / multiplying the C*s * C f because C* value is given 1350. So, we can use 1350 * the value of C f which is 1.2 that will the unit will be Newton second per kg.

So, if you do this it will come as 62 Newton second per kg. What will be the mass flow rate through the nozzle will be simply f / Isp you put the value of f which is 205 Newton given here Isp you have just found out. So, 1620 this much kg per second. So, it will come as 1.1265 kg per second. What will be the mass of the propellant? Remember it is firing for 215 second duration. So, we can simply write m dot * the firing duration t. So, it will come as you know 0.1265 * 215 this much of kg.

So, that will come as 27.2 kg. So, what will be the volume of the propellant? You simply divide this mass / the density which is 1900 kg per meter cube. So, this will be the density which will come as 0.0143 meter cube. So, from here you can easily find out the length of the grain by dividing this volume by the surface area.

So, V / Sb. So, you put the volume 1.0143 meter cube by surface area was 0.0123. So, that much meter it will come which is like 1.162 meter. So, that is the length of the grain. So, I think if you just do this carefully by applying the burn rate law equations only thing you have to remember is the consistent unit you have to use the unit very consistent then only I think you can use it properly. So, you can get the value of the length of the grain. What about the chamber pressure? Again the chamber pressure you can you can determine from the equation of the burn rate. So, first of all we know the burn rate law are = given as you know a 70 a 70 value was a value was given which was like you know 3.2 * 10^- 3 * P / 70 7 * 10 to the power 6 to the power 0.5.

Now, we do not know the value of R, but since we know the length of the grain and the operation duration of the grain we can easily find out the value of R which is the length of the grain divided / the operation the firing duration which is 215 second. So, 1.162 meter /

215 second will give us the value of R burn rate. So, now you put the value here which is going to be like 5.4 * 10^- 3 is going to be = 3.2 * 10 to the power - 3 * P / P reference which is 70 to the power 6 to the power 0.5 you can easily find out the pressure here. Pressure will come as 19.95 * 10 to the power 6 Pascal or it will it is like 19 point or close to 20 mega Pascal ok. So, that way you can easily you know solve the problem if you know it properly.

Now, it is asking about the throat area or throat diameter I guess yeah throat diameter of the nozzle. So, first of all we need to find out the throat area. Throat area you can easily find out from the thrust equation which is like CF = sorry F = CF * PC * AT. So, from here you can find out the value of AT by dividing the thrust by CF and pressure I think you can get the value here and from the AT value you can easily find out the throat diameter. So, what we have learned here is that if you do it carefully I think you can easily solve this type of problem step by step just follow step by step.

So, for the case of even if it is given for a reference pressure if A 70 value is given we can apply the equation properly only thing you have to remember is that unit has to be consistent. Once you are talking about like P is in pressure and R is in millimeter per second. So, use the unit consistently because Pascal means it is like Newton per meter square. So, we must need to use this unit in in meter per second.

So, that is why I multiply with 10 to the power - 3. So, while solving the problem we must be sure about the unit we have to use the consistent unit then I think we should get the correct values. Otherwise while solving the solving the problem also you may find some values which looks very different or kind of a odd which cannot be possible in physically then definitely you can ask the question yourself and you can also always look back that where I am doing the mistake ok and that way you can actually find out the issues and you can correct it. So, I think with that we close the discussion of our you know this module of the entire module of you know combustion of solid propellants.

So, with solving the problems I think we pretty much now aware about how to you know employ the burn rate law equation R = A P^n for solving you know this type of problems ok. Now, I think we will I mean start our next module which is like you know combustion of metals because that is also solid fuels. So, that will be start in the next lecture ok. Thank you.