Smart Structures

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Week 11

Lecture No: 58

Analysis of Electro and Magneto Rheological Fluid Flow (Continued)

So far, we have seen the analysis of ER and MR fluid flow in a rectangular passage.

Now, today we will see how the formulation looks if we consider an annular passage. So, an annular passage looks like this. So, there is two concentric cylindrical domains. The inner cylinder is a solid cylinder and the outer cylinder is a hollow cylinder. So, the inner cylinder is between inside the outer cylinder.

So, when the inner cylinder moves or remains fixed, when the inner cylinder moves it gives rise to a shear mode, when the inner shear cylinder and the outer cylinder is fixed that is a flow mode. Now, the fluid is flowing in this region, so between the inner cylinder and outer cylinder that annular region is filled with the fluid. So, when both the cylinders are fixed, and the fluid is driven by a pressure, it is a valve mode or flow mode. When the inner cylinder is moving and that is causing the fluid flow that is a shear mode.

So, we will analyze a shear mode here. And we are doing a shear mode analysis. Now to do this, again, let us take a small fluid element. So we are taking a small fluid element here which looks like this. And we have the pressure P and pressure is increased to P plus del P by del x into dx.

Now, we are taking a fluid, an element in this region of length delta x, and we are taking a cross-sectional view of the delta x region. So, from the outside, it looks like this curved, but if I take a cross-section throughout the center, through the center, it looks like this, and we are analyzing the flow there. Now this is our shear at the top delta, delta r by dr, now here our consideration is that consideration is - this is at the center and the distance from the center to the top or bottom is r and this dimension is our x and the velocity along x direction is called u. So, here we have tau. So, this is our solid cylinder inside.

This is the outer boundary of the outer cylinder, the top boundary of the outer cylinder, the bottom boundary of the outer cylinder and this is our annular region. We are studying only one annular region and that is sufficient for understanding the flow behavior. Now, here we need to find out the mass of fluid element. So, mass of the fluid here is 2 pi r dr dx rho, where rho is the fluid density. Now, 2 pi r dr dx.

So, 2 pi, so, this is our as we know this is our dr. So, this dimension is dr, and if I multiply this dr by 2 pi r, that gives me the total cross-sectional area of the annular region considering the 360-degree revolution. And then if I do 2 pi r dr dx that gives me the volume of the fluid element that means, the volume of the fluid element throughout this considering a length delta x. So, that is the volume of the fluid element and then we have after having the expression of the mass of the fluid we can now write the equation of motion. So, the force balance - so, the total force is equal to the mass into acceleration.

If we do that, then finally, the equation that we get is minus, and this quantity we may write as dm. So minus of dm multiplied by del u by del t, plus 2 pi r dr P, plus tau plus del tau by del r dr, 2 pi multiplied by r plus dr dx multiplied by tau 2 pi r dx, then we have minus P plus del P by del x into dx into 2 pi r is equal to 2 pi r dr is equal to 0.

$$-dm\frac{\partial u}{\partial t} + 2\pi r dr P + \left(\tau + \frac{\partial \tau}{\partial r}dr\right)2\pi(r + dr)dx - \tau 2\pi r dx - \left(P + \frac{\partial P}{\partial x}dx\right)2\pi r dr = 0$$

So, we are finding out the total force due to this pressure here minus the total force due to this pressure here. The total force due to this pressure is this pressure multiplied by this thickness, and then we multiply that by 2 pi, 2 pi r. Similarly, we find out the total pressure here due to the total force due to this pressure, we find out the total shear stress, the total force due to this shear stress, and that is this shear stress multiplied by this length, and then we multiplied by the periphery, 2 pi r.

And similarly, we find out the total force due to this stress and that net unbalance is equated with mass into acceleration. And after doing that, finally, the equation simplifies to minus rho del u by del t, plus tau into tau by r plus del tau by del r minus del P by del x.

$$-\rho\frac{\partial u}{\partial t} + \frac{\tau}{r} + \frac{\partial \tau}{\partial r} - \frac{\partial P}{\partial x} = 0$$

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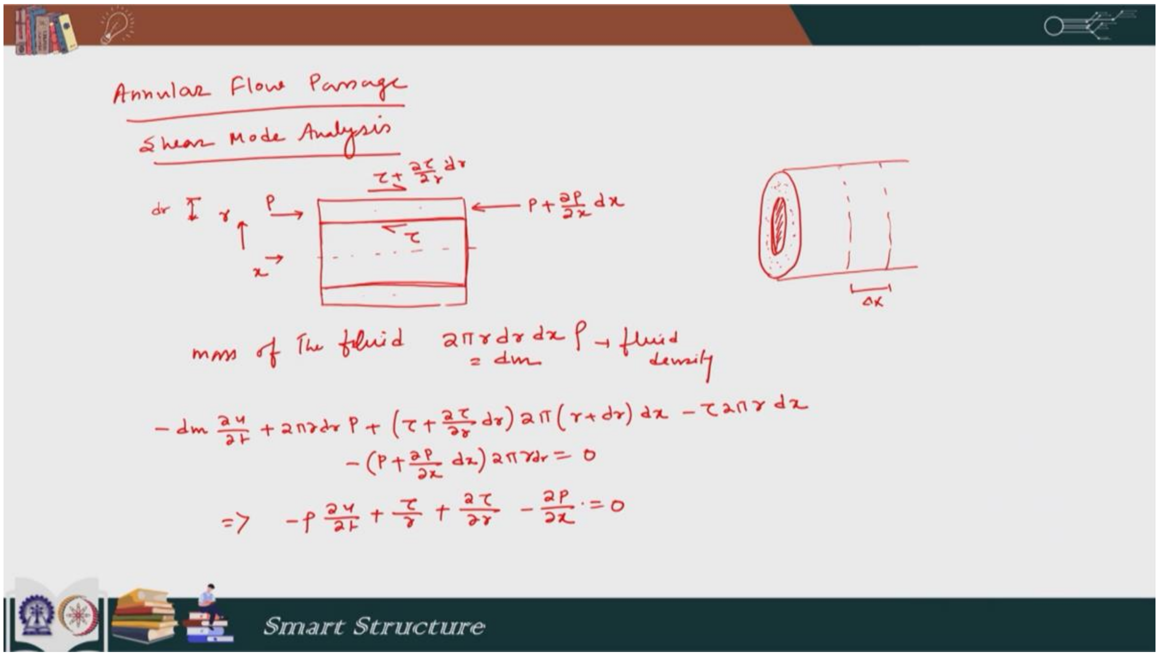

And then we assume that the flow is quasi-steady. So, del u by del t is 0.

$$\frac{\partial u}{\partial t} = 0$$

And that gives me del T by del r plus tau by r is equal to del P by del x.

$$\frac{\partial \tau}{\partial r} + \frac{\tau}{r} = \frac{\partial P}{\partial x}$$

So, this was the governing differential equation, and that is valid for both the shear mode and the valve mode, valid for both shear and mode or another mode now the only thing is that when we are in the shear mode, del P by del x is 0 because it is not pressure-driven. So, in shear mode, we have del tau by del r plus tau by r equal to 0.

$$\frac{\partial \tau}{\partial r} + \frac{\tau}{r} = 0$$

Now, first, we will do the solution for 0 applied field. Now, in the zero applied field, we know that the fluid constitutive relation is tau is equal to mu multiplied by del u by del r,

$$\tau = \mu \frac{\partial u}{\partial r}$$

and if that is so, if I put this in this equation, then the governing differential equation becomes mu into del 2 u by del r 2 plus mu by r multiplied by del u by del r equal to 0 and this can be written as del by del r of r multiplied by del u by del r. Because if I take mu out then del 2 u by del r 2 plus 1 by r into del u by del r is our derivative of r into del u by del r with respect to r. Because if I take my mu out and then if I put this r here, it becomes r into del u by del r 2 plus del u by del r, and that is equal to the derivative of r into del u by del r with respect to r.

$$\mu \frac{\partial^2 u}{\partial r^2} + \frac{\mu}{r}\frac{\partial u}{\partial r} = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = 0$$

Now, I can integrate this equation, and after integrating, we get u as a function of r as A natural log of r plus B. So, where A and B are the constants.

$$u(r) = A\ln r + B$$

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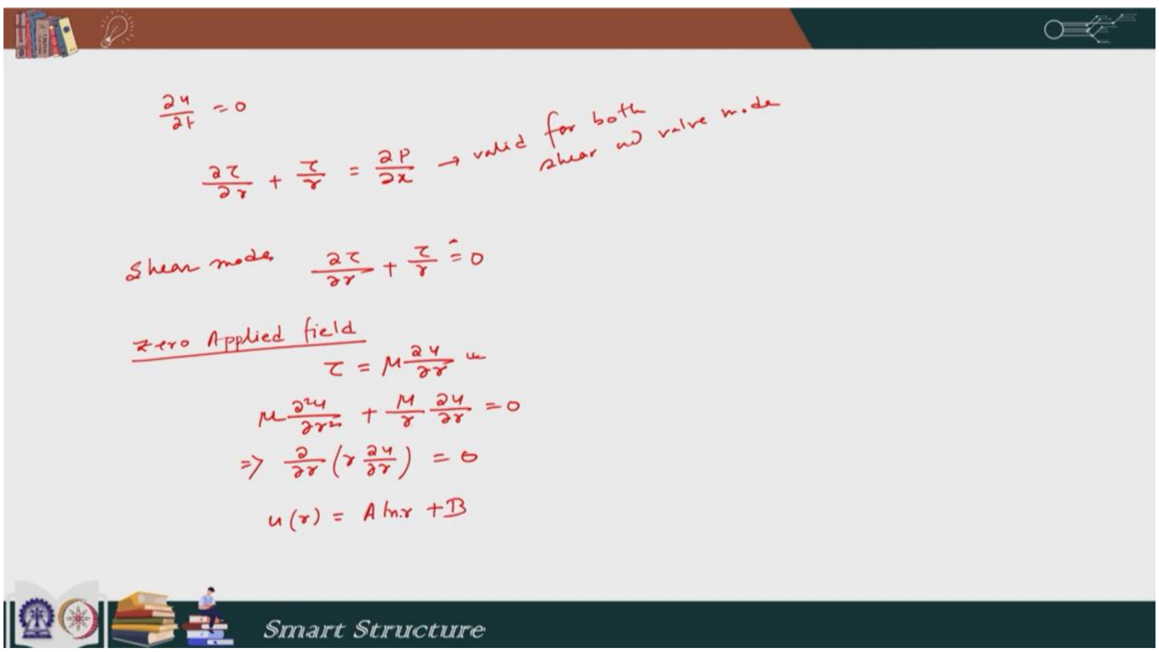

And then we can apply the boundary conditions. So, the boundary conditions are at r is equal to ri, we have velocity is equal to u0, so that is the velocity at which the inner cylinder is moving, and r is equal to r0, that means outer radius, where the velocity is 0, because if I go back to the diagram if I say that the inner radius is 0. ri and the outer radius is ro, and we say that the inner cylinder is moving with a velocity u0 and the outer cylinder is fixed. So, at the inner cylinder, because of the no-slip condition, the boundary condition is u is equal to u0 here, and here, because of the no-slip condition, the boundary condition is u equal to 0.

Now, after putting the boundary conditions, I can evaluate the constants. So, these are now a, b. So, a is equal to u0 divided by the natural log of ri by r0 and this gives me that b is equal to minus of u0 natural log of r0 divided by ri by r0. So, finally, my u as a function of r becomes u0 divided by the natural log of ri by r0 multiplied by the natural log of r by r0. Now, if this is so, then the shear stress tau is equal to mu into del u by del r, and that becomes minus mu u into u0 r into a natural log of r0 by ri.

Now, we can find out the total force F0. So, the total force F0 is the force at the surface of the inner cylinder, and that force is - to evaluate the force, I have to evaluate the shear stress at r, which is equal to ri, and then multiply by just 2 pi ri into length.

$$r = r_i \qquad u(r_i) = u_0 \rightarrow A = \frac{u_0}{ln(r_i/r_0)}$$

$$r = r_0 \qquad u(r_0) = 0 \rightarrow B = -\frac{\mu \ln r_0}{ln(r_i/r_0)}$$

$$u(r) = \frac{u_0}{\ln(r_i/r_0)} \ln\left(\frac{r}{r_0}\right)$$

$$\tau(r) = \mu \frac{\partial u}{\partial r} = -\frac{\mu u_0}{r \ln(r_0/r_i)}$$

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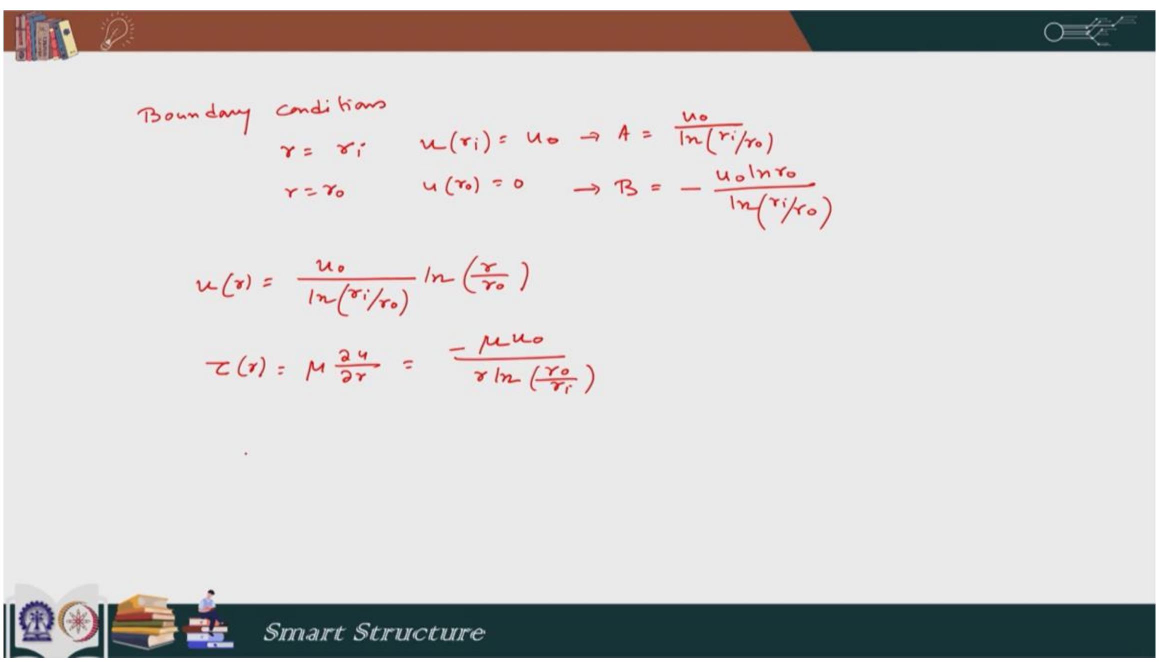

So, here we are considering that the length of the cylinder is L. If this is my fluid element dx, and if I say that this dx is a part of the cylinder.

So, the length of the cylinder is supposed as L. If I do a cross-sectional view here, that becomes our total L. So, this total L, this is dx is just this part.

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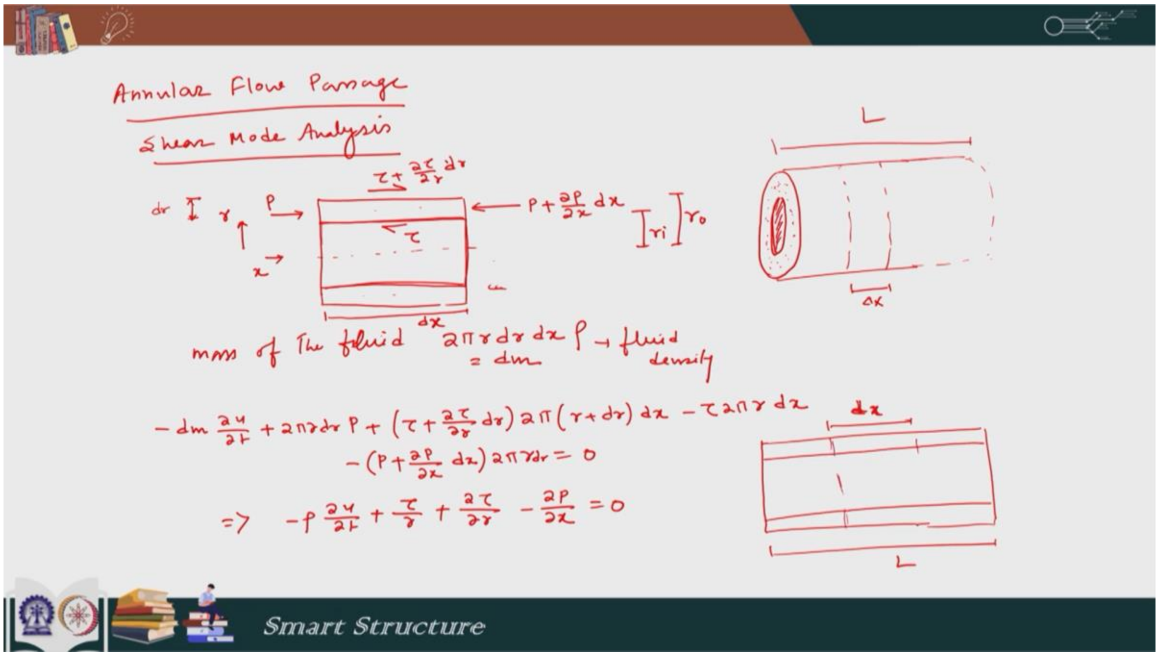

So that gives me that the force F0 is 2 pi ri multiplied by R into tau evaluated at ri, and that gives me 2 pi into L into mu into u0 divided by the natural log of r0 by ri.

$$F_0 = 2\pi r_i L \tau(r_i) = \frac{2\pi L \mu u_0}{\ln(r_0/r_i)}$$

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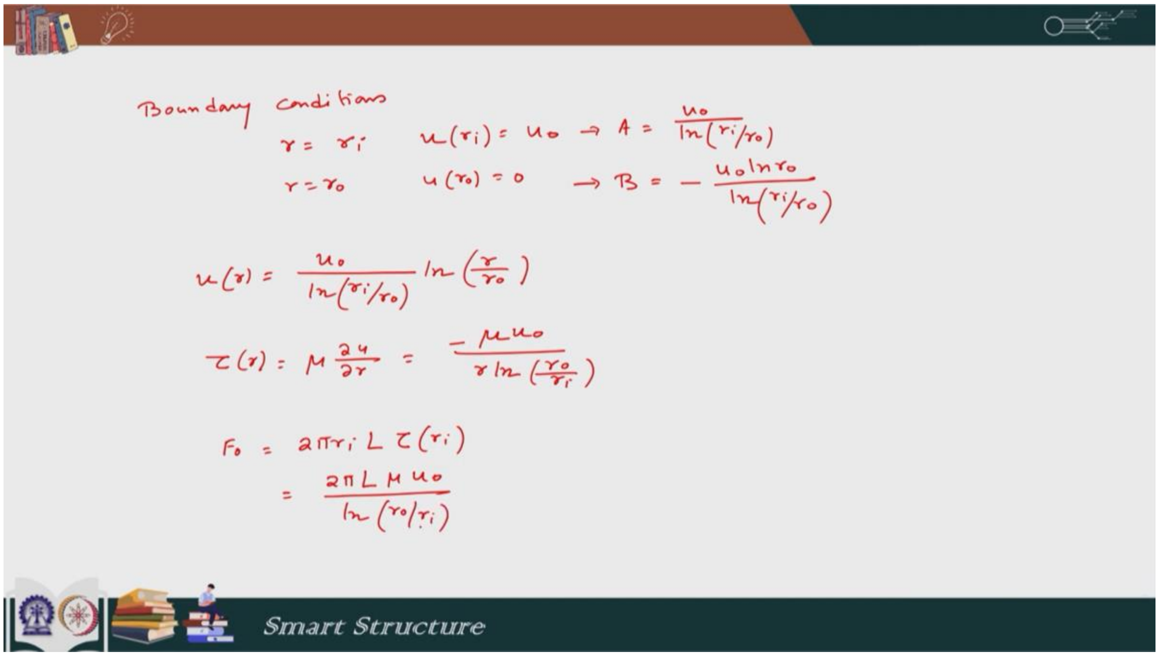

And then, with this, we can find out the expression for the active damping, the damping coefficient. So, the damping coefficient becomes F0 is equal to c-equivalent into u0.

$$F_0 = c_{e\dot{q}} u_0$$

That is the force and the damping coefficient is F0 divided by u0. And finally, the expression is capital gamma into u where capital gamma is 2 pi L divided by the natural log of r0 by ri.

$$c_{e\dot{q}} = \frac{F_0}{u_0} = \Gamma u$$

$$\Gamma = \frac{2\pi L}{\ln(r_0/r_i)}$$

So, that was for the inactive case when I have zero applied field, so yield stress is 0.

Now, we will consider a non-zero applied field. Now, in the non-zero applied field, we have the fluid constitutive relation is - tau i into, we know is the sin of gamma dot plus mu into gamma dot.

$$\tau = \tau_y \, sgn(\dot{r}) + \mu\dot{\gamma}$$

So, when the gamma dot is negative, it is minus tau. When the gamma dot is positive, it is plus tau i. So, it is tau equal to tau y plus mu into gamma dot, when gamma dot is positive and minus tau i plus mu into gamma dot, when gamma dot is negative.

$$\tau = \begin{matrix} \tau_y + \mu\dot{\gamma} & \dot{\gamma} > 0 \\ -\tau_y + \mu\dot{\gamma} & \dot{\gamma} < 0 \end{matrix}$$

Now, if I put this constitutive relation, then my governing differential equation becomes this. This is equal to tau y by r, and again, this can be written as partial of r into del u by del r with respect to r is equal to tau y by r.

$$\mu\frac{\partial^2 u}{\partial r^2} + \frac{\mu}{r}\frac{\partial u}{\partial r} = \frac{\tau_y}{r}$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) = \frac{\tau_y}{r}$$

And then finally, on being integrated, this gives me u as a function of r as tau y divided by mu into r plus C natural log of r plus D.

$$u(r) = \frac{\tau_y}{\mu}r + C\ln r + D$$

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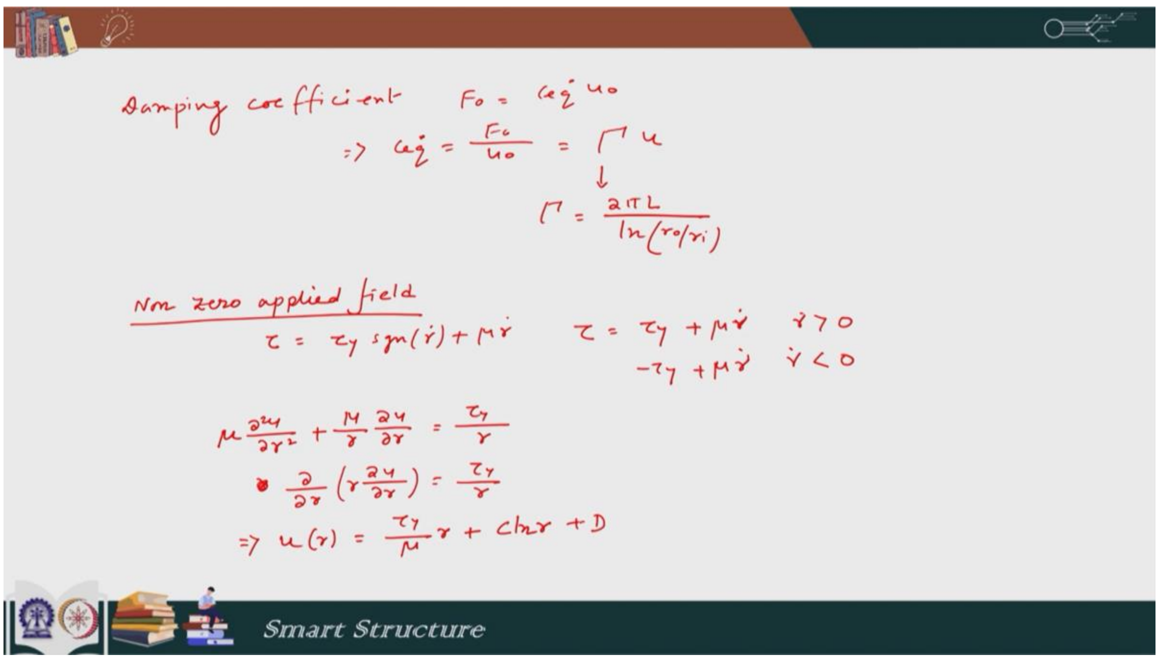

And then, we can evaluate the constants applying the same boundary conditions that at u is equal to ri, I mean, at r is equal to ri u is u0 and at r is equal to ro outer boundary u is 0. So, these boundary conditions give me that - applying the boundary conditions, applying above boundary conditions, we get C equal to minus 1 by the natural log of ro by ri multiplied by u0 plus tau I divided by mu multiplied by r outer minus r inner. And then, we have D is equal to u0 divided by the natural log of r0 by ri plus tau y by mu, and this is multiplied with minus r0 plus r minus ri divided by the natural log of r0 by ri. And then after putting that in the expression of u, we get u as a function of r as minus tau y by mu multiplied by ro minus r plus mu 0 plus tau i by mu multiplied by ro minus ri divided by natural log of r outer by r inner multiplied by the natural log of r0 by r.

$$u(r_i) = u_0 \qquad u(r_0) = 0$$

$$C = -\frac{1}{\ln(r_0/r_i)}\left[u_0 + \frac{\tau_y}{\mu}(r_0 - r_i)\right]$$

$$D = \frac{u_0}{\ln(r_0/r_i)} + \frac{\tau_y}{\mu}\left[-r_0 + \frac{(r_0 - r_i)}{\ln(r_0/r_i)}\right]$$

$$u(r) = -\frac{\tau_y}{\mu}(r_0 - r) + \frac{\mu_0 + \frac{\tau_y}{\mu}(r_0 - r_i)}{\ln\left(\frac{r_0}{r_i}\right)}\ln\left(\frac{r_0}{r}\right)$$

Now, from here, we can find out tau as a function of r as minus of mu divided by r multiplied by the natural log of r zero by ri into u zero plus tau y by mu multiplied by ro minus ri, and that finally can be written as minus mu into u0 divided by r into natural log of r0 by ri into 1 plus Bi and where we know that Bi is the Bingham number.

$$\tau(r) = -\frac{\mu_0}{r\ln\left(\frac{r_0}{r_i}\right)}\left[u_0 + \frac{\tau_y}{\mu}(r_0 - r_i)\right] = -\frac{\mu\mu_0}{r\ln\left(\frac{r_0}{r_i}\right)}(1 + \mathrm{Bi})$$

So, Bi is the Bingham number, and that Bingham number is the ratio of the yield stress by the viscous stress. So, this is tau y multiplied by d by mu u0. So, tau y is the yield stress, and mu u0 by d also has a dimension of stress, and that is the viscous stress.

$$\mathrm{Bi} = \frac{\tau_y d}{\mu u_0}$$

So, yield stress by viscous stress is our Bi, a Bingham number, and that is how we express our shear stress tau.

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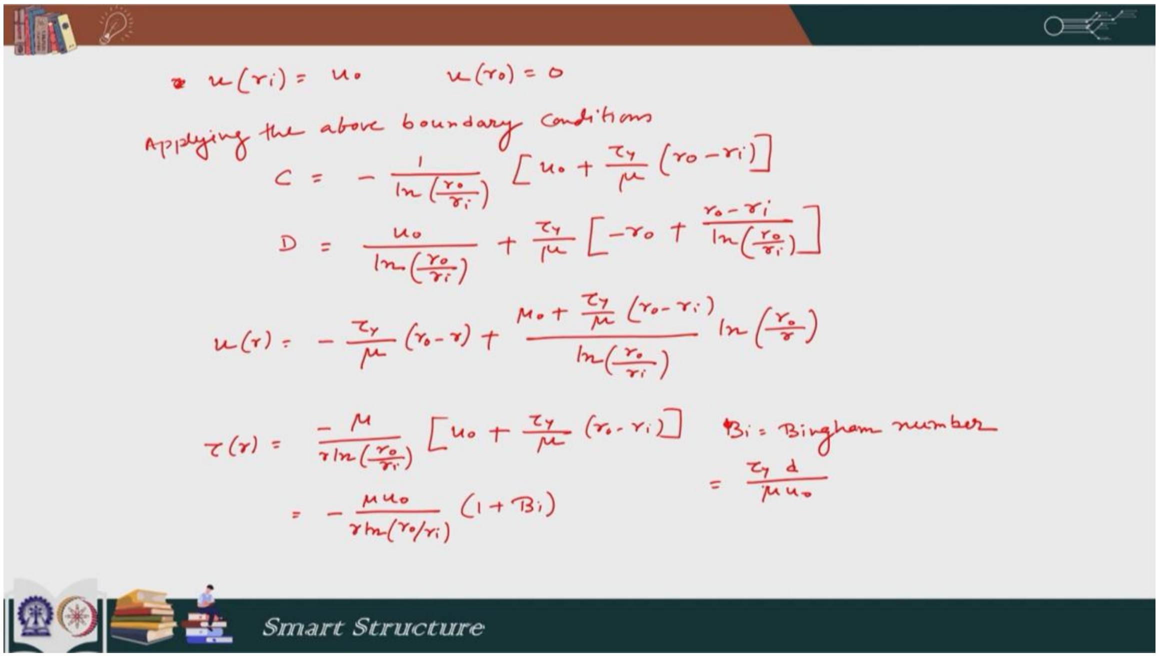

And then, from here again, we can find out the force F0 as 2 pi r0 multiplied by length into tau at ri, and that gives me this as mu into 1 plus Bi into capital gamma u0, and we have already defined capital gamma.

$$F_0 = 2\pi r_0 L\tau(r_i) = \mu(1 + \text{Bi})\Gamma u_0$$

So, this quantity helps us finding out the active damping coefficient, and we can write that as F0 by u0 and that we get as mu into 1 plus Bi into capital gamma, and then if we do c equivalent of the active case by the inactive case, we get this as 1 plus Bi because mu capital gamma was c equivalent 0, I mean, c equivalent for the inactive case.

$$c_{eq}^{a} = \frac{F_0}{u_0} = \mu(1 + \text{Bi})\Gamma$$

$$\frac{c_{eq}^{a}}{c_{eq}^{0}} = (1 + \text{Bi})$$

So, that is how we analyze it for the annular passage. So, we can see that the method is same only due to the geometry we have to be careful. So, instead of working on x and y, we are working on x and r, and accordingly, everything falls into place.

Now, this same procedure can be extended to the analysis of the flow mode or the valve mode.

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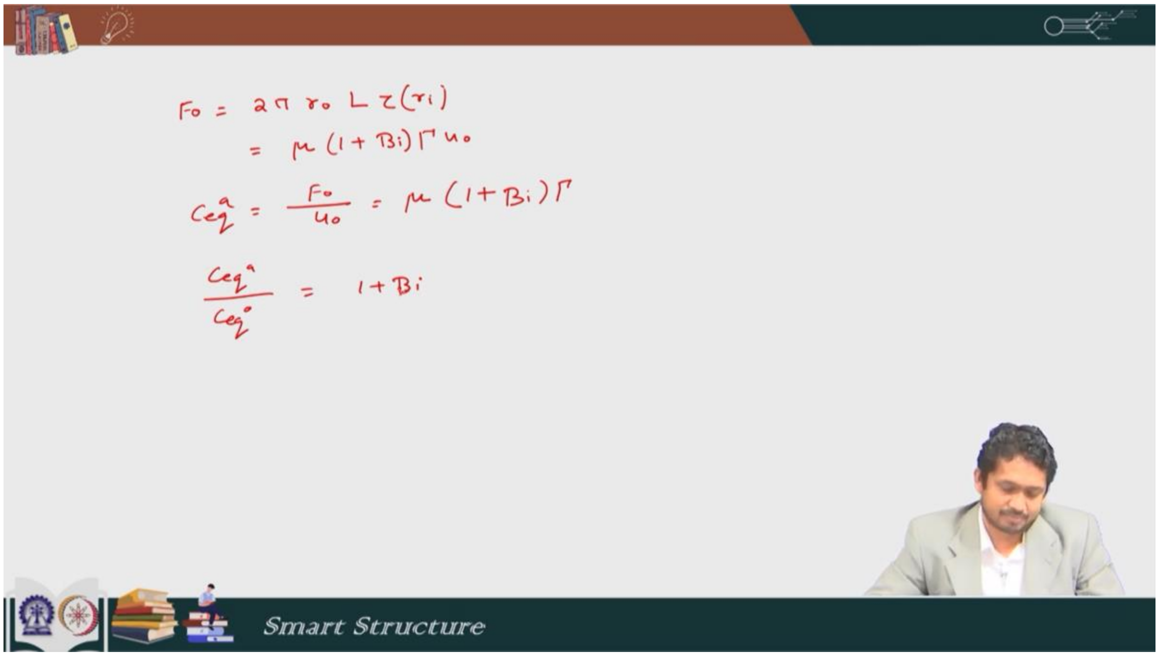

Now, in the next week, we will again look into flow mode analysis and valve mode analysis, considering again rectangular case, but using different constitutive relations.

So, this brings us to the end of this lecture and this week.

Thank you.