Course Name: Combustion of Solid Fuels and Propellants
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Lecture: 18 Evaluation of Burn Rate of DB Propellants (Contd...)

Hello everyone. So, we are continuing our discussion on combustion of solid propellants. And, there we have ah began our discussion on the burn rate of double base propellants.

We have ah discussed the mechanisms involved in the combustion of double base propellants like ah the formation of various zones depending on the chamber pressure some zones may exist may not exist. However, the relative importance of various zones we have talked about we have discussed that right after the ah propellant surface there will be a zone exist which is kind of like foam zone or solid propellant degradation zone. Then we have the feed zone and then there is a preparation zone or induction zone or dark zone where the reactions do take place, but there is no significant change in the temperature. So, temperature remains almost constant and then we have the secondary flame zone or the luminous zone. Now, depending on the pressure the dark zone may exist or the extent of the dark zone may be reduced. If the pressure is high enough more than 10 MPa then it will be completely ah reduced to almost 0.

So, the right after the feed zone the secondary luminous zone or the luminous flame zone will ah continue. And I think we have already shown this thing through some ah pictures from referred journal papers that with various pressure how the flame is coming close to the propellant surface thereby increasing the heat transfer. So, when the dark zone is very much ah you know extended to certain ah distance for example, like it may vary from few millimeter to few centimeter.

So, if the dark zone is long enough the primary heat transfer is going to take place from the feed zone ah flame to the propellant surface. And the heat conduction to the ah propellant will take place, the heat will be generated within the propellant will take place. The heat ah conduction to the ah feed from the feed zone sorry heat ah input from the feed zone will help to further degradation of the propellant and the burning will continue. In order to understand these ah processes and represent into simple mathematical ah formula we had consider a small control volume within the feed zone considering the fact that the major reactions are taking place in the feed zone.

So, we have consider the ah small control volume in the feed zone and we have discuss about the heat conduction into the control volume heat generated within the control volume and that will you know ah increase the enthalpy of the flowing gas. And based on the temperature profile we have ah discussed about the boundary conditions that at x equal to 0 which is the propellant surface the temperature is T s and at x equal to L or the extent of the feed zone we have said that the ah temperature is about ah T d. And from there we can actually try to solve the the ordinary differential equation which actually going to give us some kind of a you know temperature profile, but it is a it is a involved process.

So, ah we need to know some of the parameters in order to ah solve this equations. And then we finally, talked about that if the ah the burning rate is to be evaluated we need to know how these heat release rate is being you know influenced by the pressure and we have said that the heat release rate is very much you know influenced by the pressure the other ah parameters like the gas velocities and all those are going to be influenced by the pressure eventually the burn rate is going to be very much dependent on the pressure. And then there is one empirical relationship is formulated by the researchers which is known as the burn rate law or sent ah Roberts law. So, up to there I think we have discussed.

So, let me just quickly give you a recap of what we have discussed and we just ah continue with the equations. So, we had said that if we have this you know solid propellant and we have said that if we have which is of course, the DB propellant or double base propellant. So, right at the surface of the propellant we have one zone which is kind of a you know solid propellant degradation zone it is known as the foam zone. Then we have the feed zone then we have the dark zone and then we have the luminous zone or the flame zone. And we have also you know ah draw the typical temperature profile here like this.

So, depending on the ah pressure this dark zone may be reduced to certain extent or if the pressure is high enough the dark zone will cease to exist. Now for the consideration of the analysis we have considered a small control volume within the feed zone and we have considered the heat transfer taking place here the heat generated like chemical energy ah Q chem Q dot chem heat generated within the control volume and that increases the ah enthalpy of the flowing gases. And from there I think we have already ah found out some you know relationship through which we are telling that the we tried to you know involve the propellant density and then we said the gas density and the velocity of the gas velocity of the flowing gas. Now what we observed there is that that the rho g u g and Q dot chem depend on pressure.

$$\rho pr = \rho gug$$

And in replacing this equation and we said that R can be you know put there in the previous ordinary differential equation and if a unique solution exist for which the ah it will satisfy the temperature at the boundaries and the heat balance within the foam zone and feed zones that would give us the the value of the bonded of the propellant. Now it is not a direct solution it is going to be an involved process.

So, since these parameters are very much dependent on the pressure and also the influence of the initial temperature, temperature in the feed zone like if it is the T d the temperature of the feed zone temperature or the flame temperature the activation energy E what about the chemical composition of the propellant will also be the influencing factor what about the physical and physical and chemical property and physical and thermal properties of the propellant will also be you know influencing factor on the value of burn rate. And of course, not only the propellant also the combustion gases which are formed due to the reactions.

So, the physical property thermal property of the gases initial temperature the temperature in the feed zone the activation energy chemical composition of the propellant that will also have an influencing effect on the burn rate. Whereas, indirectly we have said that the rho g u g and Q dot chem also majorly depend on pressure. So, somehow we have to relate this you know pressure dependency and the you know these features into the equations. So, it was you know kind of very involved process there is no direct you know solution to this, but however, it has been given by the formula is like

$$r = ap^n$$

takes care of these you know T i the composition of the propellant and the chemical and physical property of the gases and other characteristics what we have just said earlier that will takes that will be taken care of by this constant A. Whereas, n brings the this is the pressure index which brings the pressure effects. So, pressure index will bring the pressure effects in this equation. Now, this equation is known as burn rate law or sent roberts law or sometime it is also known as Belize law.

So, this is very important law in the consideration of burning of the propellant and we use this very often. Now, the values of A and n generally obtain from the experiment. I will tell you some small you know numerical problem through which we can also understand how we can find the value of A and n. Generally the experiments are performed at various pressure in a you know Crawford burner I think we have already mentioned about this thing earlier. It is like a constant pressure chamber where we maintain a pressure with a inert environment N 2. So, one propellant strand is placed here and it is allowed to burn only from the top of the propellant surface. There are some fuse wire placed at a certain distance L and they are connected to a timer unit which will tell us the time required to regress this much of length of the propellant.

So, once it starts burning the first fuse wire will get cut because of the high temperature then the timer unit will start recording the time. Once this second fuse wire gets cut it will tell us that how much time taken for burning this the length of the propellant strand. Now, this is done for various pressure like once we calculate this we can get the value of R for that particular pressure. So, we can get the value of you know various pressure we can perform the experiment we can we can get the value of R 1, R 2, R 3, R 4 and so on. And if we do the curve fitting we can actually get the value of A and n. So, basically they are going to be you know obtained from the experiments and one such experiment is by doing this you know strand burning test in a Crawford burner. Now, we should talk little bit about the dark zone part because dark zone plays an important role here.

So, if the pressure is not high enough then the existence of the dark zone play an important role there because that will tell us that whether the whether the dark zone will be exist or not. Generally like the if the pressure is more than 10 MPa then you may see that dark zone will almost will almost you know cease to exist. What does that mean that if the dark zone is not available there then we can say that the luminous zone or the secondary flame zone which is directly going to provide the heat to the propellant surface for heating and degradation of the propellant. Whereas dark zone is actually differentiating between the primary flame zone and secondary flame zone.

So, if we consider the feed zone is the primary flame zone and this luminous zone or the flame zone is the secondary flame zone then dark zone is actually giving some you know gap between these two zones and actually this is going to tell us that how much ignition delay is taking place between this you know primary zone and secondary zone.

So, if the dark zone reduces the extent of the dark zone reduces then definitely the heat transfer from the secondary luminous zone is also going to increase which will help in you know increase the degradation of the propellant in the solid surface like in the foam zone and it will further enhance the burning rate. Now since it has a pressure dependency so, at a high pressure this will almost cease to exist, but at a low pressure the dark zone will be present. Now if the pressure you know falls below like chamber pressure falls below 1 MPa then the reaction will be like going to be like very slow.

Now if the pressure falls below even like you know 1 atmosphere 0.1 MPa which is like 1 atmosphere. So, it is like almost like we can say that the flame is almost like extinguish. However, the propellant surface will still have the temperature you know T s which is high enough for some finite you know for finite burning rate. So, although we may see that there is no flame, but still the temperature in the temperature on the propellant surface is high enough which is still going to give us some you know finite burn rate. Now, what is going to happen is that this is going to produce some hot gas and since the hot gas is going to form it will go to a some critical you know concentration of the gas mixture which is going to increase the certain level of the pressure and we may see that the re-ignition is possible re-ignition happens.

Let me repeat what I just said that if the pressure is low enough then you may say that that flame is almost extinguishing we may not be able to see any flame there. However, the surface temperature is still high enough that some finite you know regression is going to take place in the propellant surface and that will keep on continuing producing some gases. Now, that once the concentration of the gases reaches to certain you know critical concentration it will you know reignite the flame and we can see the flame will again reoccur. Now, what will happen since the flame will going to reoccur that it is going to have some kind of a intermittent burning.

So, it is not the burning is not continuous and this type of phenomena is known as chuffing. I think we if we recall in our previous discussions where we talked about the different ignition process like whether it is a normal ignition or you know hang fire sudden increase in pressure in the ignition like a peak sudden high ignition peak. So, if you recall what we discussed earlier I think we have already seen this type of phenomena. So, if we plot the pressure chamber pressure versus time. So, in case of a normal ignition what we said the normal ignition will be like smooth increase of pressure and we have already talked about the various processes involved there.

So, it will ignite it will you know flame spreading will happen chamber filling will happen it will go continue and reach to the equilibrium pressure. Now, depending on the size of the rocket this may be like a slower or quicker because for a small rocket the chamber filling duration will be smaller. So, it can quickly you know fill the port volume and it will reach the equilibrium pressure. So, this is kind of a normal ignition. So, this is we said that this is normal ignition. Now, what situation we have just talked about that if the pressure is low enough what is going to happen that initially it may ignite it will reach to certain pressure, but if the pressure is low enough then it may almost like we to extinguish. So, the pressure is going to fall down.

Now, since the T s or the surface temperature is still high enough to have some finite burning rate that will keep on continue to produce combustion gases and after a certain critical mixture it will reignite. So, we may see that it is going to increase the pressure again ok. So, this type of you know phenomena is of course, undesired this is not really a desired ignition process. So, that type of process is called chuffing. So, that is going to cause like intermittent burning it is going on and off, initially it is going to ignite and then if the pressure falls below certain limit it is going to go off then again you know due to regression of the propellant it is going to flame is going to re occur again and the pressure is going to rise.

So, this type of intermittent burning is known as the chuffing. So, I think you know the dark zone plays an important role in case of this you know combustion of double base propellant, but dark zone is going to very much be influenced by the chamber pressure. Now, generally you know there is also an upper limit of the pressure. In case of solid propellant combustion if the pressure is very high then there is a chance of like irregular combustion or erratic behavior of in the combustion and sometime it is very much unpredictable. So, upper pressure limit is somewhere in the range of like you know 30 mega Pascal Pascal which is very high pressure which is like 300 you know atmospheric pressure 300 atm or 300 bar.

So, in summary you can say that the pressure dependence of you know burning rate of solid propellant is very complex phenomena. So, analytically it is very difficult to give a equations which can which can tell us that this is going to depend on these parameters rather we mostly rely on the experiments. And as I just said that after conducting the experiments measuring the burn rate at various pressures if we do the curve fitting and from there we can actually find the constant a and n for double base propellant or later on we will see that same thing can be done for the composite propellant as well. The value of n how it is going to influence the burn rate of the propellant.

So, we may actually see that it has some influencing role as well like the value of n may be greater than 1 less than 1 or close to small values how do you decide the that for stable operation of the rocket what will be the value of pressure index n. So, I think will that part will come little later after discussing about the combustion mechanism of composite propellant. But before moving further let me just tell you few things which we really need to know in case of double base propellant or composite propellant that the if we plot this you know we had said that R equal to a p to the power n now if you just try to plot this you know on the basis of log plot you may see that for a normal you know burning this will almost going to give us a linear you know plot this is kind of a normal burning where the value of n is greater than 0.

Now there is process of burning where you may see that is going to decrease that is called like the mesa burning and for this case of burning the n is less than 0 there are certain cases or certain ingredients rather if you have certain ingredients in the propellant which will exhibit this type of burning that instead of increasing is going to decrease there the pressure index is going to be less than 0 or negative but there are certain ingredients which is going to show the constant burning like is going to remain almost constant. So, this type of burning after increasing it will remain constant that type of burning is called plateau burning is called plateau burning there we can see that n is going to be equal to 0.

So, these are various you know type of burning that with increase of pressure the burning rate is remaining constant in case of plateau burning whereas, with increase of pressure the burning rate is going to decrease in case of mesa burning whereas, for normal burning the burning rate is going to be linearly is going to linearly increase with increase of pressure. Now the question is that what will be the value of n which is going to give us the stable operation of the rocket that I think we will discuss in the after discussion of the composite propellant as well but in case of double base propellant typical value of n is can be like 0.5 to even 0.6 or even higher than that but of course, it is less than 1.

Now why it is less than 1 that will tell in the tell in the few after few lectures because we separately talk about the choice of index pressure index n after discussing about the composite propellant because composite propellant has a different features of burning compared to the double base propellant because if you recall our discussion on the solid propellants we have seen that the double base propellant is also known as the homogeneous propellant whereas, the composite propellants is heterogeneous propellant because the ingredients present in the composite propellants are mixture of polymeric binder and solid particles. Solid particle means they are going to present like oxidizer particles as well as some fuel particles. So, in a sense the ingredient itself is going to give us some kind of a heterogeneous mixture in the propellant grain or the propellant mixture.

So, that is going to have like heterogeneous propellant whereas, the double base propellant is also known as the homogeneous propellant. So, it is like an intimate mixing of nitrocellulose and nitroglycerin with some amount of plasticizer as well. So, with that I think we close our discussion on the combustion mechanism of double base propellant. Now I just want to close this module by showing a quick example of the burn rate what we have just said that the famous burn rate law R equal to A to the power n. So, let us quick example problem here that how this you know pressure index n and the constant A is generally calculated. As you said that after the experiment we get the value of R at various pressure. So, let us say for a given you know problem I have taken from taken this from a book that at pressure chamber pressure 7 MPa the value of R is given as 25 millimeter per second. By the way this is known as the linear regression rate also and it is generally expressed in terms of length per second. Because if you already had seen that we have said that once we measure the burn rate we actually measure the length by some you know unit time at.

$$Pc = 7Mpa\,,\ r = \ 25\ \frac{m}{s}$$

So, whatever the length is regresses over time that is denoted by L by delta t which is like R. So, certain length per unit time. So, generally it is expressed in terms of millimeter per second. So, in this example it is given that chamber pressure at 7 MPa the value of R equal to 25 millimeter per second. So, PC 1 let us say at another pressure it is given as from the experiments we have found out and it is it is a double base propellant from the strand burning experiment or the burning at

$$Pc2 = 17Mpa\,,\ r = \ 45\ \frac{m}{s}$$

Now, if we say that it is following the Saint Roberts law or the Villiers law. Now, it is asking to find out that what will be the chamber pressure PC when the burn rate is 35 millimeter per second. Now, as you already know that R equal to A P to the power N we can immediately jump on and writing that since at PC 1 it is given as R 1 this is R 2 let us say this is PC 3 this is R 3. So, we can immediately write that we have to be very consistent with our unit since it is given as point it is given as 25 millimeter per second.

$$Pc =?\,Mpa\,,\ r = \ 35\ \frac{m}{s}\,, r = ap^{n}$$

So, we can immediately like 0.025 into equal to A into 7 MPa means 7 into 10 to the power 6 Pascal to the power N. So, this is equation 1. In the second given thing is like 0.045 equal to A into 17 into 10 to the power 6 to the power N.

$$0.025\ =\ a(7 \times 10^{6})^{n} \quad ....(1)$$

So, equation 2. So, from there you can actually solve this and you can find out the value of N you can divide this equation 2 by 1. So, you can simply what you can write is if you just try to write from equations 1 and 2 what you can do you can just divide the equation 2 by equation 1. So, it will be like 0.045 by 0.025 that is going to be equal to 17 into 10 to the power 6 to the power N divided by 7 into 10 to the power 6 to the power N.

$$0.045\ =\ a(17 \times 10^{6})^{n}$$

Now if you solve for it you may actually find out the value of N which is coming out to be around 0.662 you may check the calculation once. And then any of the equation you can use because N is now known. So, just use equation 1 or 2 you can find out the value of the constant A which is going to be equal to R divided by P to the power N.

$$\frac{0.045}{0.025} = \frac{(17 \times 10^{6})^{n}}{(7 \times 10^{6})^{n}}$$

So, put the any value which is going to be like 0.025 P you can just put 7 into 10 to the power 6 to the power 0.662 if you solve for it I think you can easily find out the value of A which is like 7.35 n=0.662 into 10 to the power 7. I request the participants to check the calculation once. Now the question asked here is that if the regression rate is 35 millimeter per second what will be the chamber pressure? So, you can just simply use this use the burn rate law R is given. So, you can just simply write R equal to so, let us say this equation is A. So, from equation A you can simply write 0.035 A is given which is like 7.35 into 10 to the power minus 7 P to the power N is also known to us 0.662 you solve it and you can find out what we expect here because the burn rate is in between like 25 and 45.

$$a = \frac{r}{p^n} = \frac{0.025}{(7 \times 10^6)^{0.662}}$$

$$= 7.35 \times 10^{-7}$$

So, since the burn rate is in between we are expecting the pressure will be in between of 7 and 17 MPa. So, if you solve for this you may get chamber pressure is going to be equal to 11.57 MPa. You do the calculation correctly and then I think you can easily find out the answer.

$$0.035 = 7.35 \times 10^{-7}(p)^{0.662}$$

$$Pc = 11.57\ Mpa$$

So, this is these are all your all our answers. So, the very simple equations we have used here the burn rate law and we are able to find out the value of A and N and also we are able to find out the value of chamber pressure at a particular regression rate where it is taking place like 35 millimeter per second. So, in the similar approach this we have done it for two, but in order to get a better value of the N and N we actually conduct the experiments for you know for various you know occasions and we can actually find out the pressure index A and N for different you know propellant ingredients like if you change the propellant ingredients we mix we mix some other ingredients into the propellant formulations we can actually perform the experiments again using the Crawford burner and we can find out the value of A and N for that particular propellant and we can actually compare the propellant formulations having different you know type of oxidization and fuels and with that we can actually compare the the performance of various propellant combinations.

So, I think with that we close the discussion on the combustion mechanism of double base propellants. In the next module we will begin our discussion on the combustion of composite propellants. Alright, thank you.