Smart Structures
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Week 12
Lecture No: 61
Analysis of Electro and Magneto Rheological Fluid Flow (continued)
Part 03

So, today we will continue our discussion on flow mode analysis using Bingham biplastic model. This was the Bingham biplastic model that we discussed.

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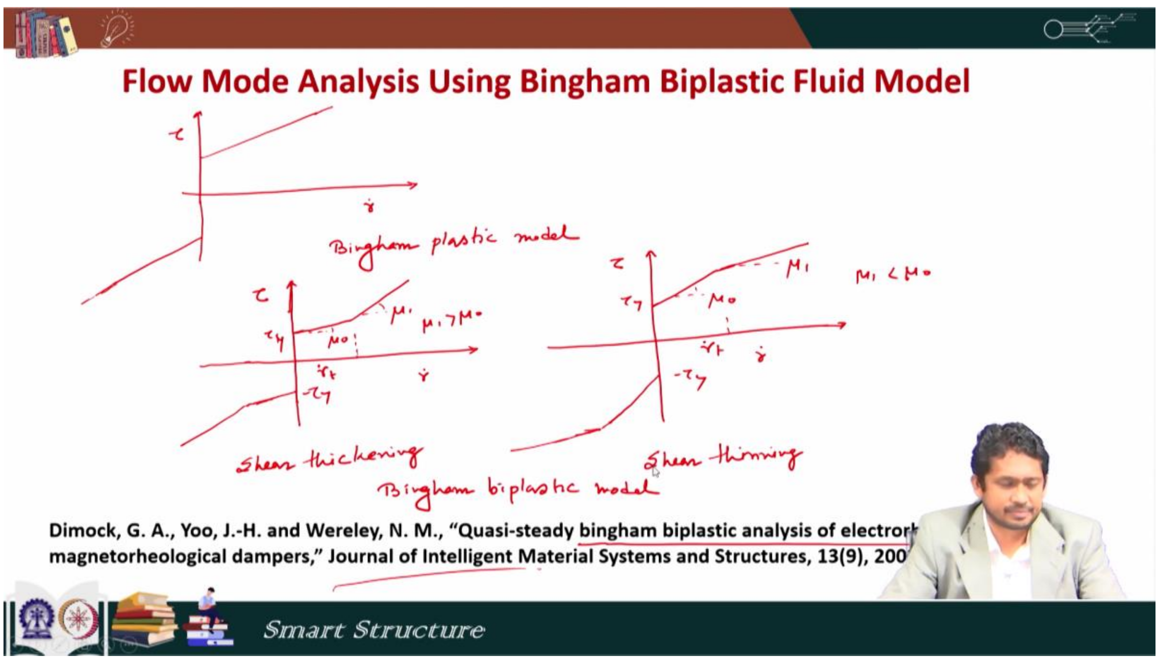

And then, we saw that this kind of model divides the entire region into a several parts. So, at the center there is the plug region where the shear strain rate is 0. And then, at the two sides of it there is a region where the shear strain rate is - the magnitude of the shear strain rate is less than gamma dot, gamma_t dot and then beyond that there is a region where the magnitude of the shear strain rate is more than gammat dot. And we are analyzing only half of it, half of the domain because of the symmetry. So, in the last class, we found out the velocity equation in region 5.

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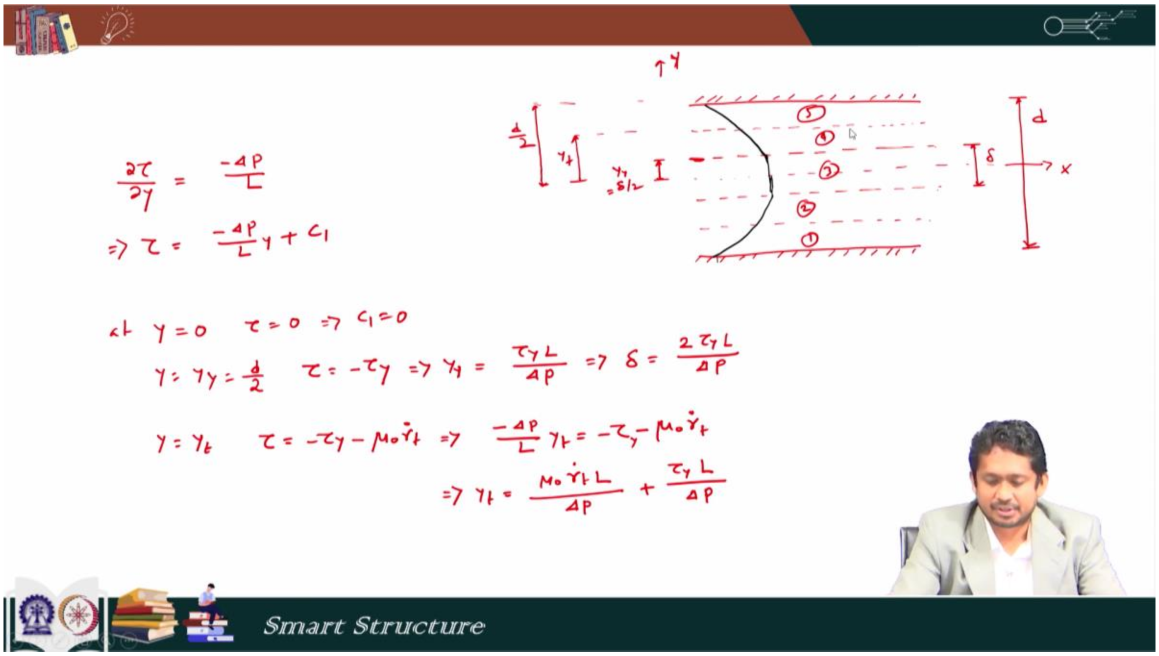

Today, we will continue from that. So, we will find out the equation of velocity in region 4. Now, in region 4, the equation of shear stress is tau y equal to minus of tau_y plus mu_0 into del u_4 by del y. So, u in region 4 is denoted as u_4, and that is equal to minus del P by L multiplied by y. Now, from here, we can write del of u_4 by del y equal to tau_y minus del P by L into y. And then, we can integrate the equation that gives us u_4 equal to tau_y. So, there was a mu also here, mu_0 which was missing. So, after integrating, we find tauy by mu_0 into y minus del P by mu_0 L multiplied by y square by 2, and then we have plus C_2.

$$\tau(y) = -\tau_y + \mu_0 \frac{\partial u_4}{\partial y} = -\frac{\Delta p}{L} y$$

$$\Rightarrow \mu_0 \frac{\partial u_4}{\partial y} = \tau_y - \frac{\Delta p}{L} y$$

$$\Rightarrow u_4 = \frac{\tau_y}{\mu_0} y - \frac{\Delta p}{\mu_0 L} \frac{y^2}{2} + C_2$$

Now, we have to evaluate the constant, C_2 and that we have evaluated by putting the condition that - in this region, in this dash region, the junction between region 5 and region 4, the velocity obtained from the equation that we are dealing with now, is the same as the velocity that was obtained from the, that can be obtained from the velocity equation for region 5. So, the velocity is continuous between region 5 and region 4. If we apply that condition so, that just can be written as u_4 at y_t is equal to u_5 at y_t. Now, if we apply that condition, then we can evaluate the constant C_2. And after evaluating the constant C_2, if we put it back in the equation - so, this helps us evaluating C_2. And then, after evaluating C_2, we can find out our u_4 y as y minus y_t by mu_0 multiplied by tau_y minus del P by 2 L multiplied by y plus y_t, and then we have plus y_t minus d by 2 divided by mu_1 multiplied by tau_y plus mu_0 minus mu_1 into $gamma_t$ dot plus y plus d by 2 multiplied by minus del P by 2 L.

$$u_4(y_t) = u_5(y_t)$$

$$u_4(y) = \frac{y - y_t}{\mu_0}\left[\tau_y - \frac{\Delta p}{2L}(y + y_t)\right] + \frac{y_t - \frac{d}{2}}{\mu_1}\left[\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t + \left(y + \frac{d}{2}\right)\left(-\frac{\Delta p}{2L}\right)\right]$$

Now, that becomes our expression for u_4 y. And then, we know that y_t is equal to mu_0 $gamma_t$ dot multiplied by L by delta P plus tau_y multiplied by L by delta P.

$$y_t = \mu_0\frac{\dot{\gamma}_t L}{\Delta p} + \frac{\tau_y L}{\Delta p}$$

So, we can put this here and here also. If we do that, then we can find out our - so, we can put this y_t here as well as here. After doing that, our expression for u_4 y becomes just this - 1 by mu_0 multiplied by tau_y minus delta P divided by 2 L, y multiplied by y plus 1 by 2 mu_1 L multiplied by delta P d square by 4 minus L d into tau_y, we can put another - multiplied by tau_y plus mu_0 minus mu_1 into $gamma_t$ dot, and then, plus L square by delta P multiplied by tau_y plus $gamma_t$ dot mu_0 whole square of it and then 1 minus mu_1 by mu_0. That is the entire expression for u_4 as a function of y.

$$u_4(y) = \frac{1}{\mu_0}\left[\tau_y - \frac{\Delta p}{2L}y\right]y + \frac{1}{2\mu_1 L}\left[\frac{\Delta p d^2}{4} - Ld\{\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t\} + \frac{L^2}{\Delta p}\left(\tau_y + \dot{\gamma}_t\mu_0\right)^2\left(1 - \frac{\mu_1}{\mu_0}\right)\right]$$

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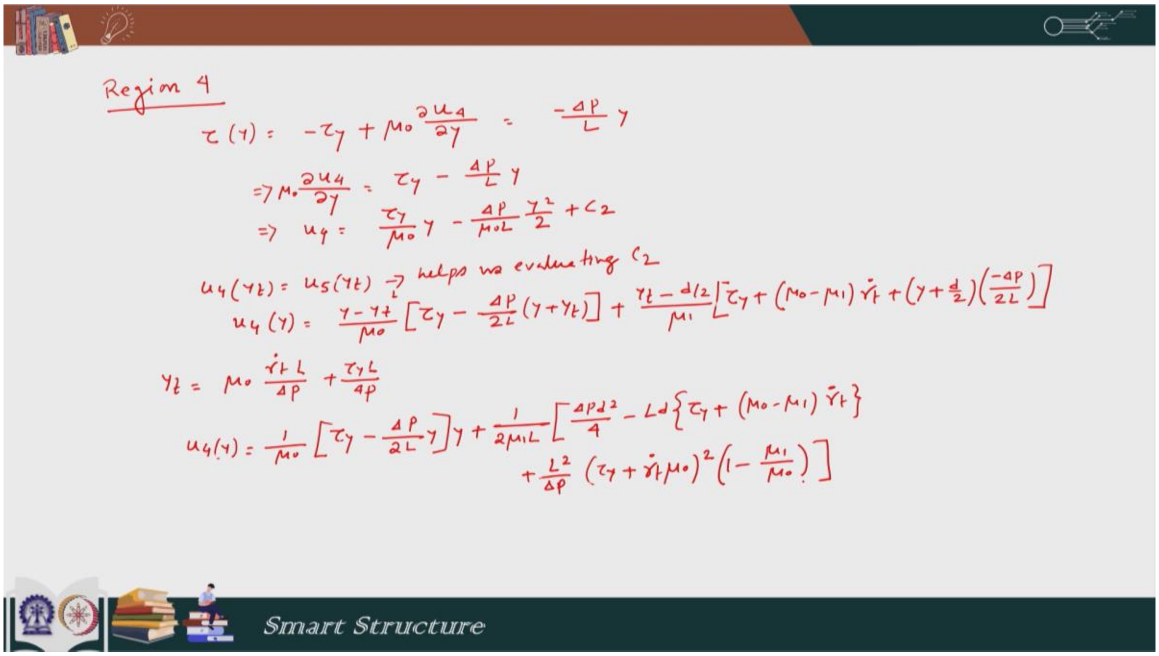

Now, we want to look at the plug region, and in the plug region, we know that the velocity is continuous. So, whatever the velocity as u_4, whatever the velocity that we get at y is equal to y yield, that means, here, whatever the velocity we get at y_y, that means, here, the junction between regions 3 and 4, based on the velocity equation at region 4, that same velocity is valid throughout the entire third region. So, we can say that u at region 3, we have u_3 as a function of y is just u_4 that is obtained at y_y, and we know that y_y is equal to delta of 2 which is tau_y L by delta of P, delta P.

$$u_3(y) = u_4(y_y) \qquad \Rightarrow y_y = \frac{\delta}{2} = \frac{\tau_y L}{\Delta p}$$

And then, u_3 becomes tau_y square L divided by 2 mu_0 delta P plus 1 by 2 mu_1 L multiplied by delta P d square by 4 minus L d multiplied by tau_y plus mu_0 minus mu_1 into gamma dot, and then, plus we have L square by 4 P multiplied by tau_y plus $gamma_t$ dot mu_0 whole square of that multiplied by 1 minus mu_1 by mu_0. So, that is our u_3 and that is the constant velocity that is valid over the entire third region.

$$u_3(y) = \frac{\tau_y^2 L}{2\mu_0 \Delta p} + \frac{1}{2\mu_1 L}\left[\frac{\Delta p d^2}{4} - Ld\{\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t\} + \frac{L^2}{\Delta p}\left(\tau_y + \dot{\gamma}_t \mu_0\right)^2\left(1 - \frac{\mu_1}{\mu_0}\right)\right]$$

Then what we can do is - we can do some non-dimensionalization. So for that, again we can write delta bar as delta by d. Then we define $delta_t$ as a new variable which is just twice of y_t, which means delta in this region, this dimension from here to here, whereas delta t is the dimension from here to here. The distance between here to here. So, the distance between the junction of regions 1 and 2, and the junction of regions 4 and 5. And then again, we can further non-dimensionalize $delta_t$ bar equal to $delta_t$ by d.

$$\bar{\delta} = \frac{\delta}{d}, \qquad \delta_t = 2y_t \qquad \text{and} \qquad \bar{\delta}_t = \frac{\delta_t}{d}$$

So, after all this, we can write u_1 as a function of y in a non-dimensional form as delta P d square by 8 L, multiplied by 1 minus delta t bar square divided by mu_1 plus $delta_t$ bar minus $delta_y$.

So, these are the non-dimensional terms that we will be using. Now with this, these velocities can also be written in a non-dimensional form and we would not do that.

So, after that we will find out the equivalent damping coefficient, and for that again we have the same approach. We first find out the total flow rate, and then we equate the total flow rate with a flow rate which would have been there, if there was a uniform velocity u_m. And that would give us equivalent uniform velocity u_m. Then we divide that velocity by the force and that would give us the damping coefficient. So, for that the first we have to find out the total volumetric flow rate. So, total flow rate Q is equal to Q_1 plus Q_2 plus Q_3 plus Q_4 plus Q_5. And from that, we can write this as - and we know that Q_2 and Q_4 are same, Q_1 and Q_5 are same because of the symmetry. So,with that, we can write our total flow rate. So, we can write this as 2 multiplied by y_t to d by 2, and we have b u_5 d y. So, if we again look at the diagram, it is between y_t and d by 2, where u_5 is valid. So, the corresponding flow rate is just u_5 multiplied by b integrated over the region.

We multiply this by 2 because of the symmetry. And then, we have 2 and then, a region from y_y to y_t, and here we can integrate b u_4 d y, and then we have 0 to y_y, and here, we integrate just b multiplied by u_3 d y.

$$Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$$

$$= 2\int_{y_t}^{d/2} bu_5 dy + 2\int_{y_y}^{y_t} bu_4 dy + 2\int_{0}^{y_y} bu_3 dy$$

Because 0 to y_y is half of the plug thickness, so, if we just multiply by 2, we get the total flow rate due to the entire plug thickness. And here, u_3 is constant over the plug, so, we can just multiply by the thickness of the region and that will give the corresponding flow rate. So, after doing everything, the total flow rate comes as - delta P, then we have b, d cube by 12 mu_0 L, then 1 minus $delta_y$ bar multiplied by 1 plus $delta_y$ bar by 2 minus 1 minus $delta_t$ square. So here, $delta_y$ bar is just our delta bar because that is defined between the region y_y and minus y_y. So, we can call it $delta_y$ bar also. And then, we have 1 plus $delta_t$ bar by 2 multiplied by 1 minus mu_0 by mu_1 and that is our total flow rate Q.

$$Q = \frac{\Delta p b d^3}{12\mu_0 L}\left[(1-\bar{\delta}_y)\left(1+\frac{\bar{\delta}_y}{2}\right) - (1-\bar{\delta}_y)^2\left(1+\frac{\bar{\delta}_y}{2}\right)\left(1-\frac{\mu_0}{\mu_1}\right)\right]$$

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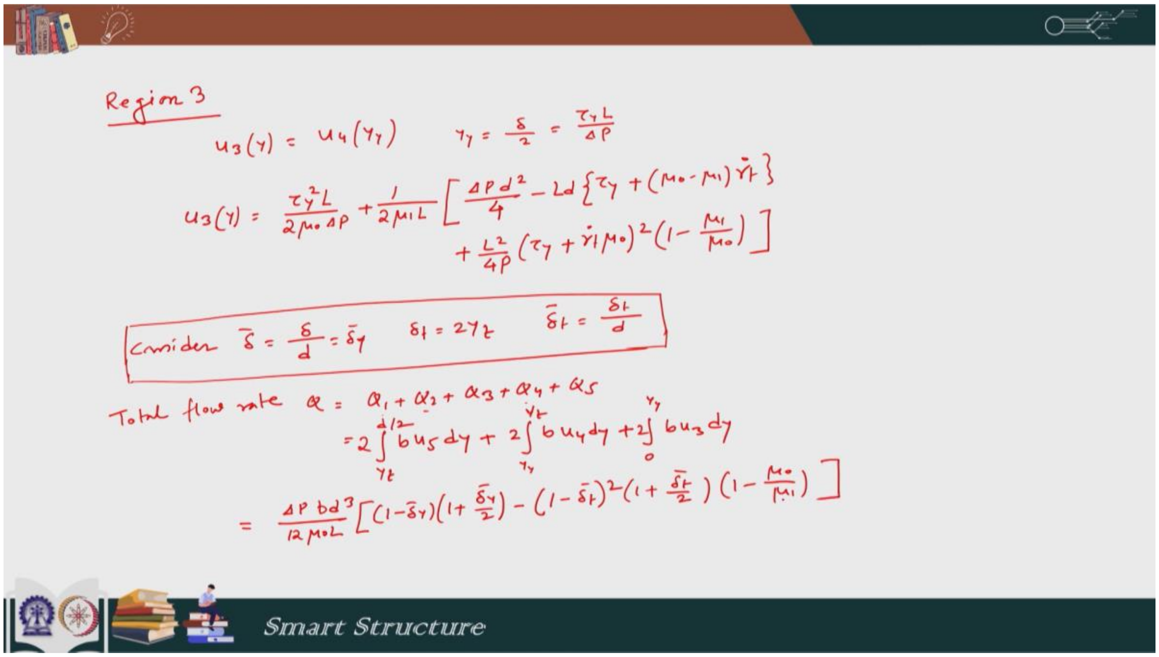

And then, we equate that to an equivalent, the total - so, this gives us the total flow rate and then we equate that too. Flow rate that would have been there because of uniform velocity u_m. So, we equate this Q with u_m b d.

And from there, we can find out u_m. Also, we have the force F is equal to delta P multiplied by b d. So, now if I divide F by u_m, that gives me the effective active damping coefficient. So, that becomes our delta P b d divided by Q by b d. And if we do that finally, the active damping coefficient comes to be 12 mu_0 L into our b square d square divided by b d cube. And then, we have here, 1 by 1 minus $delta_y$ bar whole square into 1 plus $delta_y$ bar by 2 multiplied by 1 minus $delta_t$ bar whole square, and then we have 1 plus $delta_t$ bar by 2 and 1 minus mu_0 into mu_1.

$$Q = u_m bd$$

$$F = \Delta pbd$$

$$C_{eq}^a = \frac{F}{u_m} = \frac{\Delta pbd}{Q/_{bd}} = \frac{12\,\mu_0 b^2 d^2}{bd^3} \frac{1}{\left(1-\bar{\delta}_y\right)^2\left(1+\frac{\bar{\delta}_y}{2}\right) - \left(1-\bar{\delta}_y\right)^2\left(1+\frac{\bar{\delta}_y}{2}\right)\left(1-\frac{\mu_0}{\mu_1}\right)}$$

So, this is our equivalent damping in the active mode and then we can find out the effectiveness of this active damping by dividing that by the active damping in the inactive mode. And for that, we can just find that out by C_{eq} a divided by C_{eq} 0. And that again, using the same approach, comes just as 1 by 1 minus $delta_y$ square, multiplied by 1 plus delta$_y$ bar by 2, just this denominator should come here, multiplied by this, into 1 plus delta$_t$ bar by 2 multiplied by 1 minus mu$_0$ by mu$_1$.

$$\frac{C_{eq}^{a}}{C_{eq}^{0}}=\frac{1}{\left(1-\bar{\delta}_y\right)^2\left(1+\frac{\bar{\delta}_y}{2}\right)-\left(1-\bar{\delta}_y\right)^2\left(1+\frac{\bar{\delta}_y}{2}\right)\left(1-\frac{\mu_0}{\mu_1}\right)}$$

So, what we are doing here is to find out the inactive damping, we are just setting these terms delta$_y$ bar and delta$_t$ bar to 0, because when the fluid is not active, at that time, we do not have the concept of yielding. So, the constitutive relation is just that of the Newtonian fluid. That means that the region does not get divided into plugs or the region where, I mean, regions 3 and 4 do not come into the picture. All these divisions do not come. So, the entire region has just 1 constitutive relation, which is mu into the shear strain rate.

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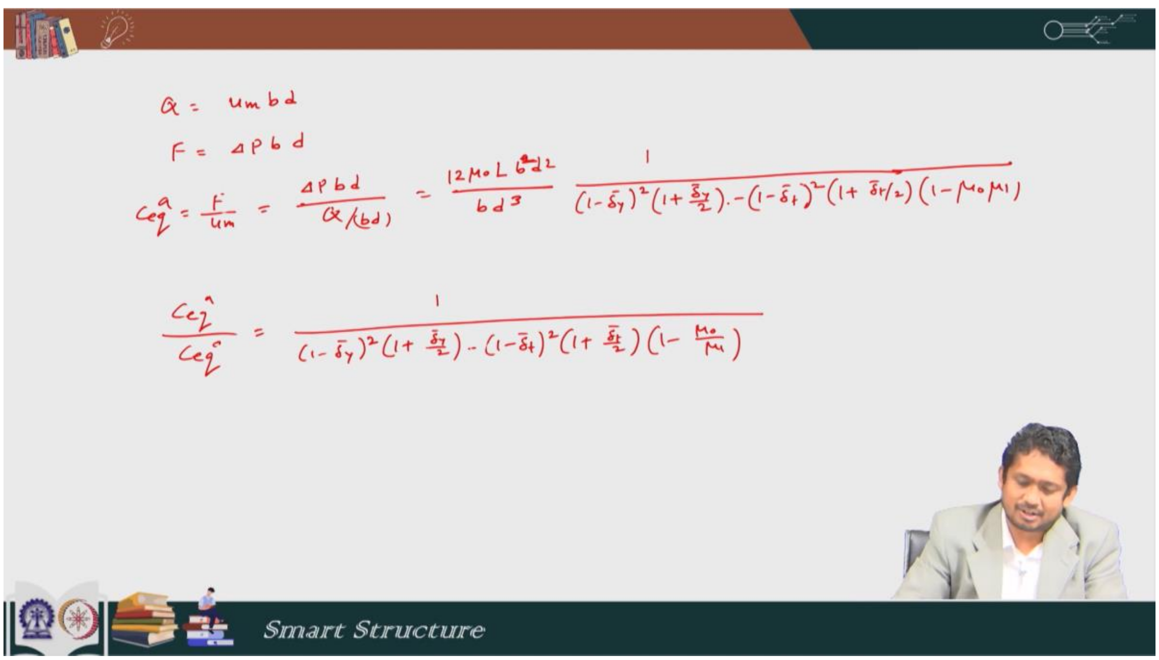

Now, because these quantities are 0. So, C_{eq} at 0 is - just this. So, if I divide this quantity by just this, we have this as - so, if I divide this quantity just by this. So, we have the C_{eq} active by C_{eq} 0 as this. Here we can see that because the constitutive relation is piecewise linear, this damping coefficient does not depend on the velocity. In the previous case, when we are dealing with the Herschel Berkeley model, there the constitutive relation was non-linear.

So, because of that, the equivalent damping coefficient was a function of u_m. Here, it is piecewise linear. So, here we do not have that as a function of u_m.

So, with that I would like to conclude this lecture here.

Thank you.