Smart Structures
Professor Mohammed Rabius Sunny
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Week - 12
Lecture No - 60
Analysis of Electro and Magneto Rheological Fluid Flow (Continued)

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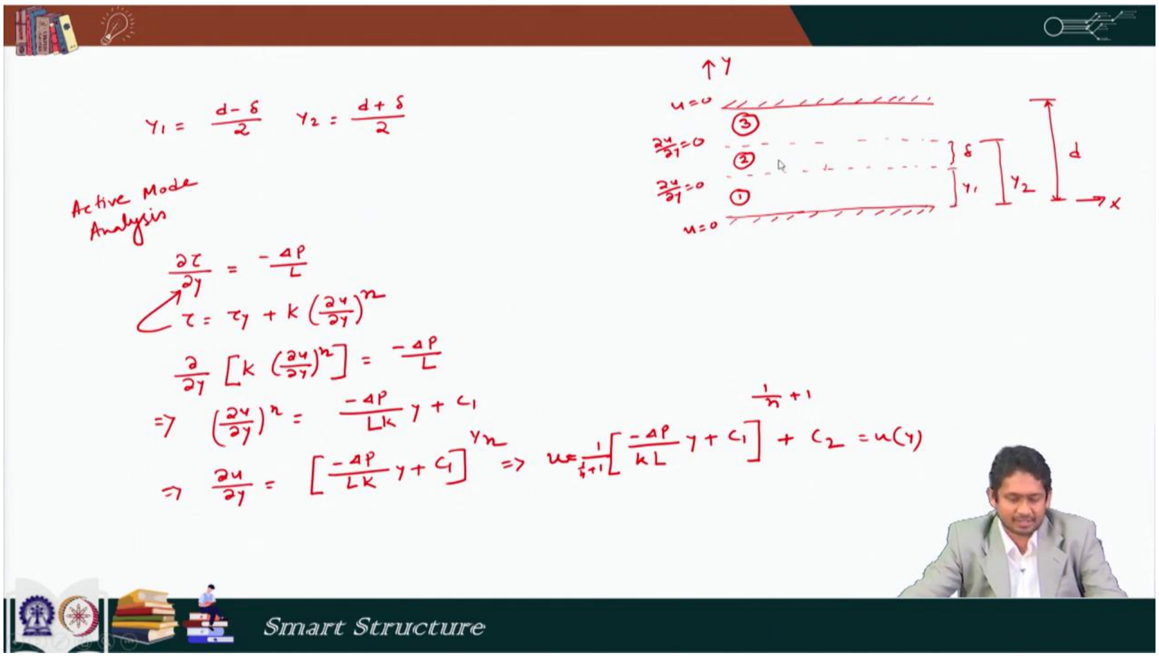

Today, we will continue our discussion on the flow mode analysis using the Herschel Bartlett fluid model. So, we saw that our region has three sub regions, I would say 1, 2 and 3 where the velocity profile is different and the shear strain variation we saw them to be linear and we found out the velocity distribution over each of these 1, 2 and 3 region.

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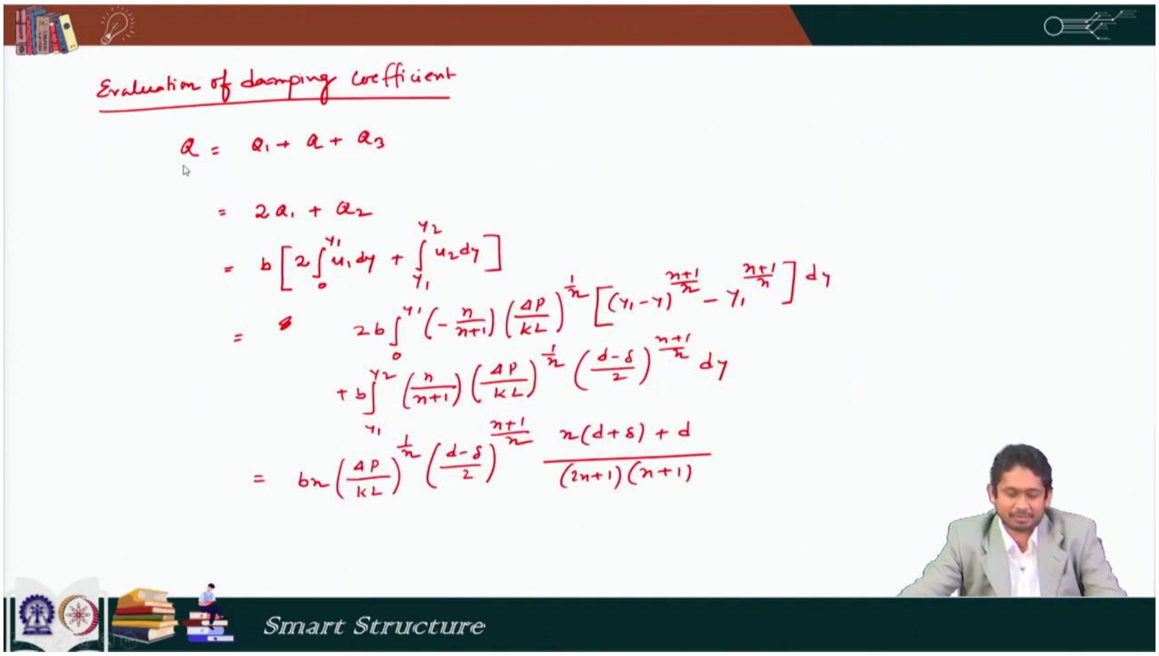

And then we are finding out the effective damping and for effect finding out the effective damping we found out the total flow rate Q. And then we have to equate this total flow rate Q with the total flow rate obtained from a equivalent a constant velocity Um.

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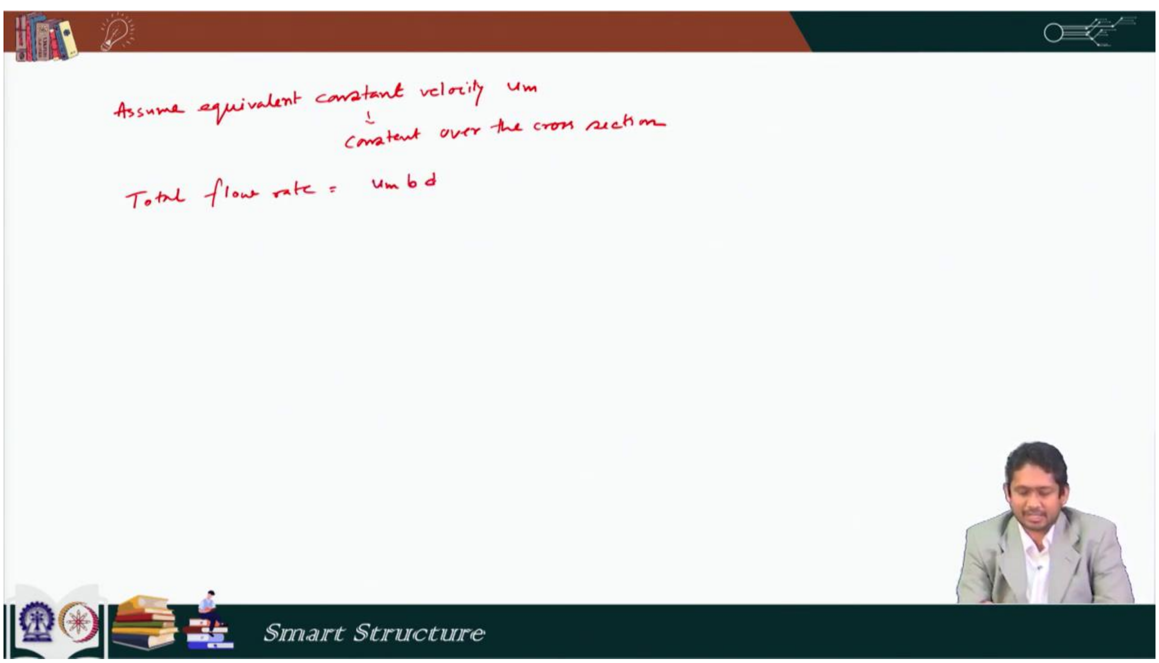

So, assume equivalent constant velocity Um. So, it is constant over the cross section. So, it is constant means constant over the cross section. And then if it is so, then our total flow due to this is Um multiplied by b into d and as we know b is the dimension over the other direction.

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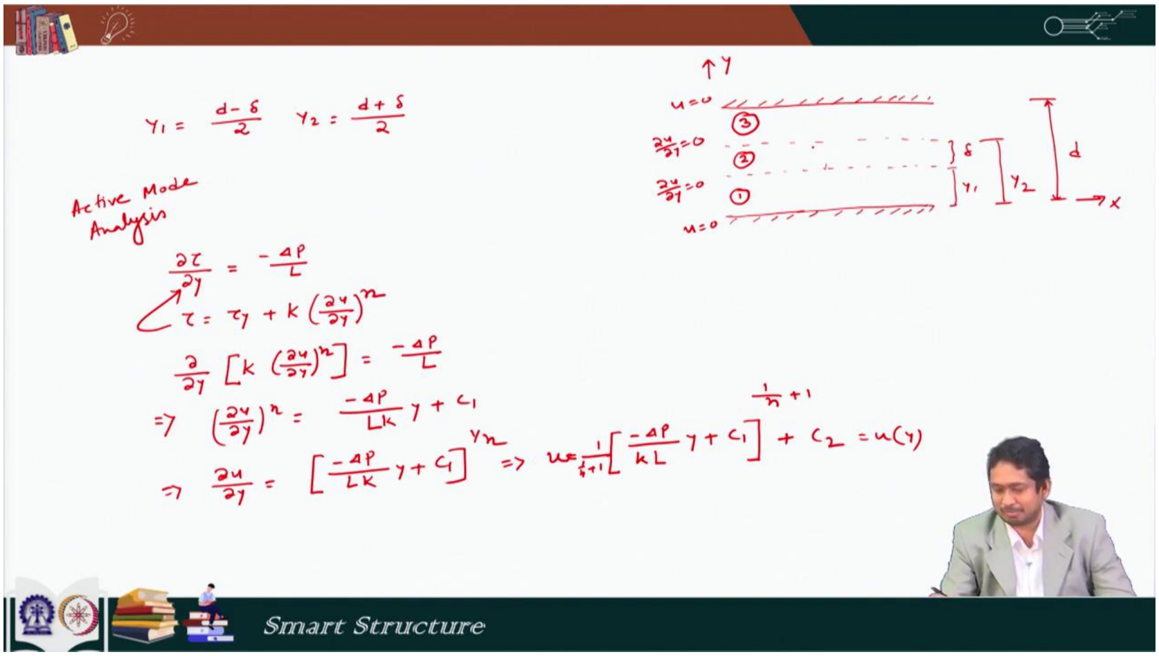

So, this is our x. This is our y. So, width of the plate we can consider that to be b. So, b is our dimension over the z direction which is perpendicular to x y.

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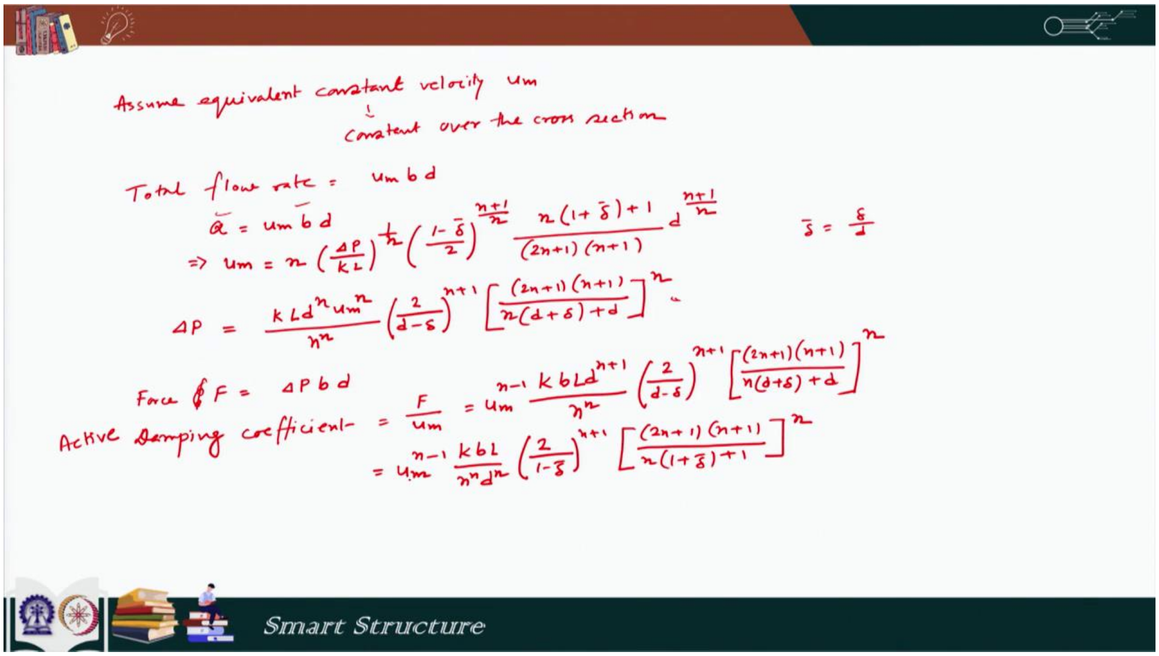

So, we have Um into b d then we can equate this with Q. So, if we equate Q is equal to Um b d and that helps me evaluating Um. So, Um becomes n multiplied by delta P by k L to the power 1 by n and then we have 1 minus delta bar divided by 2 n plus 1 by n and we have n multiplied by 1 plus delta bar plus 1 divided by 2 n plus 1 by n plus 1 multiplied by d into n plus 1 by n.

$$u_m = n\left(\frac{\Delta P}{kL}\right)^{\frac{1}{n}}\left(\frac{1-\bar{\delta}}{2}\right)^{\frac{n+1}{n}}\frac{n(1+\bar{\delta})+1}{(2n+1)(n+1)}d^{\frac{n+1}{n}}$$

So, here delta bar is our del by d which we defined before also. So, this delta has been non-dimensionalize with respect to d and the expression is written. So, we got that just by equating Q with Um b d. Now, here we can see that our Um is a function of delta P to the power 1 by n. So, in such a case if I want to find out the equivalent damping then it would come out as a function of delta P.

So, we can do that or we may want to express our equivalent damping as a function of Um also. So, if you want to do in the other way which means if you want to find out our equivalent damping in terms of Um then better you express delta P in terms of Um. If we do that then it becomes delta P equal to k L d to the power n Um to the power n divided by n to the power n multiplied by 2 by d minus delta n plus 1 and then within a bracket we have this quantities 2 n plus 1 multiplied by n plus 1 divided by n into d plus delta plus d to the power n. And again we can take out d and write this entire expression in terms of delta bar as well like we did before. So, this relation is just an inversion of this relation where I am expressing delta P in terms of Um.

$$\Delta P = \frac{kLd^n {u_m}^n}{n^n} \left(\frac{2}{d-\delta}\right)^{n+1} \left[\frac{(2n+1)(n+1)}{n(d+\delta)+d}\right]^n$$

Now, our finding out the equivalent damping coefficient is simple. We write a force corresponding to this delta P and the force is delta P multiplied by b d. So, force F which we can say the force responsible for causing the flow that is delta P multiplied by b d. So, now this damping coefficient is in the active mode when the fluid is active. So, we can call it active damping coefficient and that is equal to F by Um.

So, F by Um finally, comes out to be Um to the power n minus 1 multiplied by k b L d to the power n plus 1 divided by n to the power n multiplied by 2 by d minus delta to the power n plus 1 multiplied by 2 n plus 1 multiplied by n plus 1 by n into d plus delta by d and we have to the power n. And then this quantity can be written in a non dimensional form as Um to the power n minus 1 multiplied by k b L divided by n to the power n, d to the power n multiplied by 2 by 1 minus delta bar where delta bar is delta by d to the power n plus 1 and then we have the same thing within the bracket 2 n plus 1, n plus 1 divided by n into 1 plus delta bar plus 1 to the power n. So, this is our active damping coefficients. Now, here we can see that the active damping coefficient as we expected it is a function of Um which was not the case previously when we are using the Bingham plastic model because in that case n was equal to 1 and if I put n equal to 1 then this quantity just becomes 1. So, the damping coefficient becomes independent of the Um equivalent flow velocity.

$$\frac{F}{u_m} = {u_m}^{n-1} \frac{kbL}{n^n d^n} \left(\frac{2}{1-\bar{\delta}}\right)^{n+1} \left[\frac{(2n+1)(n+1)}{n\left(d+\bar{\delta}\right)+d}\right]^n$$

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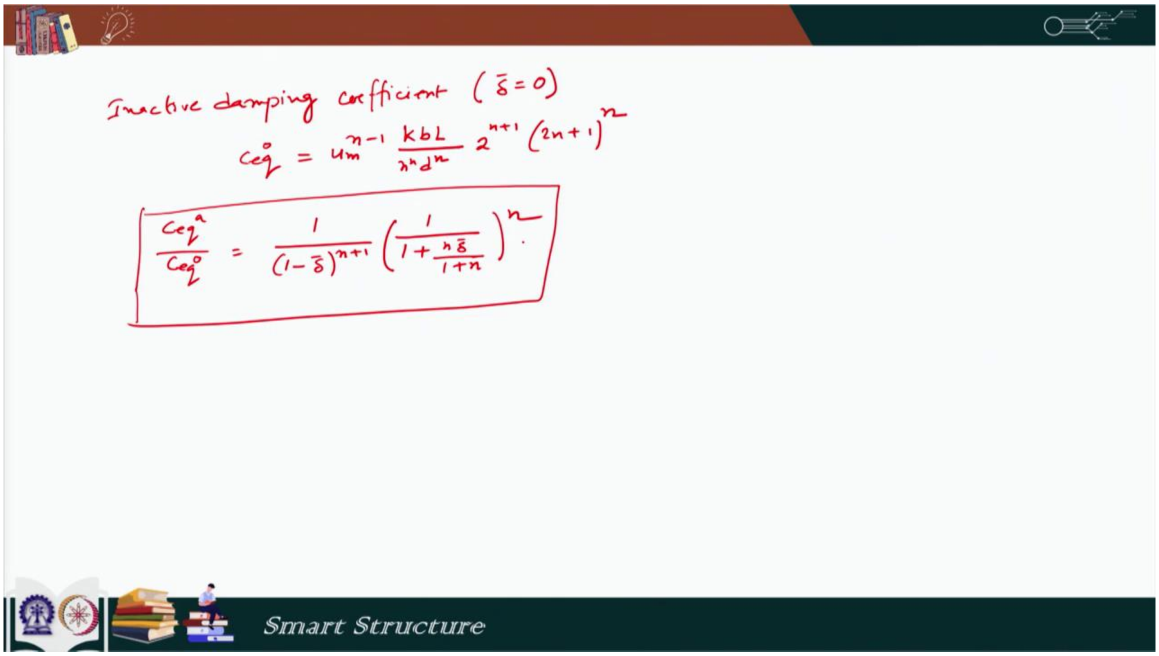

So, if I want to compare this with inactive damping coefficient, we should compare for the same Um and inactive damping coefficient can be found out just by setting delta bar to 0. So, inactive damping coefficient which means the damping coefficient when there is no electric or magnetic field applied which means when tau y is 0 and we know that at that time delta bar is equal to 0 because there is no concept of yielding at that time and if we find out the inactive damping coefficient which we can do just by setting delta bar is equal to 0 in the expression of active damping coefficient, the inactive damping coefficient becomes Um to the power n minus 1 by k b L divided by n to the power n, d to the power n multiplied by 2 to the power n plus 1 multiplied by 2 n plus 1 to the power n. So, the ratio of active and inactive damping coefficient can be obtained from here in terms of this non dimensionalized plug thickness delta bar and that becomes 1 minus delta bar to the power n plus 1 multiplied by nth power of 1 by 1 plus n delta bar divided by 1 plus n. So, this is the ratio of the active and inactive damping coefficient. Now here if we set n is equal to 1, we will get all the quantities that we got using the Bingham plastic model.

$$C_{eq}{}^{o} = u_m{}^{n-1}\frac{kbL}{n^n d^n}2^{n+1}(2n+1)^n$$

$$\frac{C_{eq}{}^{a}}{C_{eq}{}^{o}} = \frac{1}{(1-\bar{\delta})^{n+1}}\left(\frac{1}{1+\frac{n\bar{\delta}}{1+n}}\right)^n$$

So, it is as advisable that the learners try to plot the flow profile and try to evaluate these expressions for various values of n. So, that will give an idea of how the flow profile changes when we incorporate n more than 1 that means, the shear thickening case or we incorporate n less than 1 in case of in the shear thinning case and which can be easily done with the help of platforms like MATLAB.

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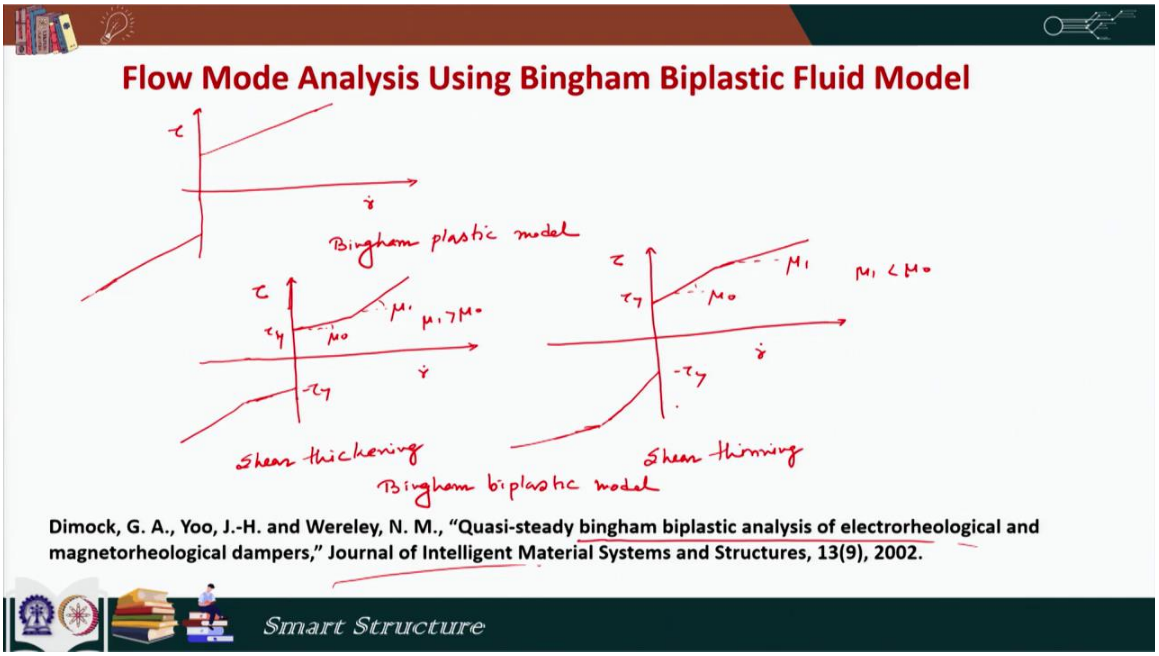

Now this brings us to the end of this flow analysis using the Herschel bulk lay model. Next we will do the similar analysis using Bingham Biplastic Model. So, Bingham Biplastic Model is a modified version of the Bingham plastic model to incorporate shear thinning or shear thickening behavior and the little bit more details about the model can be read in this paper that is referred here.

Now in Bingham plastic model we have seen that the variation of shear stress with shear strain is just this. So, this is Bingham plastic model. So, in the entire region, I would say, postal region our del tau by del gamma dot is constant. So, there is no shear thickening or shear thinning. What the Bingham biplastic model says is that this postal region, they divided into two parts.

So, depending on whether it is shear thinning or shear thickening, the graphs would look like this. So, if it is shear thinning let us do it for shear thickening first. So, if it is shear thickening the graph looks like this and again it is same whether we are in the positive side or negative side just the sign changes. So, we can see that after some value of shear strain rate, the slope of the stress versus shear strain curve increases which means the viscosity increases. So, it is shear thickening.

So, this is shear thickening and then we have something like this when I have shear thinning effect. In that case the slope is more before and less after. So, this is shear thinning. We can call this slope as mu 0 and this is mu 1 here this is mu 0 here this is mu 1. So, in this case our mu 1 is greater than mu 0. In this case our mu 1 is less than mu 0 and this is Bingham biplastic model. And when you have mu 1 and mu 0 same it becomes the Bingham plastic model and again please understand that the graph here and the graph here are same. So, mu 0 here and the mu 0 in the negative side is same. mu 1 here and the mu 1 here are same. Similarly, tau y yield stress is same at the positive and negative side.

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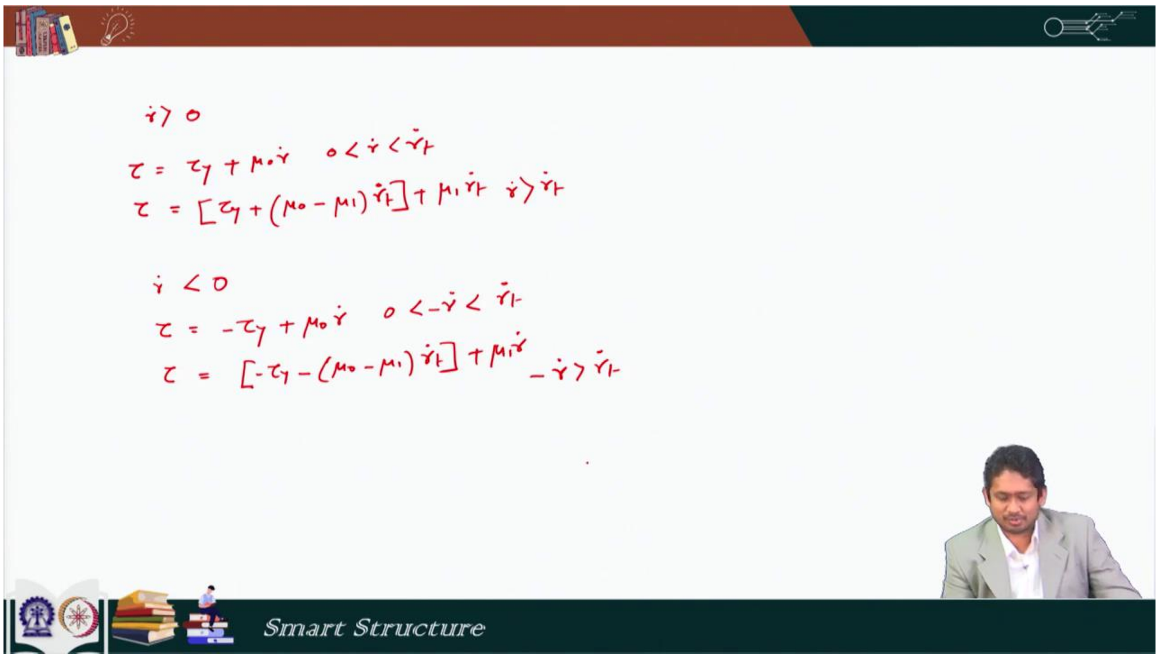

So, the whether the fluid is moving towards the positive direction or negative direction it does not matter. Now, we have to look at the equations how to write this entire thing in the form of equation. Now, when we have gamma dot equal to 0 which means the flow velocity gradient is positive we have tau equal to tau y plus mu 0 gamma dot given that gamma dot lies between 0 and gamma t dot. Now, gamma t dot is the strain rate at which the slope change occurs. So, that is denoted as gamma t dot.

$$\tau = \left[\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t\right] + \mu_1\dot{\gamma}_t$$

$$\tau = \left[-\tau_y - (\mu_0 - \mu_1)\dot{\gamma}_t\right] + \mu_1\dot{\gamma}_t$$

Now, irrespective of the electric or magnetic field that we apply that means, irrespective of the tau y, gamma t dot always remain same. So, if we apply high electric field, this graph just shifts parallelly up. So, the gamma t dot remains same and the gamma t is same at the negative side also. And then we have tau equal to tau y plus mu 0 minus mu 1 multiplied by gamma t dot plus mu 1 gamma t dot when our gamma dot is more than gamma t dot that means, when we are in this region or in this region. Now, what we saw just now is based on the fact that gamma dot is more than 0 that means, we are looking at this graph or this graph.

Now, we would look at the negative side negative side of the graph that means, when our gamma dot is less than 0. So, gamma dot is less than 0 and then the equation looks like this tau is equal to minus tau y plus mu 0 gamma dot gamma dot is a negative quantity now. And when the magnitude of gamma dot which is minus gamma dot for this case is less than gamma t dot and then tau is equal to minus tau y minus mu 0 minus mu 1 into gamma t dot plus mu 1 into gamma dot and that is when minus gamma dot is greater than gamma t dot that means, the magnitude of gamma dot is greater than gamma t dot. So, we will use this relation now for analyzing the flow mode or valve mode. Now because the model is biplastic, if we look at the flow region instead of seeing three regions we would see five regions.

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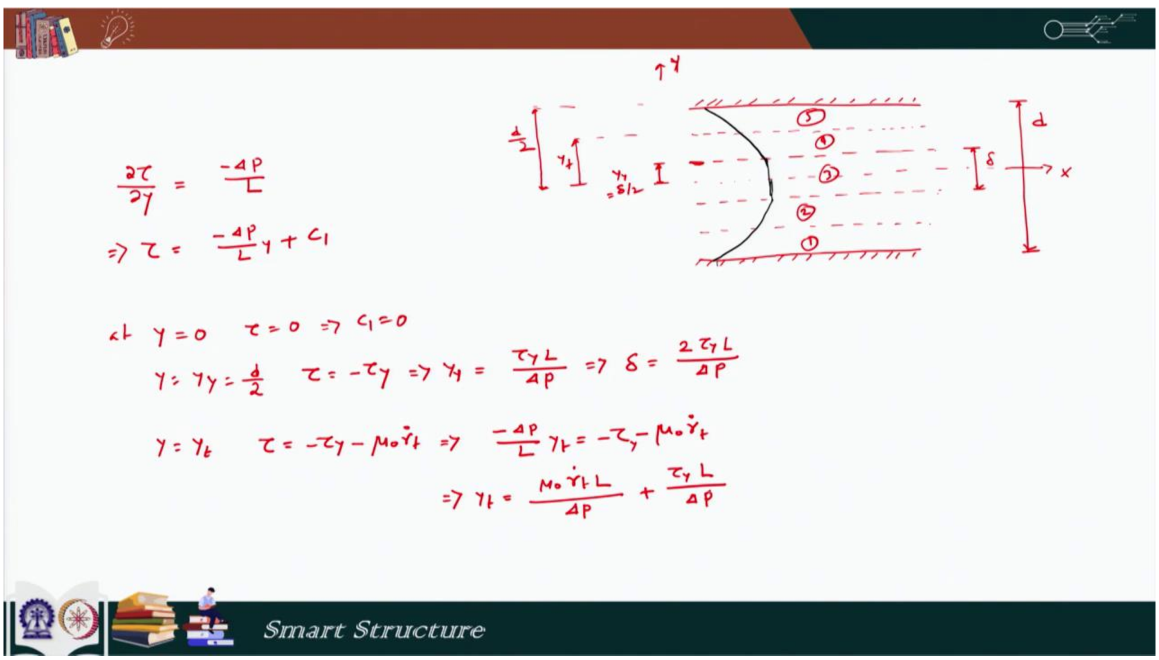

So, again we have two plates which are fixed at their position and we can take an axis at the midline now instead of the bottom line x and these are y. So, here we have a plug region. So, these are plug region between these two dashed lines and where our velocity gradient is 0 and then between the end of the plug region and the plate, there is a junction where below which we have magnitude of gamma dot less than the gamma t dot and above which we have the magnitude of gamma dot greater than gamma t dot and same thing is at the bottom half also. This is symmetric. Now because of the symmetry we will do the analysis starting from the mid region.

$$\tau = -\frac{\Delta P}{L}y + C_1$$

$$\delta = \frac{2\tau_y L}{\Delta P}$$

$$y_t = \frac{\mu_0 \dot{\gamma}_t L}{\Delta P} + \frac{\tau_y L}{\Delta P}$$

Now this is our delta. This is our d. Now we call this distance as y_γ is equal to delta by 2 and then we call this distance, from the mid region midline to the line where the two postal region meets that means, where gamma dot is just gamma t dot and we call that as y_t and then we have from the mid region to the upper plate that is d by 2. The velocity profile if we see it would look something like this. We can show it in a different color may be. Again this is constant here and this is symmetric. So, like this. Now this is what we have to analyze. So, here our approach is that at first, we write the shear equation and from there we will find out this y_γ y_t and then we will start solving for the velocity. Now while solving for the velocity we will start from the uppermost region. So, we name the regions like this region 1, region 2, region 3, 4 and 5. So, we will start from upper region 5.

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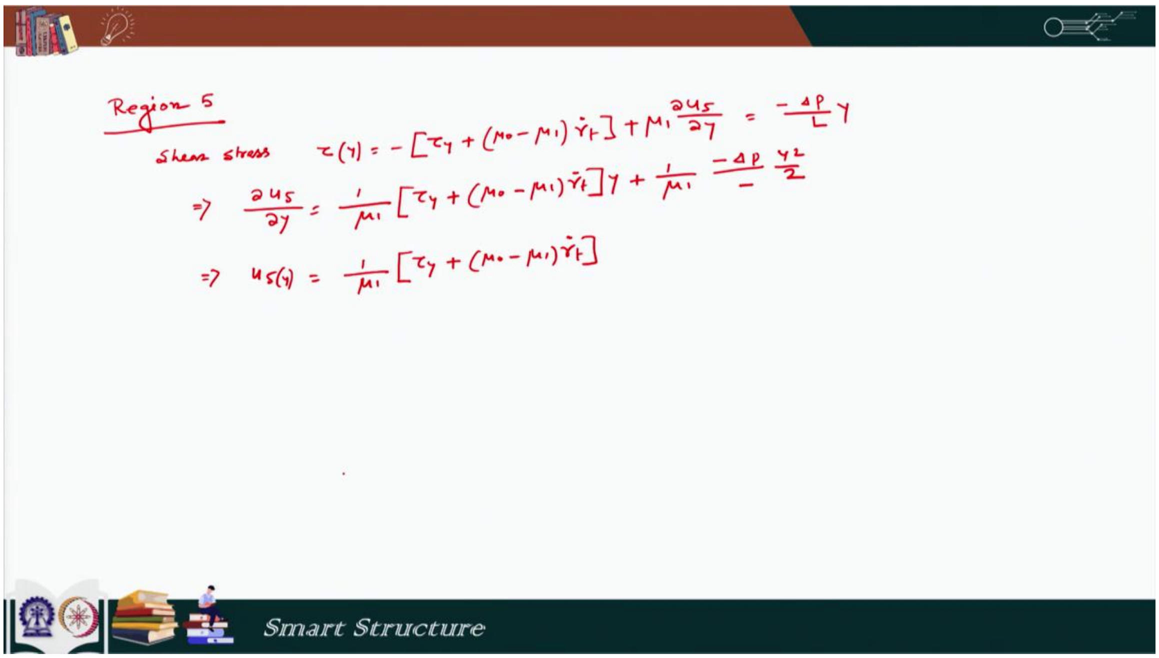

So, we will put the boundary condition at y is equal to d by 2 and then we will put the continuity condition at the junction between region 4 and 5 and that will help us finding out the velocity profile at 4 and if I get the velocity profile at 4, we know that the velocity between the junction of region 3 and 4 is the velocity in the region 3 and that helps us finding out all the velocities and then from there, we can find out again using the same approach the equivalent constant velocity and that can help us finding out the damping coefficient. Now our governing differential equation is del tau by del y is equal to minus del P by L same equation and that gives us tau is equal to minus del P by L into y plus C_1. Then we put the boundary conditions. So, here our y is equal to 0 is the midline, that means, the middle of the plug region and we know that at that region, our shear stress is 0. So, we can say that at y equal to 0, tau equal to 0 and from there we get C_1 equal to 0 and then at y equal to y_1 which means here, we have at y equal to y_γ we call that at y equal to y_γ is equal to d by 2.

We have tau equal to minus of tau y. It is minus because the velocity gradient is negative after that. So, it is minus of tau y and that gives me that y_γ equal to tau y multiplied by L by delta P and from here we can say that delta, the thickness of the plug region, is equal to 2 tau y L divided by delta P. Next, we will put the condition that at y equal to y_t, we know that tau equal to minus tau y minus mu 0 gamma t dot and that tells me that minus del P by L y_t is equal to minus tau minus mu 0 gamma t dot and by solving this, we can find out y_t as mu 0.

So, it is tau y. So, it is mu 0 gamma t dot L divided by delta P plus tau y L divided by delta P. So, we have now characterized all the regions in terms of their dimensions. Now we have to look into the flow profile. So, for the flow profile we solve the governing differential equation in terms of the velocity. And as we said before we will start with region 5.

So, in region 5, the shear stress is, tau as a function of y, is equal to minus of tau y plus mu 0 minus mu 1 into gamma t dot plus mu 1 into del u_5 by del y. So, u_5 means u at region 5 and that gives me, I mean, that quantity is equal to minus del P by L into y. So, from here we can write that del u_5 by del y, if we just in this equation if I just write everything in terms of del u_5 by del y that becomes 1 by mu y of tau y plus mu 0 minus mu 1 into gamma t dot y plus 1 by mu 1 into minus del P by L y square by 2. And then we can integrate this quantity and that will give us u_5 as a function of y equal to 1 by mu 1 into tau y plus mu 0 minus mu 1 into gamma t dot. So from this expression if I express everything in terms of del u_5 by del y L get del u_5 by del y equal to 1 by mu 1 multiplied by tau y plus mu 0 minus mu 1 into gamma t dot minus 1 by mu 1 into delta P by L into y.

$$u_{5(y)} = \frac{1}{\mu_1}\left[\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t\right]$$

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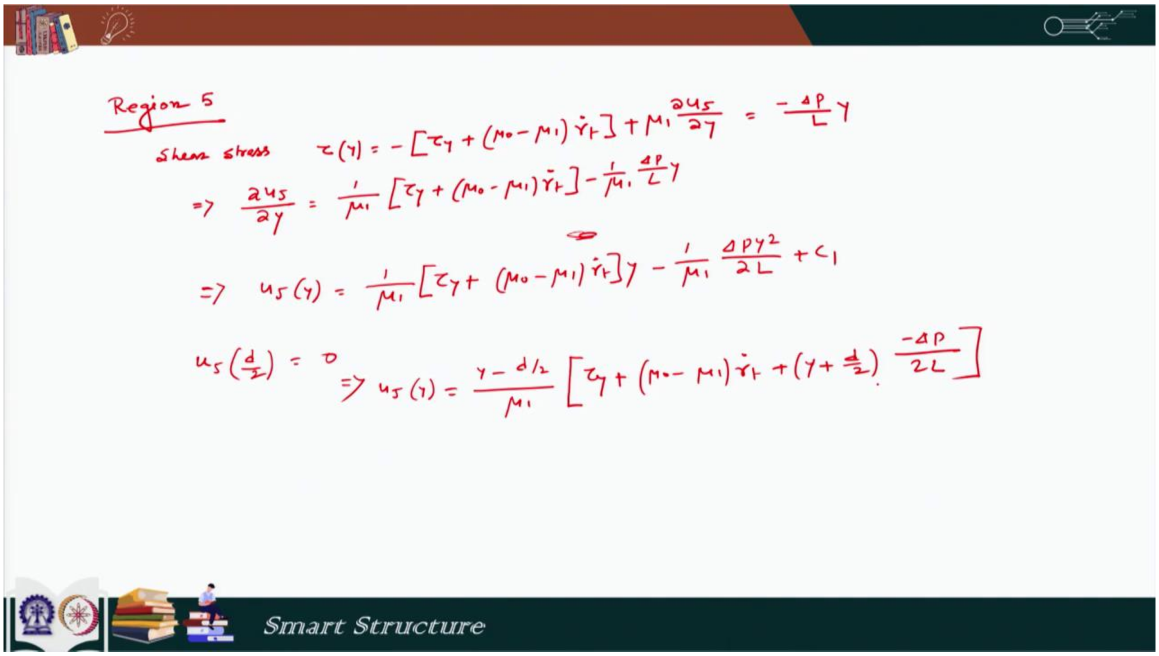

$$u_{5(y)} = \frac{1}{\mu_1}\left[\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t\right]y - \frac{1}{\mu_1}\frac{\Delta P y^2}{2L} + C_1$$

$$u_{5(y)} = \frac{y - \frac{d}{2}}{\mu_1}\left[\tau_y + (\mu_0 - \mu_1)\dot{\gamma}_t\right] + (y + \frac{d}{2})\frac{-\Delta P}{2L}$$

And then we can integrate. Now we can integrate the equation and that will give me u_5 as a function of y as 1 by mu 1 multiplied by tau y plus mu 0 minus mu 1 gamma dot into y minus 1 by mu 1 into delta P y square by 2 L plus a constant C_1. Now, we need to evaluate the constant and that we can do because we know that at the upper plate which means at y equal to d by 2, u_5 is 0. So, if we put this boundary condition here then finally, we can evaluate our C_1 and the final expression that we get is y minus d by 2 divided by mu 1 multiplied by tau y plus mu 0 minus mu 1 gamma t dot plus y plus d by 2 multiplied by minus delta P by 2 L. So, this is our velocity profile in region 5. After that we will find out the velocity profile in region 4 and region 3 and then from there we will proceed for finding out the equivalent damping the damping coefficient.

So, we will do that in the next lecture. Let us end the lecture here.

Thank you.