Smart Structures
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Week - 12
Lecture No - 59
Analysis of Electro and Magneto Rheological Fluid Flow

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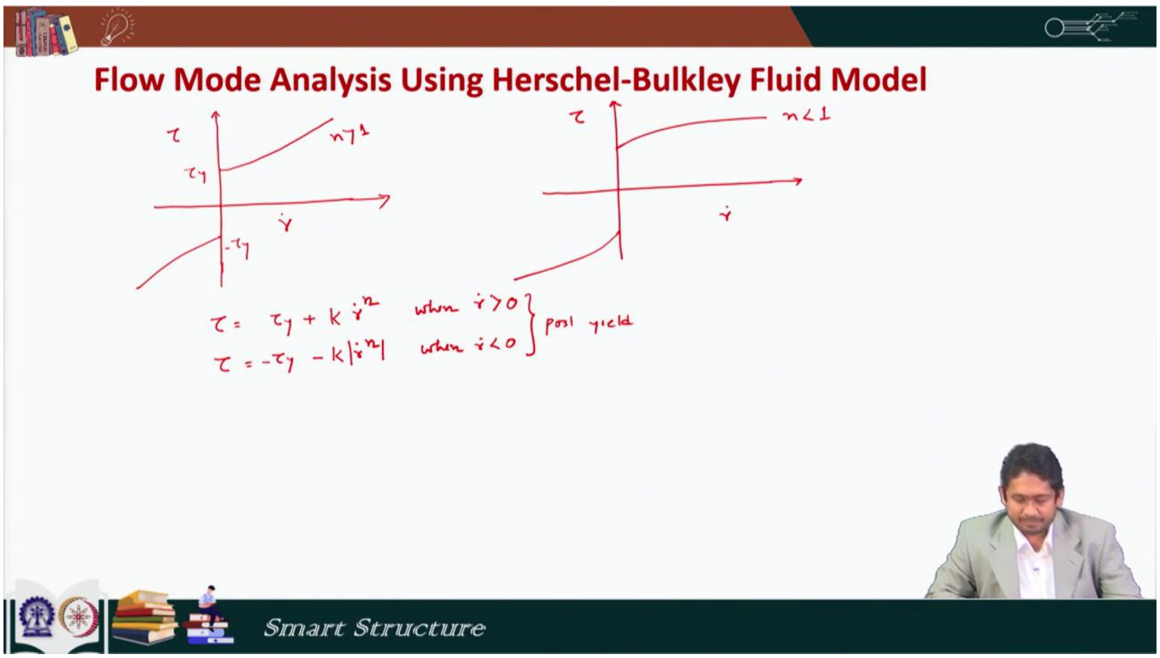

Welcome to the 12th week. This week we will start with the discussion of flow mode analysis, but instead of using the Bingham plastic model that we did before, here we will use Herschel-Buckley fluid model. Now as per the Herschel-Bulkley fluid model as we have seen before that this fluid model is non-linear. So, the post yield behavior is modeled using non-linear equation. So, the model looks like this. After the yield point, the shear stress versus shear strain graph looks like this.

So, this is our tau y and this we can say minus tau y. So, the equation is tau equal to tau y plus k multiplied by gamma dot to the power n. And this is when our gamma dot is greater than 0 which means this part of the graph, the upper part the positive part of the graph. And then we have tau equal to minus tau y plus k gamma dot to the power n and this is when our gamma dot is negative.

$$\tau = \tau_y + k\dot{\gamma}^n$$

$$\tau = -\tau_y - k|\dot{\gamma}^n|$$

Now here we have to be careful. So, it is better that we put a mod sign here and put a negative sign here. So, that makes the graph same as the graph at the negative side same as at the positive side. Now here this entire thing is post yield behavior and this graph here we have n greater than 1. So, that is why we can see that when increase in gamma dot, the slope of the graph is also increasing which means the viscosity is increasing with gamma dot and this is called shear thickening behavior.

If I draw the graph tau versus gamma dot here with n less than 1 then the graph looks like this and here it is n minus 1. So, it is shear thinning behavior. So, the effective viscosity is reducing as the shear strain rate is increasing. Now in general, the ER or MR fluids are shear thinning fluid which means in most of the cases, they have n less than 1, but in our formulation is more generic here whether it is greater than 1 or less than 1, does not matter. The same formulation holds.

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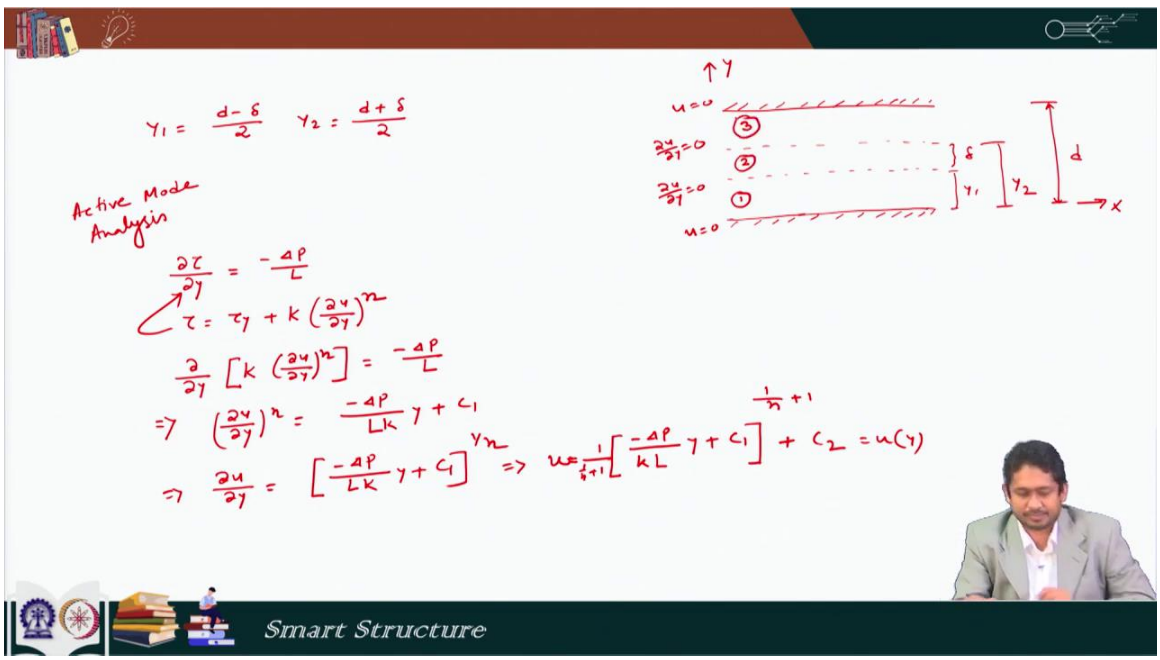

So, we will do it using the flow mode. So, in the flow mode as we have seen before, we have two plates. There is a upper plate and there is a lower plate and both the plates are fixed and in between them, the velocity varies. Now we have seen that there is a region in between around the midline of it there is a region where the fluid behaves like a solid in the sense that there is no velocity gradient in this region and that we call as plug and from here to here the distance we denote as y_1 and from here to here the distance we denote as y_2 and we have seen that this distance between these two plates we can denote that as d. This is our x axis and this is our y axis and we have seen that our y_1 is d minus delta by 2 and y_2 is d plus delta by 2.

$$\frac{\partial u}{\partial y} = \left[\frac{-\Delta P}{Lk} y + C_1\right]^{1/n}$$

$$u = \frac{1}{\frac{1}{n}+1}\left[\frac{-\Delta P}{Lk}y + C_1\right]^{\frac{1}{n}+1} + C_2 = u(y)$$

The velocity varies here from y equal to 0 to y equal to y_1 and again from y equal to y_1 to y equal to d and in between y equal to y_1 and y equal to y_2 the velocity is constant. This is when the flow is in active mode. So, here we will do active mode analysis only. That means, our tau y is non-zero. There is non-zero yield stress because of non-zero field that is applied. So, we will start with the governing differential equation and the governing differential equation is del tau by del y is minus del P by L and our constitutive relation we have written tau equal to tau y plus k del u by del y to the power n. Now, here see if you substitute the governing differential equation, the equation of motion becomes del by del y k del u by del y n equal to minus del P by L and then if we integrate the equation, so, this is obtained just by putting this here and then if we integrate the equation, we get a equation of the form which is del u by del y to the power n minus del P by L k y plus c_1. Actually if we integrate it we get minus del P by L plus c1 first and then we divide by k. So, c_1 also gets divided by k, but because c_1 is a constant, we can observe that k within c_1. So, we can write it in this form and then after that from here we can find out the root of it and we get del u by del y and then that gives us minus del P by k L y plus c_1 to the power 1 by n and then from here again after integrating we get minus del P by k L y plus c_1 to the power 1 by n plus 1 and this quantity also has to be divided by 1 by n plus 1.

So, that gives us 1 and then we have plus c_2. So, that is our expression for y. So, u as a function of y is this. After that our procedure is same. So, we apply the boundary condition and the boundary condition is velocity is 0 here. So, u equal to 0 here we have u equal to 0 and we have del u by del y equal to 0. We have del u by del y equal to 0 and that gives us the velocity profiles. Then our next job is to find out the plug thickness that we get from the shear equation here. So, only thing is that because of the power n the flow profile looks different. So, we put the boundary condition. Before that if we call this as region 1, we call this as region 2 and we call this as region 3.

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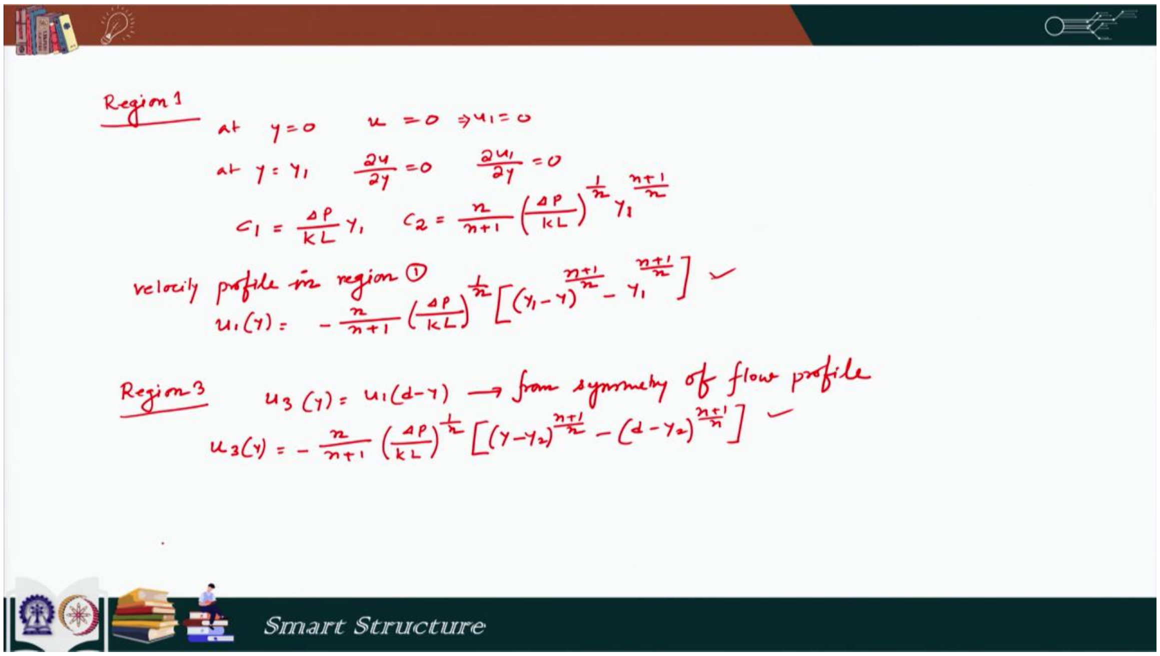

$$u_1(y) = -\frac{n}{n+1}\left(\frac{\Delta P}{kL}\right)^{\frac{1}{n}}\left[(y_1 - y)^{\frac{n+1}{n}} - y_1^{\frac{n+1}{n}}\right]$$

$$u_3(y) = -\frac{n}{n+1}\left(\frac{\Delta P}{kL}\right)^{\frac{1}{n}}\left[(y - y_2)^{\frac{n+1}{n}} - (d - y_2)^{\frac{n+1}{n}}\right]$$

Now we put the boundary condition. So, in region 1 our boundary conditions are that at y equal to 0 we have u equal to 0 or we can call it u_1 equal to 0. So, u_1 means u at region 1 and then we have at y equal to y_1, the velocity gradient is 0. So, we can say del u by del y equal to 0. This we also write as del u_1 by del y. In region 1, u is just u_1. So, the first boundary condition gives us if we put these boundary conditions or satisfy the boundary conditions then the first boundary condition gives us C_1 equal to delta P divided by k L multiplied by y_1 and then we get C_2 equal to n by n plus 1 delta P divided by k L to the power 1 by n into y_1 to the power n plus 1 by n. So, these are the two constants that are evaluated after applying the boundary condition. So, after evaluating the constants we can put them in the governing differential equation in the equation for u and that gives us the velocity profile.

So, velocity profile in region 1 looks like this u_1 as a function of y becomes minus n by n plus 1 into delta p k L to the power 1 by N multiplied by y_1 minus y n plus 1 by n minus y_1 into n plus 1 by n. So, that is our velocity profile in region 1. So, we can see here that if our N is equal to 1 then it becomes a quadratic equation of y. Here depending on the n the the degree would change. Next we have to go to region 3.

So, region 3 is the uppermost region. So, if we look here now whatever we see the flow profile here that is symmetrically can be seen in region 3 also. So, just by using the symmetry we can find out the flow profile here. Now please understand the velocity gradient in region 3 is negative. So, just by using symmetry we can say that u_3 y is equal to u_1 d minus y. So, u_3 is the velocity profile in our region 3. So, this comes from symmetry of flow profile or velocity profile. Now if that is so, then our expression for u_3 is minus n by n plus 1 into delta P by k L to the power 1 by n multiplied by y minus y_2 to the power n plus 1 by n minus d minus y_2 to the power n plus 1 by n. So, these two profiles here this and this, they are symmetric with respect to the midline.

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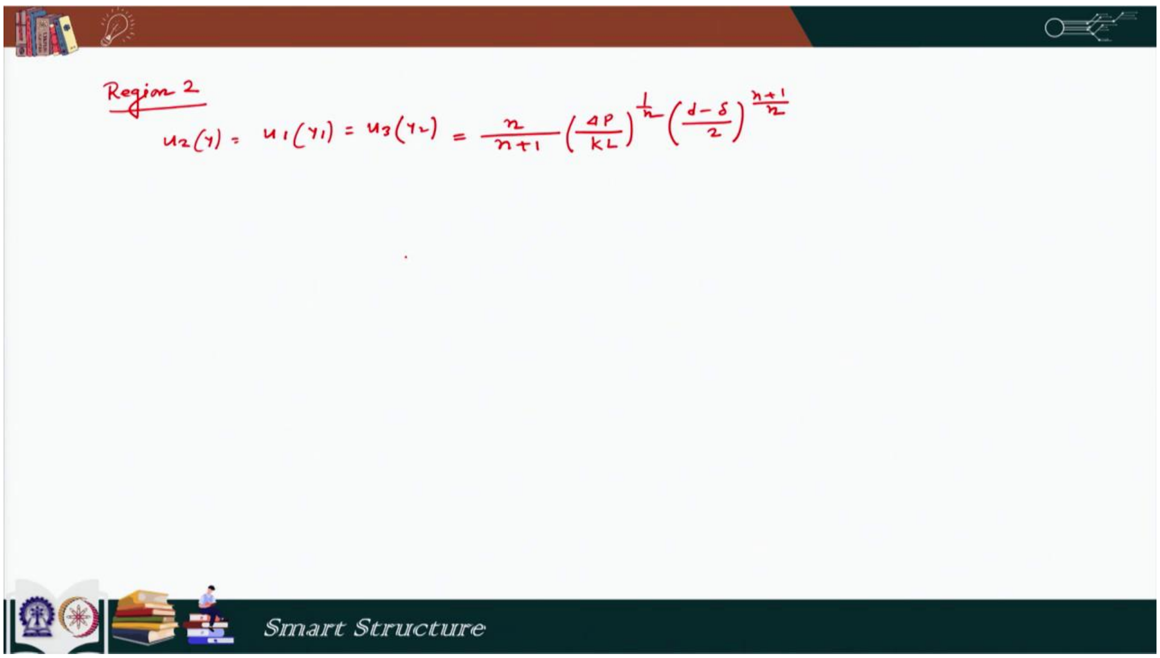

Now we need to go to region 2 we have already evaluated 1 and 3. Mid region is region 2 and that is our plug region. So, in region 2 first we find out the velocity profile and again if we go back to the figure, whatever the velocity here the same velocity is here and that velocity is same within that entire plug region. So, if we evaluate the velocity here by using the expression of u_1 and if we evaluate the velocity here by using the expression of u_3 we get these two velocities to be same and that is the constant velocity that is maintained within that plug region. So, we can say that u_2 as a function of y which is eventually not a function of y is equal to u_1 evaluated at y is equal to u_3 evaluated at y_2. So, u_1 evaluated at y_1 is equal to u_2 evaluated at y_2.

$$u_2(y) = u_1 y_1 = \frac{n}{n+1}\left(\frac{\Delta P}{kL}\right)^{\frac{1}{n}}\left(\frac{d-\delta}{2}\right)^{\frac{n+1}{n}}$$

Now if we do that evaluation then the expression becomes n by n plus 1 multiplied by del P by k L to the power 1 by n multiplied by d minus delta by 2 which is our y_1 to the power n plus 1 by n and that is the velocity at which the plug region moves. .

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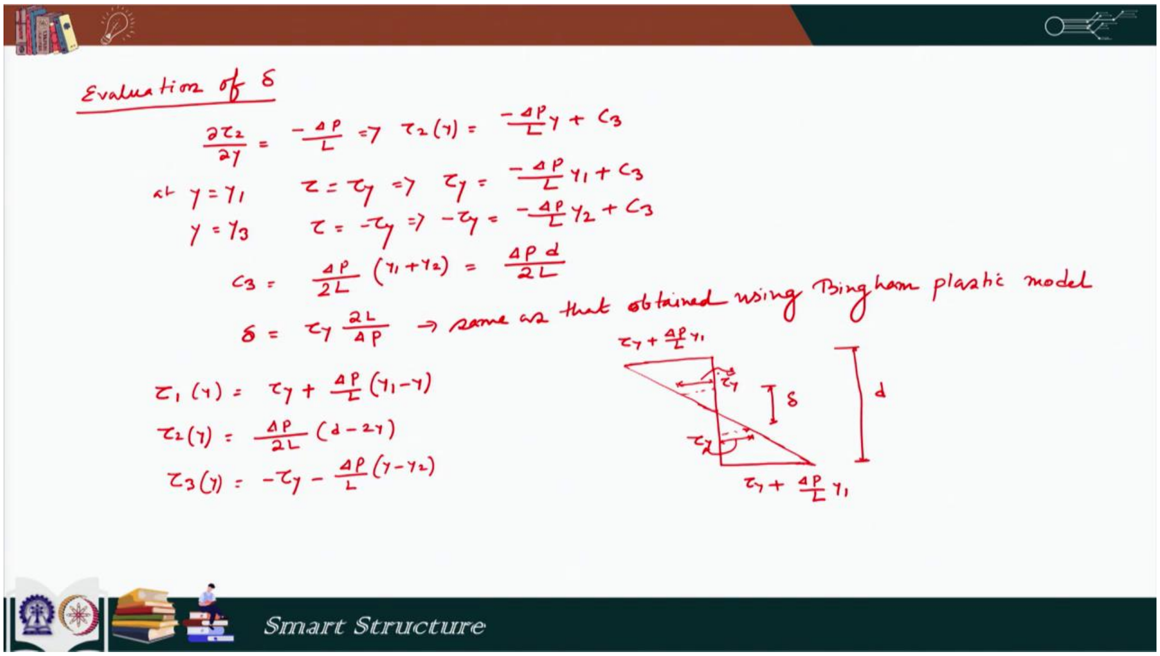

$$\delta = \tau_y \frac{2L}{\Delta P}$$

$$\tau_1(y) = \tau_y + \frac{\Delta P}{L}(y_1 - y)$$

$$\tau_2(y) = \frac{\Delta P}{2L}(d - 2y)$$

$$\tau_3(y) = -\tau_y - \frac{\Delta P}{L}(y - y_2)$$

Now, we have to find out the thickness of the plug region which means delta. So, evaluation of delta. So, for that also you will use the same approach that we used for the Bingham Plastic Model. Now, our equation is del tau 2 by del y is equal to minus of del P by L and then if we integrate the equation, so, tau 2 is the velocity in region 2, if we integrate the equation it gives us tau 2 as a function of y is equal to minus del P by L y plus c_3 and then we know that at y equal to y_1, our yield stress is tau y and stress is tau y and at y equal to y_2, the stress is minus tau y.

If we go back to the image once again, so, here where the region 1 moves the junction between region 1 ends the junction between region 1 and 2, here our tau is equal to tau y because here the velocity gradient mean in region 1, the velocity gradient is positive. So, here tau is equal to tau y. In region 3, the velocity gradient is negative. So, in between region 2 and region 3 that means, at the upper end of the delta region or the plug region, tau is equal to minus tau y. If we put these two boundary conditions that can help us find out the constants and the thickness of the plug region.

So, at y equal to y_1, we have tau equal to tau y and this gives us tau y, the yield stress is equal to minus del P by L into y_1 plus C_3 and then at y equal to y_3, we have tau equal to minus tau y and that gives me this equation. Minus tau y equal to minus del P by L multiplied by y_2 plus C_3 Now, if we add these two equations we can evaluate C_3. So, adding these two equations we get C_3 as delta P by L multiplied by y_1 plus y_2 divided by 2 and we know that y_1 plus y_2 is d, the distance between the two plates. So, that gives us delta P d by 2 L. Now, we can put this C_3 in one of the equations and that can help us with the evaluation of delta or we can subtract these two equations also then C_3 gets eliminated and delta can be evaluated because our y_1 and y_2 both are linear function of delta.

So, from that we can find out delta as tau y multiplied by 2 L divided by delta P. So, we can see that the thickness delta is as per this model is same as the thickness delta that we got from the Bingham plastic model. So, same as that obtained using Bingham plastic model. Now, that we have got delta. So, if we know delta we know y_1 and y_2.

So, accordingly we can write all the velocity profile and the shear profile properly. So, with that we can say that tau as a function of y becomes tau y plus delta P by L multiplied by y_1 minus y and then tau 2 y tau, in the region 2, is delta P by 2 L d minus 2 y and tau 3, as a function of y, is minus tau y minus delta P by L y minus y_2. So, our governing differential equation says that shear del tau by del y is equal to minus del P by L and integrating that we get that our variation of tau along y is linear and that is what we see here. So, if you want to plot the shear distribution, it would look like this. So, when y is equal to 0 our tau is tau y plus delta P by L into y_1 which is like this and when our tau is equal to to d then if we put d we get the same shear, but with a negative sign.

So, if you put y equal to d here it becomes d minus y_2 and d minus y_2 as we know is equal to y. So, the shear here and shear here are same, but opposite and in between them it varies linearly. So, at d by 2, the shear is 0. Shear stress we are talking about here and it is this. So, here it is tau y plus delta P by L multiplied by y_1 and this value is also tau y plus delta P by L y_1 here it is 0 we have some plug thickness delta and so, this amount is delta and this entire distance is d and if you want evaluate tau at y_1, so, it is we if we just put y equal to y_1 then this quantity becomes 0. So, tau 1 at y_1 is tau y and here it is minus tau y. So, this quantity is minus tau y. So, this is plus and this is minus. Now please understand tau y is not the location along y here. It is this dimension to be more specific. So, this is tau y and this quantity is tau y.

Next our goal is to find out the effective damping and for that again we apply the same procedure our procedure is that we find out the total flow and then we equate that total flow to a flow which we would have obtained if I would have assumed an uniform flow throughout and by equating that we get the equivalent uniform flow and from that equivalent uniform flow, we can find out the damping coefficient. We can find out the total force from the pressure and then dividing that force by the equivalent uniform flow which we generally denote as Um we can find out the damping coefficient.

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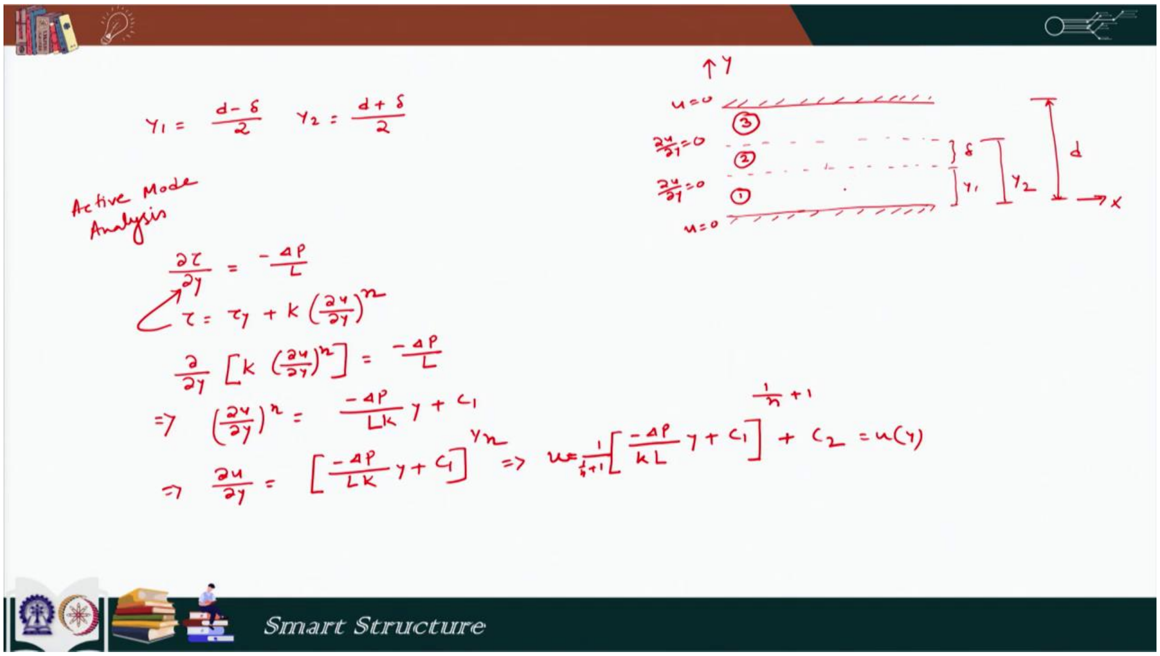

So, our total flow is here consisting of the contribution from these three regions Q_1, Q_2 and Q_3 and we know that the Q_1 and Q_2 are same.

$$u = \frac{1}{\frac{1}{n}+1}\left[\frac{-\Delta P}{kL}y + C_1\right]^{\frac{1}{n}+1} + C_2 = u(y)$$

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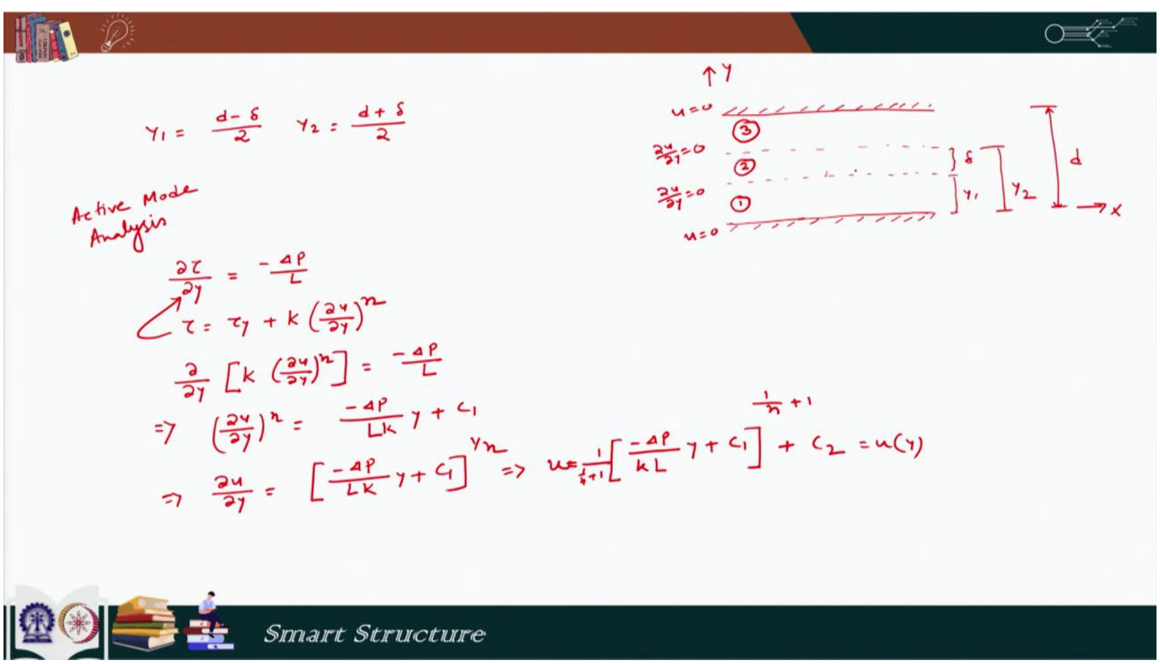

So, it becomes 2 Q_1 and then we have Q_2. So, if I add 2 Q_1 plus Q_2 that gives me total flow.

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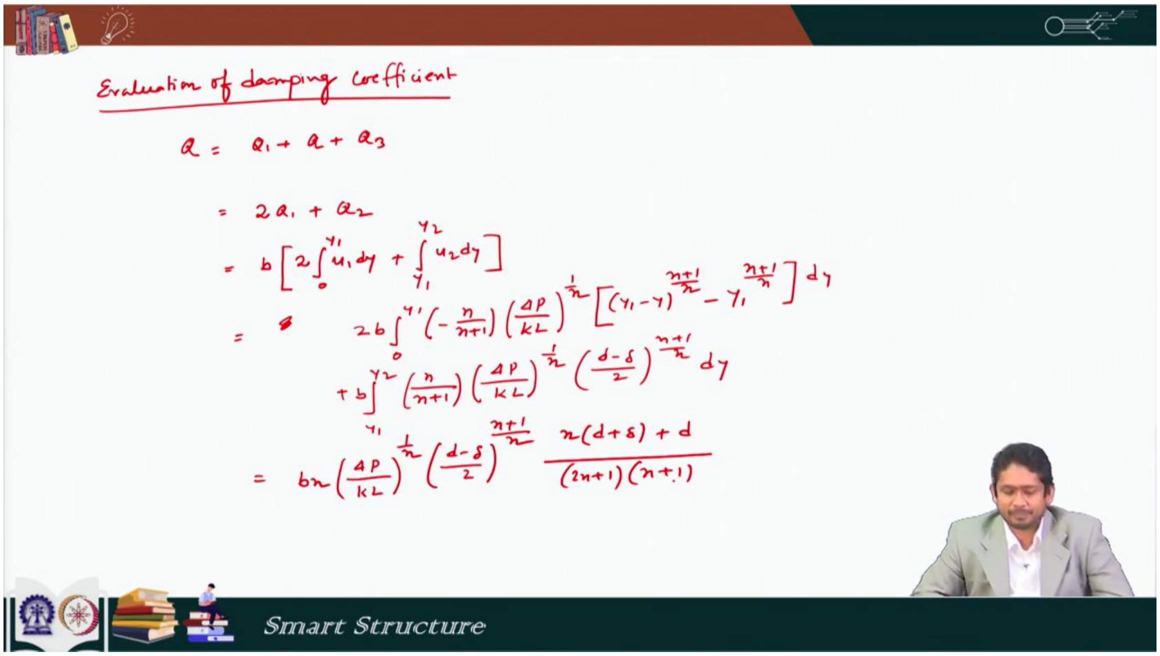

So, here we do evaluation of damping coefficient. So, for that the total flow Q is Q_1 plus Q_2 plus Q_3 which we write as 2 Q_1 plus Q_2 from the symmetry and then we have $2Q_1$ equal to b multiplied by some quantities in a bracket and then we have 2 multiplied by integral of u_1 over 0 to y_1 and we have integral of u_2 over y_1 to y_2 and then we know that we can rewrite the expression in a better way. So, we have 2 multiplied by integral of this quantity over 0 to y_1.

$$Q = b_n \left(\frac{\Delta P}{kL}\right)^{\frac{1}{n}} \left(\frac{d-\delta}{2}\right)^{\frac{n+1}{n}} \frac{n(d+\delta)+d}{(2n+1)(n+1)}$$

So, and here we have b and y_1 to y_2. So, integral of this quantity and finally, after doing all the integrations the flow rate Q becomes b n multiplied by delta P by k L to the power 1 by n multiplied by d minus delta to the power n plus 1 by n multiplied by n into d plus delta Plus d divided by 2 n plus 1 by n plus 1. So, this is our total flow rate Q. After that from here we will find out the equivalent constant velocity and from there we will find out the damping coefficient and then we will find out the ratio of the damping coefficient in the active mode and in the inactive mode.

So, we will do that in the next class we will end this lecture here.

Thank you.