

Course Name: Combustion of Solid Fuels and Propellants
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Lecture: 17 Evaluation of Burn Rate of DB Propellants

Welcome back. We were discussing about the combustion of double base propellants. In this lecture, we will try to see if we can derive some mathematical equation of the linear burn rate of double base propellant. But before moving * the evolution of burn rate, we will try to spend few more minutes on the dark zone, which is also known as the induction zone. Let us try to see the residence time of the dark zone, sometime it is termed as the reaction zone of the reaction time of the dark zone. And what are the influencing factors of this dark zone? And then I think we can move on to the evaluation of burn rate of double base propellants.

So, in the previous lecture, I think we had already seen that the dark zone is very much you know dependent on the pressure and temperature. But we also need to remember that the dark zone is very much dependent on the concentration of NO not the not the NO₂ not the NO₂. So, if we try to see how the concentration of NO is going to influence the residence time in dark zone, in dark zone sometime it is denoted as τ_d , which is determined from the length of the dark zone divided / the average gas velocity of the dark zone u_d . So, here we are saying that L_d is the thickness of the dark zone and u_d is the average gas velocity in the dark zone.

Now, the residence time can also be considered as the reaction time to initiate the luminous flame zone or the ignition delay time between the feed zone and the luminous flame zone as we have already discussed in the previous lecture. Now, if we apply the overall continuity equation, we can actually find out the gas average gas velocity of the dark zone is going = to the density of the propellant * burn rate by the density in the gas density in the dark zone. Now, we can also get some more relationship by using the equation of state where we can see the $\rho = P / R T$. So, if we just apply the equation here we can actually get the residence time or the reaction time of the dark zone is going = to d is going = to $P * L_d / R * \rho$ here we can write $R_d * \text{temperature } T_d$. So, here we can say the temperature in the dark zone is R_d gas constant in the dark zone is R_d and here R is the burn rate or linear burn rate or linear regression rate.

P_d is the density of the gas in the dark zone, ρ_p is the propellant density, R_d is the gas constant. Now, as we told earlier that the dark zone reaction products the dark zone reaction producing luminous flame is kind of you know dependent on reaction involving NO not the NO₂ the reaction time in the dark zone can be plotted as a function of the concentration of NO. Now, if you look at the reaction time in the dark zone τ_d in millisecond one can

see as we increase the concentration of NO the dark zone reaction time or dark zone residence time is going to decrease because the concentration of NO is going to influence the reactions in the dark zone. So, eventually that will reduce the reaction time in the dark zone. Now, if we increase the pressure for a increasing pressure we can see this is also going to change.

So, that is like p increases as you already mentioned earlier that if the pressure increases in fact, for a high pressure dark zone cease to exist dark zone will be almost eliminated. So, the concentration of NO and the pressure are going to influence the residence time in the dark zone or eventually this will influence the influence the ignition delay time between the primary reaction zone and the secondary luminous zone or the luminous flame zone. So, this part we also need to remember in the context of the dark zone that how the you know residence time of the dark zone is going to be influenced by the pressure and the concentration of NO. Now, with this discussion I think we can proceed further and try to find out some you know mathematical relationship of the burn rate what we just said that the linear regression rate or the burn rate which is generally you know given in terms of like millimeter per second because if you recall in our discussion that in case of solid propellant it is generally burn normal to the surface and the burn rate you know is considered to be like how much length is regresses over unit time. So, if you consider like a n burning propellant.

So, it is going to regresses over time. So, the R is going to be like $L / \Delta t$ like how much amount of length has regresses over a time period Δt . So, it is basically the length divided by time. Now, there are various instruments which are various you know experimental setup have been used to determine the regression rate or the linear burn rate one such instrument is called Crawford burner I think we have already mentioned earlier that we can conduct the experiment by considering a closed chamber filled with you know nitrogen and we can let the one small strand of propellant burns and the length is going to regress over time if we inhibit the propellant surface circumferentially if we let the burning to take place only from the you know top of the propellant then at a certain time period after a certain time period it will burn a length of you know L where we have put the fused wire. So, based on the timer you know unit is going to give us the time period Δt where the first and the second fuse wire will get cut and since they are already impregnated at a particular length from there we can actually find out the regression rate $L / \Delta t$ and that we can do it for various pressure.

So, this is like a different pressure it may be like P 1 P 2 P 3 and so on and one can actually plot it and we can find out the relationship between the pressure and the relationship between the regression rate and the pressure. Now, how the regression rate is going to very much you know dependent on the pressure we try to understand through some simple analysis. So, in order to do that we will just need to consider a small control volume in the feed zone because we have said that the major reactions will take place in the feed zone.

So, feed zone is kind of a very important consideration and the heat release is going to take place in the feed zone. So, if we consider a small control volume in the feed zone and if we consider the energy balance there and from there actually we can try to find out how the regression rate is going to be you know dependent on the pressure.

So, let us try to look at that. So, let me just draw one simple you know zone wise diagram what we have already did earlier. So, we have like preheated zone, we have foam zone or solid state reaction zone, then we had feed zone, then we had dark zone and finally, we had the luminous zone. So, for the evolution of burn rate we are trying to consider a small control volume within the feed zone within the feed zone. What we said that let us say we consider a control volume at a distance x the length of the control volume is dx the cross sectional area of the control volume is unit cross sectional area.

We will try to see the we consider the energy balance across the control volume in the feed zone. Now, how do you do that? We know that the heat transfer is going to take place / you know through conduction. We can consider the heat conducted heat conduction increases from like sorry heat conduct heat conduction * the control volume. We can determine from like assuming the heat conducted along the x axis and the heat conduction is going to increase is going to increase from like this surface is going to be like $-k_g dT/dx$ at x is going to increase to $-k_g dT/dx + d dx$ of $k_g dT/dx * dx$. This is going to happen at $x + dx$ because this is the distance x and this is like $x + dx$.

So, this is what about the heat release over a distance dx . If you consider the volumetric heat release is like let us say q_{chem} which is like volumetric heat release rate joule per meter cube second. We can get the heat release over distance x we can and we know the unit cross sectional area. So, this is going to be $q_{chem} * dx$. Now, what about the net heat transfer to the control volume? You can simply this - this.

So, you can get the net heat transfer from the sorry this should be outside of the bracket negative sign. So, the net heat transfer into the control volume is going $= -dx k_g dT/dx * dx$. By the way k_g is the thermal conductivity of the gas conductivity of the gas all right. So, the heat transfer to the control volume and the heat generated in the control volume what is going to do is, is going to increase the enthalpy of the gas flowing through it. So, we said that heat transfer to the control volume heat generated in the control volume $q_{chem} * dx$ heat transfer was.

This, this is going to increase the enthalpy of the gas flowing through it. So, if the enthalpy of the gas is like we say like $\rho_g * u_g * c_p T$ that is going to increase to $\rho_g * u_g * c_p (T + dT)$. Now, I will tell you what is u_g and what is ρ_g . $\rho_g u_g$ you can understand that that they are the you know density of the gas and the so ρ_g here is the density of the gas u_g is the one dimensional velocity of the gas. So, basically the enthalpy of the gas flow through

the control volume increases from $\rho g u g c T$ to $\rho g u g c * T + dT$ where $T + dT$ has is the increased temperature and c is the specific heat of the gas ok.

Now, from the steady flow energy balance for the control volume if we just considering the steady flow for the $c v$ control volume what you can write? We can write the heat transfer * the control volume which is $d d x kg dT d x * d x$ + the heat generated within the control volume which is $q k m q \dot{com} came * d x$ is going = to $\rho g u g c T + d T$ - the flowing enthalpy is like $\rho g * u g * c * T$ ok. Because the enthalpy of the gas has changed from $\rho g u g c * T$ to $\rho g u g c * T + dT$. Now, if you simplify this we will end up getting kg we we are considering kg as the constant you know thermal conductivity of the gas this will come as $d \text{ square } d x \text{ square } + \rho g * u g * c * d T d x = q \dot{ k m}$. So, we have this equation now we have this ordinary differential equation. So, if we want to know the boundary condition here if you want to solve this equation we need to know the boundary condition.

So, what are the boundary condition we have? We can look at what we already understood in the previous lecture that we said that at $x = 0$ which is you know part of the propellant. So, we can consider that foam zone is still the solid phase reaction is taking place in the foam zone. So, it is here we can say it is $x = 0$. So, the beginning of the feed zone is kind of $x = 0$ which is right at the end of the end of the foam zone. So, that is $x = 0$ and $x = 0$ we said the temperature was surface temperature which is T_s .

So, we can say the surface temperature is T_s and end of the feed zone which we have said that temperature may be T_d because the dark zone has just started. So, we can say the temperature at $x = 0$ T is considered to be T_s at $x = L$ let us say this is you know end of or edge of feed zone edge of feed zone this we can say temperature is going = to T_d let us say edge of feed zone means starting of the dark zone. These are the two you know boundary conditions we have. So, basically the temperature profile will be like this we have like this is like the distance x . So, we had the starting temperature from T_i to initial temperature to it will reach to propellant surface temperature T_s and then eventually it will go to like the T_d sometime it is termed as T_1 also since we use the dark zone temperature T_d .

So, let us consider this as the T_d . So, this is the end of the feed zone or the edge of the feed zone after that the temperature will remains almost constant in the dark zone under the condition of lower pressure ranges and then in the luminous zone the pressure will sorry the temperature will again increase. But since we are considering only up to the feed zone. So, we will considering the edge of the feed zone which is like let us say length of the feed zone is L . So, at $x = L$ the temperature is $T = T_d$.

Now, in order to solve this what we need. So, we know the temperature distribution like this for a given I mean we can have this temperature distribution for a given value of $U g$

and if we have this you know if we have this given value of U_g $Q_{k,m}$ is also you know if it is specified we can try to solve this and we can actually get the temperature profile something like this. Now we have to understand certain things here that heat release rate which is like $Q_{k,m}$ is kind of you know very much influenced / the chamber pressure or the pressure and temperature and of course, is going proportional to the reaction rate. So, if we look at the Arrhenius reaction Arrhenius equation there that $Q_{k,m}$ is going to be finally, is going to be very much you know influence or it will depends on pressure and temperature and is going to be proportional to somewhat like $A P^m e^{-E/RT}$ where you know E is activation energy you I mean we I think we know this equation from the Arrhenius equation where $k = A * e^{-E/RT}$ that is the equation I am writing here. So, this is E is the capital E is the activation energy R is the gas constant and $A e^{-E/RT}$ kind of m are kind of constant for the given reaction.

So, we can see here that it is very much you know influenced / the pressure of the chamber which is the P . Now what about the value of U_g or the gas velocity? The gas velocity will be decided / how the gas evolves you know from the degradation of the solid propellant in the foam zone because ultimately degradation of the solid propellant is going to decide the gas evolved in the foam zone because if you recall in the different zones of the combustion of the double phase propellant foam zone is very much part of the propellant surface where the degradation or the solid phase reactions are taking place in the foam zone. So, as the reactions taking place here they are basically breaking the bonds between like CO_2 and NO_2 and they are actually going to liberate the gas phase species here from the foam zone. So, the the gas velocity U_g will be kind of decided by the rate at which the gas gets evolved in the foam zone due to the heat transfer from the hot gases from at the propellant surface. So, basically this is going to be influenced by the heat transfer from the propellant sorry heat transfer to the propellant surface from the feed zone and the degradation of the propellant will eventually lead to the formation of gas gaseous species.

So, in a simple term we can say that the gas velocity U_g is going to be very much you know depend on on the gases being evolved in the foam zone due to the heat transfer from the you know heat transfer to the propellant surface or at the propellant surface. So, we can say if we know the linear regression rate let us say R that R is the regression rate at which the propellant is regresses or the length of the propellant regresses over a unit time period is R . We can actually you know make the mass balance equation we can actually write the the mass balance rate at the surface that the surface is going to regresses. So, for a unit cross sectional area we can say that mass balance rate at the surface what you can say? We can say that density of the propellant is ρ_p into unit cross sectional area 1 is going equal to the gas evolved like gas flowing through the feed zone the density of the gas is ρ_g U_g is the gas velocity into unit cross sectional area 1 where R is kind of you know

representing of just like a velocity like meter per second or millimeter per second. So, in a sense R is regressing over time like the propellant is regressing over time.

So, millimeter per second. So, if you look at the you know cigarette like strand or just if you consider the cigarette burning what is going to happen here? As the person pops it through it. So, this will keep on burning you know. So, length is going to regress over time. So, it is eventually like this. So, the regression rate is something like that which is giving you the linear burn rate.

So, if you just do the mass balance rate at the surface we can actually end up getting the relationship that $\rho_p \cdot R$ is going to be $\rho_g \cdot U_g$. So, eventually we are going to relate the R with respect to other parameters. Now, somehow if we replace U_g in terms of $\rho_p \cdot R / \rho_g$ we can actually get into the equation here where my R is going to involve. Now, one can actually understand that a unique solution for R . So, let us say if we try to you know put this in this equation and we can write $K_g \frac{d^2}{dx^2} + \rho_g \cdot$ instead of U_g we can write here U_g as $\rho_p \cdot R / \rho_g$.

So, we can write $\rho_p \cdot R / \rho_g \cdot C \frac{dT}{dx}$ which is going to do $Q_{dot} \cdot \text{chem} / \rho_g$ you can cancel out. So, here we can actually include the R in the equation. In this equation now we have involved R there which is the linear regression rate. Now, we should understand here one thing that this equation is very much involved like it is not so easy to solve this equation straight forward way. Rather if you look at the $Q_{dot} \cdot \text{chem}$ which is the heat release rate or volumetric heat release rate which is very much you know dependent on the pressure and temperature or in a sense we can say it is very much is going to be very much influenced by the pressure.

The other parameters which are like so, the heat transfer the thermal conductivity K_g is involved there. So, we can actually have the value of gas thermal conductivity, but without knowing the property of the gas it is very difficult what are the different type of gas mixtures present there, what is the temperature profile we said here in getting the temperature profile is also kind of very difficult. So, in a sense we can say that unique solution for R is kind of you know is going to be very very you know involved or rather we can say that the unique solution of R which is going to satisfy the temperature profile at the boundaries and heat balance within the foam and feed zone is going to give us the bond rate of the propellant. If we somehow solve this equation which is going to which is going to you know satisfy the criteria of the temperature at the ends and the you know heat balance within the foam and feed zones that would give us the linear bond rate or the regression of the propellant. Now, the process of you know having the solution is going to be very very involved, but one thing we can notice that the $\rho_g \cdot U_g$ and $q_{dot} \cdot k \cdot m$ are somehow very much depend on the chamber pressure or the pressure that the density of the gas, the velocity of the gas, the heat release rate $q_{dot} \cdot k \cdot m$ is somehow is going to be influenced / the chamber pressure.

So, in a sense we can say that R is going to be very very strong function of pressure. So, we can say that my regression rate linear regression R linear regression rate R is going to be a strong function of pressure. So, from there actually we can say that it is going to be it is like a indirect way of realizing that the regression rate or the linear bond rate is going to be influenced / the chamber pressure. And that way it is generally written that R is going to be $A p$ to the power n . This is the bond rate law very popular bond rate used for solid propellant, where A takes care of the you know initial temperature composition of the propellants and other characteristics and n brings the pressure effect whereas, the n is the pressure index.

So, n brings the pressure effects of the propellant which are kind of you know dependent of the combustion process. So, this is the famous bond rate law or the Villiers law sometime it is termed as the Sen Robert's law. The value of A and n are going to be obtained by considering the experiments and using the curve fitting as we have already said that using the Crawford burner test one can actually try to get the data of bond rate for various pressure p_1 p_2 p_3 and so on. And from there if we do the curve fitting we can try to find out the value of A and n ok. So, I think with this note we will close this lecture today.

I think we can continue the bond rate law in the next class onward ok. We will try to understand what are the other factors the pressure range, how do you really know that at what pressure range, how is going to work, how we plot this you know bond rate versus pressure, is there any difference for different type of you know value of n , whether there is any influence of that. So, we will look at it look at those parameters in the next lecture alright. Thank you.