Course Name: Combustion of Solid Fuels and Propellants

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Lecture: 02 Basic Principle of Rocket (continued)

Hello everyone, welcome back. So, we are starting our second lecture on the course. If you recall in the previous lecture, we discussed about the rocket equation which is known as Tsiolkovsky's equation and there we found out that the incremental velocity is given as:

$$\Delta V \ = V_J \ln\left(\frac{M_i}{M_f}\right)$$

Where V_J was defined as the efflux velocity, Mi was the initial mass of the rocket, and Mf was the final mass of the rocket. And we know that to give the high velocity to the vehicle we need to give in incremental order and that is why this delta V incremental ideal velocity is an important equation. Now, we should also appreciate that this incremental velocity cannot be given in one stage or a single stage. If it can be given it is of course, beneficial like the single-stage to rocket or even in two-stage SSTO, single-stage to orbit or two-stage to orbit those are like an efficient vehicle, but in reality, it is very difficult to give a high incremental velocity at once.

Instead, the incremental velocities are given in stages which is why there is a concept called multistaging. So, we will talk about multistaging here briefly because you have already learned that thing in the previous course. Now, we have also discussed the mass ratio of the rocket (Rm) which is defined as the:

$$Rm \ = \frac{M_f}{M_i}$$

Then we have also talked about the various mass fractions like payload, structural, and propellant mass fractions.

And we have rewritten the delta-V equation in terms of those mass fractions. Now, if you talk about multistaging, multistaging in a rocket is the design concept where we are you know making the velocity increments step-wise or stage-wise. So, here, actually, the strategy is to make ascent missions more efficient by dividing the total rocket into several sub rockets or stages. Typically, each sub rocket is a complete rocket by itself, and at the end of the burnout, the shell and other inert mass are discarded from the rocket. Now, after each stage is over, the second stage will start and keep running. It is going to provide the velocity increment.

Now, also, if you can recall, in the previous courses or your basic courses there are rockets at the bottommost stages where they are tied up with the main rocket or we can say the booster rocket. So, if this is the booster stage, if this is the booster stage some rockets are tied up with this booster stage these are called clustering of rockets. So, we can call this parallel staging, some way you can say it is a parallel staging. Now, the multistaging is done in this serial manner. So, this is the concept of multistaging.

So, this is the first stage, this is the second stage and this is the third stage. So, the total velocity increment will be like the velocity increment provided by the first stage plus the velocity increment provided by the second stage plus the velocity increment provided by the third stage. Now, one can derive a generic equation considering the n number of stages and you can write down the velocity increment, and total velocity increment as V j 1. So, we can just be writing the rocket equations that we already have in terms of the mass ratio which is going to be like this:

$$\Delta V = V_{J1} ln\left(\frac{1}{R_{m1}}\right) + V_{J2}\ ln\left(\frac{1}{R_{m2}}\right) + \ V_{J3}\ ln\left(\frac{1}{R_{m3}}\right) + \cdots + V_{Jn}\ ln\left(\frac{1}{R_{mn}}\right)$$

So, this is like a generic way of writing.

One can simplify it by considering the efflux velocity that V J 1 equal to V J 2 equal to V J 3 equal to V J n equal to average V J, and one can write this in terms of the following equation,

$$\Delta V = \bar{V}_J \ln\left(\frac{1}{R_{m1}}\right)\left(\frac{1}{R_{m2}}\right)\ldots\left(\frac{1}{R_{mn}}\right)$$

One can further simplify this by considering the final mass of the first stage is going to be the initial mass of the second stage, the final mass of the second stage is going to be initial mass of third stage and so on. So, if we do so, we can get

$$\Delta V = \bar{V}_J \, ln\left(\frac{M_{i1}}{M_{fn}}\right)$$

So, this will give the ratio of the initial mass of the first stage to the final mass of the last stage of the rocket or you can say it represents the overall mass ratio of the rocket which is going to be equal to

$$\Delta V = \bar{V}_J \, ln\left(\frac{1}{R_m}\right)\hat{}n$$

Where Rm is the overall mass ratio of the rocket. So, this is going to be to the power n. So, it will become

$$\Delta V = n \, \bar{V}_J \, ln\left(\frac{1}{R_m}\right)$$

Now, I am following these equations from one of the textbooks given all given already in the course curriculum which is the rocket propulsion book by Professor Ramamurthy. So, one can refer to it for more details even the NPTEL video lecture is also available. So, these are just a revision of what we have already learned.

Now, if you recall we had already told ourselves that the in the delta-V equation or the ideal rocket incremental velocity equation or rocket equations the V j or the jet velocity was an important parameter. So, can we just spend some time and look at the what are the influencing factor of this V j. But before going to that some of the assumptions we should you know remember or we should recall in terms of the ideal rocket. So, some of the assumptions we generally follow for ideal rocket. Now, why we really look at this assumption because you know the concept of ideal rocket is useful because the relevant some of the basic thermodynamic principle we can express those principle in a simple mathematical relation.

And of course, that will help us to derive certain equations more simply. Some of these equations will describe the quasi-one-dimensional nozzle flow which we always consider that is, of course, an idealization and simplification of three-dimensional flow. And of course, it involves two-dimensional relations and real aero thermo mechanical behavior. However, this relationship will give us a starting point for the designer to understand how we can represent the flow behavior or the overall aero thermochemical behavior in terms of simple mathematical relations. Being so, the overall error if we can see it will not be more than like you know 5 to 6 percent.

So, it will be a good estimate to begin with these ideal rocket assumptions with the simplified equations and we can get some performance parameters which will not be that much off from the actual performance parameters maybe between like 1 to 6 percent. So, let us talk about the various assumptions involved in the ideal rocket concept. So, the first one we consider that the working substance or sometimes we say the working fluid, the working substance we say it is homogeneous. So, what is the working substance? Working substances are nothing, but they are chemical reaction products. So, they are combustion products.

So, they are homogeneous. We can consider that all the working species are gaseous. Now, there is a chance that the species can be in the form of you know some condensed phase, and there may be some liquid or solid phases of combustion products, but for our assumptions, we are considering that all the species are gaseous. So, any kind of condensed phases are kind of negligible. Of course, one can argue on these things how can we consider this one you see if you consider a solid rocket, if this is the typical solid propellant rocket it consists of the propellant grain.

In the propellant grain if you look it look it very closely it consists of various you know metal particles as fuel, it may have like it must have the oxidizer crystals. So, if we consider these are aluminum particles, the bigger one if we consider they are aluminum those are crystals and they are bounded together with the help of the HTPB or the polymer. So, once they burn, they will create the combustion products which will go and expand in the nozzle. So, according to our assumptions we are considering that there will be no condensed phases present in the product, but there is a possibility that some of these aluminum particles will convert into aluminum oxides. So, there is a chance of some of these small oxide particles present in the gas and maybe there will be some if the temperature is not high enough, if it is not more than its volatilization temperature they may become some molten oxides.

So, some of the phases may be liquid, but for the sake of simplicity for the ideal rocket concept, we are considering that all gases all species present are gaseous. So, this is another assumption. Now, once we say all species are gaseous, we also say that our working substance obeys the perfect gas law. This is a straightforward assumption. Now, we say that no heat transfers across the rocket walls.

So, in a sense, we say the flow is adiabatic. So, this is another assumption. I think many of the assumptions you have already learned during the aerodynamic course if you have taken like while considering the isentropic flow through nozzle some of the assumptions we have already learned there. Still, I just want to repeat those just to start the course with a background that we already know familiar with this stuff. The number 5 assumption is there is no appreciable friction, friction, and of course, no boundary layer effects. We consider that there are no shock waves or flow discontinuities present in the nozzle flow.

Again, probably you have already done this thing in your previous aerodynamics course and this assumption was already there. So, probably you have considered that we consider that the propellant flow is steady and constant. Now, if you look at the previous slide here what you are saying is propellant flow means that whatever the propellant is burning there, there will be some mass flow rate like let us say m dot f for the solid rocket. So, there will be some burning of the propellant layer and they will form some product gases. So, it will be some regression of the propellant.

So, that will form the combustion gases after burning. So, we say that the propellant flow is constant and steady. The same thing is applicable for the liquid rocket because liquid rocket it is coming from two separate tanks fuel tank and oxidizer tank they are passed through some pump and they are coming to the thrust chamber. So, what we are saying through this assumption that the propellant flow is steady and constant. The transient effects like during the starting and stopping there may be some changes in the flow rate of course, but we are neglecting the transient effect.

So, we are neglecting the transient effects such as during starting start, or stop. One more assumption we already considered while we learned about the isentropic flow through the nozzle which is like the exhaust gas leaving the nozzle is axial means it is following like one dimensional. So, what I am saying here is if you look at the nozzle here, we are saying the flow is axially directed means one-dimensional. But if you look at look at very closely near the wall the flow cannot be exactly axial because it is going to follow the like this like this. So, the assumption we are making here that all the gases are flowing through the nozzle axially or it is parallel to the nozzle axis.

So, one-dimensional flow is considered. This was already there if you recall in recall the assumptions used in the isentropic flow through the nozzle. So, it is the same assumption. Now we also consider the gas velocity, pressure, temperature, and density to be uniform across any cross-section, any section normal to the axis. What does it mean? It means let us say if we consider our section here like at A A B B.

So, at any section, if you look there are various locations like at different radial locations. So, according to these assumptions, the gas velocity, pressure, temperature, and density are uniform across any section normal to the axis. So, if I say the velocity at this location and this location are the same although there is a radial it is radially at a different location. So, this location is not the same as this, but since we are considering one section normal to the axis we are considering at this station at station A the velocity, pressure, temperature, and density are uniform. Whereas, same thing at station B the velocity here and velocity here are the same.

So, this is another assumption. So, it is almost similar if you recall you have done some cycle analysis for air-breathing engines like turbojet, and turbofan you have considered the pressure, and temperature at different locations like at the compressor inlet, at the compressor outlet, at the combustor outlet, at the turbine inlet, at the turbine outlet you did not consider the I mean the change of pressure, temperature at different radial location at the particular section. So, it is a similar type of assumption. And also, we say that the chemical equilibrium is established. We say that there is no further change in the gas composition whatever the gas is passing through the nozzle there is no further change in gas composition. So, there might be some reactions taking place in the nozzle itself, but we are not considering that even if it is taking, we are considering it has a negligible effect on the species composition.

So, we say that chemical equilibrium is established, and no chemical composition will change when the gas passes through the nozzle. So, these are the various assumptions and other assumptions that allow us to do some simplified analysis and make the equations. I would say represent the equations in simpler mathematical terms through which we can actually do the calculations. We can begin our derivations of what we have already learned about the V j. If you recall, our important terminology in the rocket equation was delta V, which is equal to

$$\Delta V = V_J \ln\left(\frac{M_i}{M_f}\right)$$

So, we said that we should at least look at the parameters influencing V_J.

So, let us try to derive this. So, we need to consider the flow through the nozzle here. So, the nozzle is nothing but just a vent. You can say one high-pressure, high-temperature chamber is there. So, we can consider, you know, gas expansion from a high-pressure chamber. We can treat one high-pressure chamber; you can take any shape does not matter. So, if you look at the nozzle, it will look like this.

So, we can treat this the same way: as if it is a high-pressure chamber with a small vent. So, this is the vent. So, if you consider a control volume of this vent, you can apply it if you consider the inlet and outlet condition of the vent. So, let us say we have the chamber condition as you know, the h c, p c, and T c. So, enthalpy is h c, the chamber pressure is p c, and chamber temperature is T c at the exit; we can say we know already the velocity is V_J we can consider the pressure p e, enthalpy h e.

Now, if you apply the steady flow energy equation for this control volume, what you can write here is, let us say we consider that there is a heat input and work output by this control volume. So, if the heat input is q dot work output is W e, we can write the equation as:

$$\frac{\dot{Q} - \dot{W}e}{\dot{m}} = \left(he + \left(\frac{V_J^2}{2}\right) + g.ze\right) - \left(hc + \left(\frac{V_c^2}{2}\right) + g.zc\right)$$

Now, here, the m dot is the mass flow rate through the nozzle hc; we have already said the hc and he are the enthalpy of gas in the chamber here it is in the chamber here it is at the exit of the nozzle; ze and zc we have considered that heights above the datum at entry and exit you know we have already considered the adiabatic flow. Since there is no work the We dot is zero.

So, we can also consider Q dot as 0 (due to adiabatic flow). So, eventually, this equation will become

$$\left(he + \left(\frac{V_J^2}{2}\right) + g.ze\right) = \left(hc + \left(\frac{V_c^2}{2}\right) + g.zc\right)$$

Also, the difference between the ze and zc are minimal. So, we can also take out these two terms. Further, we can see that the velocity Vc is kind of, you know, very small compared to the velocity at the exit. So, this can also be negligible because the inside the chamber flow is almost like having an insignificant velocity. Eventually, this equation will become:

$$\left(V_J^2/2\right) = (hc - he)$$

So, if you expand this further, we can write this in terms of Cp Tc and Te.

$$\left(V_J^2/2\right) = Cp\ (Tc - Te)$$

Remember, we had begun this exercise just to know the influencing factors on the efflux velocity V_J.

So, we are trying to write V j in terms of others. So we can write,

$$V_J^2 = 2\,Cp(\,Tc - Te)$$

$$V_J^2 = \frac{2\gamma RuTe}{Mw(\gamma - 1)}\,(1 - Te/Tc)$$

So, can we again know what Cp is?

$$Cp = \gamma.Ru/Mw(\gamma - 1)$$

$$Cp = \gamma R/(\gamma - 1)$$

Where R is the specific gas constant. So (Ru), is the universal gas constant which is given as the 8.314 kilojoule per kilo mole Kelvin. So, suppose you know the molecular weight of a particular gas. In that case, you can easily find the gas constant for that particular gas because if you have to apply this equation, you need to find the value of R. Now, one can think in terms of the isentropic flow. We can simplify this equation further by considering that this Te by Tc we can write in terms of the pressure ratio Pe by Pc as follows:

$$\frac{Te}{Tc} = \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

So, applying this relation, we can modify the V_J equation, and we can write V_J square, or we can directly write V_J as :

$$V_J = \sqrt{\frac{2\gamma Ru}{(\gamma - 1)M_w}Te\left[1 - \left(\frac{Pe}{P_C}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

So, this is the equation for the efflux velocity or the jet velocity. In the following lecture, we will try to understand the parameters that are more important and influencing the value of V_J because V_J is very important in terms of the rocket equation. So, we will try to understand the various influencing factor for V_J. With that we close this lecture. Thank you.