Smart Structures
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Week 11
Lecture No: 56
Analysis of Electro and Magneto Rheological Fluid Flow (continued)
Part 03

So, today we will continue our discussion on flow mode analysis of electro and magnetorheological fluids.

In the last lecture, we did the analysis considering the fact that there is no applied electric or magnetic field. And today, we will discuss about what happens when there is a magnetic field or electric field applied like what is shown here. And in the last class towards the end, we saw that because the shear stress is close to 0 near the mid zone. So, there is always a region here, where I have no strain rate, shear strain rate. So, this part, the thickness of which is delta that behaves like a solid. So, it just moves as it is without any shear strain rate. Now, the other two part, the above this region of delta thickness that part has shear strain rate and the below part also has a shear strain rate. So, we divide this entire zone into three parts maybe - zone 1, zone 2 and zone 3. Now, because of the symmetric nature of the problem, this zone 2 is placed symmetrically. So, based on the dimensions, we can say that the thickness of zone 1 is d minus delta by 2 and thickness of zone 3 is also d minus delta by 2. And that tells me that the value of y_1 is d minus delta by 2 and accordingly I can find the value of y_2 difference i.e., between this point and this point and that is d plus delta by 2.

$$y_1 = \frac{d-\delta}{2} \qquad y_2 = \frac{d+\delta}{2}$$

So, we have discussed the boundary conditions also. At these two plates, we have no slip boundary condition and at the two ends of this mid-region, we have strain rate as 0. Now, based on this boundary conditions, we can analyze the flow. Now, we have three regions and we have to look at the behavior separately.

So, let us look into region 1. In region 1, again our governing differential equation is - mu del 2 u by del y 2 is equal to minus of del p by L.

$$\mu \frac{\partial^2 u}{\partial y^2} = -\frac{\Delta p}{L}$$

And then if we integrate that, if we integrate that, that gives me u_1 of y. If we just integrate twice, that gives us two constants and finally, it comes as C_2.

$$u_1(y) = -\frac{\Delta p}{2\mu L} y^2 + C_1 y + C_2$$

So, there are two constants and these two constants. And these two constants can be evaluated by putting the condition that at y equal to 0, u is 0, and at y equal to 1, del u by del y is 0. So, at y equal to 0 we have u_1 equal to 0, and that tells me that C_2 is equal to 0. And then at y equal to y_1, we have del u_1 by del y which is evaluated at y equal to y_1 is equal to 0. That tells me that C_1 equal to delta p, y_1 by mu L.

$$at\ y = 0, u_1(0) = 0 \qquad \Rightarrow C_2 = 0$$

$$at\ y = y_1, \left.\frac{\partial u_1}{\partial y}\right|_{y=y_1} = 0 \qquad \Rightarrow C_1 = \frac{\Delta p y_1}{\mu L}$$

So, just to show it, we have del u by del y equal to minus delta p, L y by mu plus C_1. So, in this expression, when I put y equal to y_1 and the left-hand side as 0, C_1 comes out.

$$\frac{\partial u}{\partial y} = -\frac{\Delta p}{\mu L} y + C_1$$

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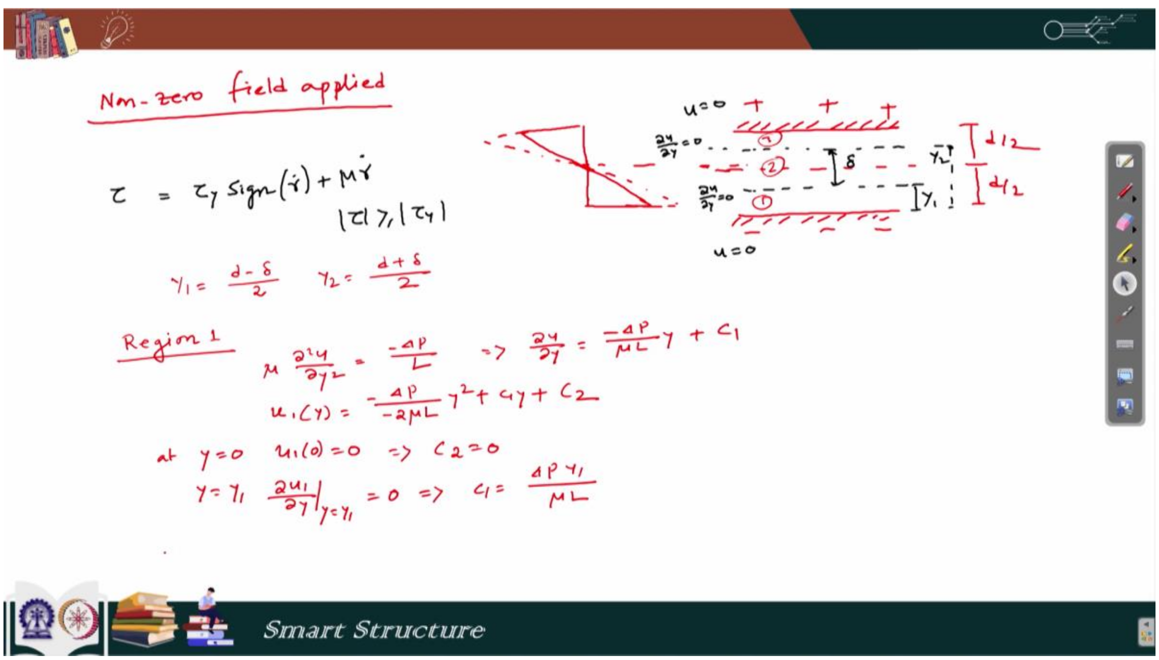

So, that shows me a profile of the velocity along the thickness in region 1.

So, we can write that as u_1 as a function of y becomes minus del p by 2 mu L, y square plus del p y_1 by mu L, y. And then, the same thing can be represented as - del p by 2 mu L multiplied by y into d minus delta minus y. Because we know, y_1 in terms of d and delta or this is also del p by 2 mu L multiplied by y into 2 y_1 minus y. So, y_1 is d minus delta by 2. So, it is 2y.

$$u_1(y) = -\frac{\Delta p}{2\mu L} y^2 + \frac{\Delta p y_1}{\mu L} y$$

$$= \frac{\Delta p}{2\mu L} y(d - \delta - y)$$

$$= \frac{\Delta p}{2\mu L} y(2y_1 - y)$$

Then, now let us go to region 3. In region 3, we can just say that the flow profile would look symmetric, but still, we can derive that. So, in region 3, if we denote the velocity as u_3. So, u_3, y becomes minus del p by 2 mu L, y square plus C_3 y plus C_4 or we can say del u_3 by del y, we are not evaluating at anywhere, we are just writing the expression. So, del u_3 by del y is equal to minus delta p by mu L y plus C_3.

$$u_3(y) = -\frac{\Delta p}{2\mu L} y^2 + C_3 y + C_4$$

$$\frac{\partial u_3}{\partial y} = -\frac{\Delta p}{\mu L} y + C_3$$

Now again, we would do the same set of exercises to find out C_3 and C_4. So, here we apply the condition that strain rate del u_3 by del y evaluated at y equal to y_2 is 0 and that tells me that C_3 is equal to del p y_2 by mu L. And then we can put the other boundary condition which is u_3 which is evaluated at d and that is - if we do it, we get the expression like this minus del p by 2 mu L into d square plus del p by mu L into d y_2 plus C_4 equal to 0 and that helps me finding out C_4 as del p by 2 mu L d into d minus 2 by 2.

$$\left.\frac{\partial u_3}{\partial y}\right|_{y=y_2} = 0 \qquad \Rightarrow C_3 = \frac{\Delta p y_2}{\mu L}$$

$$u_3(d) = -\frac{\Delta p}{2\mu L} d^2 + \frac{\Delta p}{\mu L} d y_2 + C_4 = 0 \qquad \Rightarrow C_4 = \frac{\Delta p}{2\mu L} d(d - 2y_2)$$

So, if we plot the velocity profile, we would see that the value is 0 here and it goes up and get some value and again the value is 0 here and if y reduces it gets the same value at y equal to y_2. Now, we have to just ensure that if we evaluate u_1 at y equal to y_1, we get the same velocity when we evaluate u_3 at y equal to y_2.

So, before doing that let us write down the final expression for u_3 after applying all the constants and the expression becomes minus del p by 2 mu L y square plus del p by mu L y y_2 plus del p by 2 mu L d square minus 2 d y_2 and on simplifying, the expression comes as del p by 2 mu L multiplied by d minus y multiplied by y minus delta.

$$u_3(y) = -\frac{\Delta p}{2\mu L}y^2 + \frac{\Delta p}{\mu L}yy_2 + \frac{\Delta p}{2\mu L}(d^2 - 2dy)$$

$$= \frac{\Delta p}{2\mu L}(d-y)(y-\delta)$$

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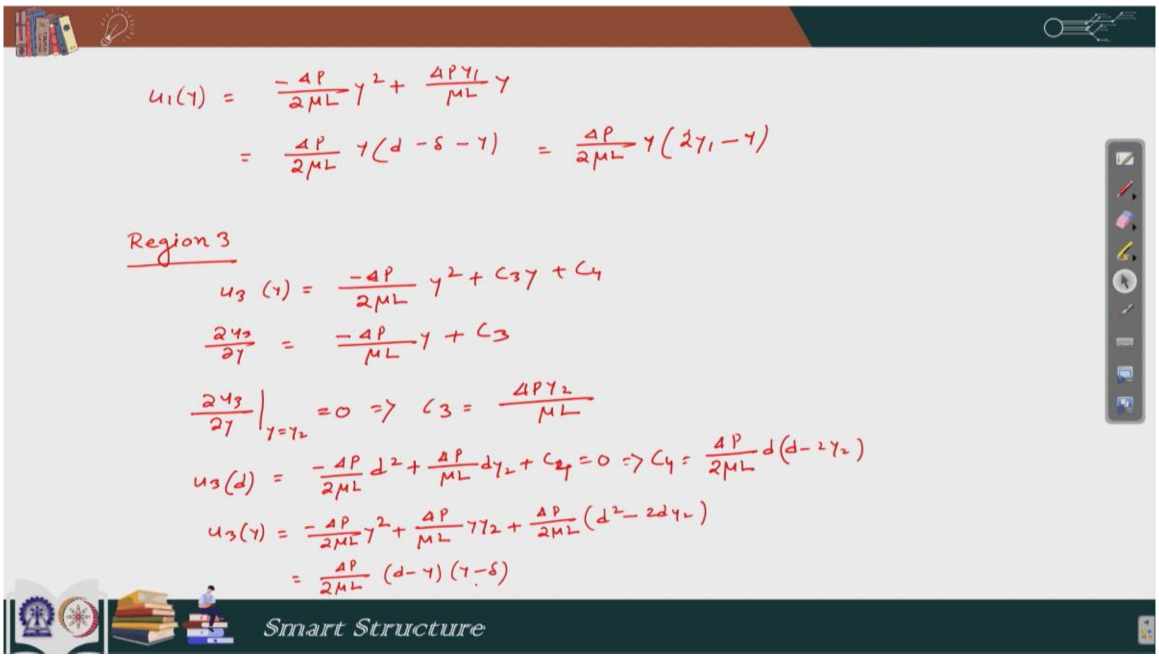

Now, let us evaluate those velocities at y equal to y_1 considering u_1, and at y equal to y_3 considering u_2. So, that would give me the velocity at which the region 2 travels. So, region 2, see if I evaluate u_1 at y_1, that comes as delta p by 2 mu L multiplied by y square and this again can be written as delta p by 8 mu L multiplied by d minus delta square.

$$u_1(y_1) = \frac{\Delta p}{2\mu L}y_1^2 = \frac{\Delta p}{8\mu L}(d-\delta)^2$$

Similarly, u_3 at y_2, if we evaluate that comes as delta p by 2 mu L multiplied by d minus y_2, multiplied by y_2 minus delta, and that is delta p by 8 mu L multiplied by d minus delta square.

$$u_3(y_2) = \frac{\Delta p}{2\mu L}(d-y_2)(y_2-\delta) = \frac{\Delta p}{8\mu L}(d-\delta)^2$$

So, that shows us that u_1 at y_1 is equal to u_3 at y_3 is equal to some velocity we call it u_p.

$$u_1(y_1) = u_3(y_2) = u_p$$

So, now, if we plot the velocity profile and this is our region with thickness delta. So, the velocity is 0 and 0 here and the velocity increases and gets a value u_p. And here also the velocity increases and the value are u_p and this u_p is maintained in the region, in the mid region, in region 2. So, region 1, region 2, region 3.

Now, this constant velocity u_p, u_1 u_3 equal to 0. Now, this constant velocity up is the velocity at which the mid region travels and that is called a plug region. So, this region as we said there is no velocity gradient. So, that travels as a solid with one velocity. So, that is called the plug region. So, we can call it region 2 is known as plug region. Now, that we know the velocity of the plug region, we have to find out the thickness delta of this region. Now, to do that, first we solve the governing differential once again in terms of shear stress and that helps us doing that.

Now, we know that we have del tau_2 by del y is equal to minus del p by L, that is the differential equation that in terms of the shear and del 2 is the shear stress in region 2. So, if we solve the equation that gives me tau_2 as a function of y is minus del p by L into y plus C_5.

$$\frac{\partial \tau_2}{\partial y} = -\frac{\Delta p}{L} \qquad \Rightarrow \tau_2(y) = -\frac{\Delta p}{L} y + C_5$$

Now, we have to evaluate C_5 first. So, for that again we apply the boundary conditions. If we have evaluated this del 2 at y equal to y_1, it should be equal to the yield stress, because immediately below this y equal to y_1. The stress is more than the yield stress. So, at here, the stress is yielding stress, equal to yield stress and here the stress is equal to the negative of the yield stress because here we can see that the velocity gradient is positive, here the velocity gradient is negative.

So, here in this region tau_2, we can say tau_2 y_2 equal to minus of tauy and here we can say tau_2 y_1 equal to tau_y.

$$\tau_2(y_1) = -\tau_y \qquad \text{and} \qquad \tau_2(y_1) = \tau_y$$

So, we have to put these two boundary conditions. If we do that, we can write that at y equal to y_1, we have tau_2 equal to minus delta p by L into y_1 plus C_5 and we also have at y equal to y_2, all right. So, it is not this value is, now tau_y, the yield stress. So, it is tau_y, and here we have minus of tau_y is equal to minus of delta p by L into y_2 plus C_5.

$$\text{at } y = y_1 \qquad \tau_y = -\frac{\Delta p}{L} y_1 + C_5$$

$$y = y_2 \qquad - \tau_y = -\frac{\Delta p}{L} y_2 + C_5$$

So, if we add these two equations, then this gives me C_5 as delta p by 2 L multiplied by y_1 plus y_2.

$$C_5 = \frac{\Delta p}{2L}(y_1 + y_2)$$

If we just add these two equations, then tau_y cancel each other and we have two C_5 here, and finally, solving that we get this. And again, that tells me that if we put that same value then tau_2 y. So, if we put that same value of C_5 here, then we get tau_2 y is equal to minus del p by L into y, plus we get - so, we get tau_2 of y is equal to minus del p by L into y plus del p by 2 L multiplied by y_1 plus y_2 and we know that y_1 plus y_2 is d, because our y_1 is d minus delta by 2 and y_2 is d plus delta by 2 if we add them, we get d. So, we have d. So, again this expression can be written as del p by 2 L into d minus 2 y.

$$\tau_2(y) = -\frac{\Delta p}{L} y + \frac{\Delta p}{2L} d = \frac{\Delta p}{2L}(d - 2y)$$

So, that is the expression of shear strain in the region, no, shear stress in the region, in the plug region.

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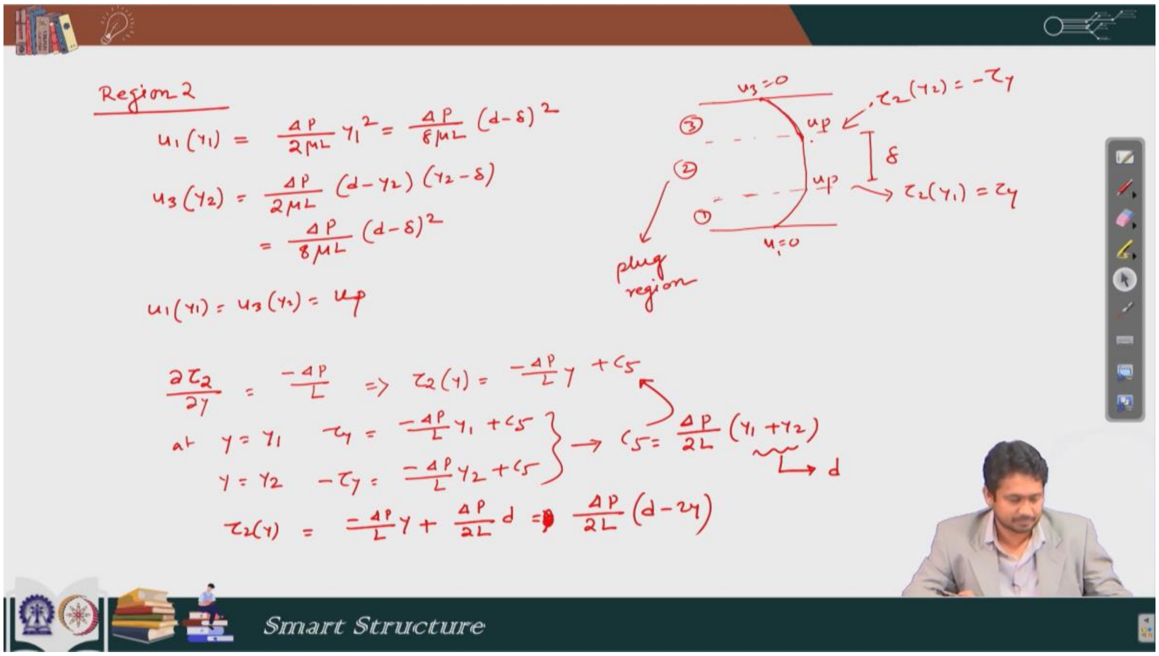

Next, we have to find out delta. So, for that what we can do is - we can put the value of this τ_2 y at any of y maybe y_1 or y_3 at any of the boundaries and from there we can find the value of either y_1 or y_2. If we know y_1 or y_2 we can find out delta.

So, let us write at y equal to y_1. We can write, minus del p by L plus del p multiplied by d by 2 L equal to τ_y and from here. So, here it is evaluating y equal to y_1. So, y_1 should be there. And finally, after simplifying, we know that this quantity y_1 is d minus delta by 2. So, finally, after solving we can find out delta, the plug thickness and that comes as τ_y multiplied by 2 L divided by delta p.

$$\text{at } y = y_1 \qquad -\frac{\Delta p}{L} y_1 + \frac{\Delta p}{2L} d = \tau_y$$

$$\Rightarrow \delta = \tau_y \frac{2L}{\Delta p}$$

Then we can define a non-dimensional quantity del by d. So, the plug thickness divided by the total thickness of the region and that comes as - τ_y multiplied by 2 L divided by delta p d. Let us call that delta bar. So, that is a non-dimensional quantity.

$$\bar{\delta} = \frac{\delta}{d} = \frac{\tau_y 2L}{\Delta p d}$$

Now, from here, we can see that when this non-dimensional quantity is 1, that means, the entire region, there is no flow, that means, the entire region is in the pre-yield region. In other words, we can say that, if τ_y is greater than delta p d by 2 L, when τ_y is greater than delta p by 2 L. So, that denotes that delta is 1, which means that in that yield stress there is no flow possible, that means, if have given the pressure difference and then if I want to close the flow by the valve action, then I have to put an electric or magnetic field in such a way that we at least get this amount of τ_y and that that blocks the entire flow.

$$If\ \tau_y \geq \frac{\Delta p d}{2L} \qquad \Rightarrow \bar{\delta} = 1$$

Now, we have to find out the equivalent damping and for finding out the equivalent damping, we would proceed with the same. And for finding out the equivalent damping, we proceed with the same approach. So, we will assume that there is an equivalent velocity u_m may be and we will find out the flow corresponding to that in terms of u_n, and then we will find out the actual flow, and then we equate both the terms and that would give me u_m.

Now, if we draw it once again, we have these two plates x y z and this is our d. We say that the dimension in the z direction is b. So, if the equivalent velocity is u_m, then flow rate is equal to u_m multiplied by b multiplied by d.

$$\text{flow rate} = u_m bd$$

We have to now equate that with the actual flow. So, the flow actual is Q, which is equal to 2 Q_1 plus Q_2.

$$Q = 2Q_1 + Q_2$$

Again, there are three regions region 1, region 2, region 3. So, the flow in region 1 is Q_1, flow in region 1 is Q_3 and if I add those two up, I get the summation as 2 Q_1, because Q_1 and Q_3 are same because these two regions are symmetric.

$$2Q_1 = Q_1 + Q_3$$

So, Q_1 and Q_3 are same. So, these two regions give me a flow of 2 Q_1, and Q_2 is different that gives me a flow of Q_2 and their total is the total actual flow. So, to find this out, we integrate of integration from 0 to y_1 and then we multiply here y_1 of y and then we multiply that quantity by b and then we integrate from 0 to y_1. And then, for Q_2, I just need u_m, sorry, not u_m that is the u_p, the plug flow here, plug velocity u_p multiplied by the cross-section dimension that is delta multiplied by b.

$$Q = 2\int_0^{y_1} u_1(y)bdy + u_p \delta b$$

So, if we evaluate this quantity, finally the quantity becomes delta p b by 12 mu L multiplied by d minus delta cube plus we have delta p b by 8 mu L multiplied by d minus delta square delta. So, that is the total flow Q. And this should be equated with mu m b d and then once we do that we get, sorry, it is not mu m, it is u_m.

$$Q = \frac{\Delta p}{12\mu L}(d-\delta)^3 + \frac{\Delta pb}{8\mu L}(d-\delta)^2\delta = u_m bd$$

So, once we do that, we get u_m as delta p d square 12 mu L multiplied by 1 minus delta bar square, delta bar is this quantity the non-dimensionalized plug thickness multiplied by 1 plus delta bar by 2.

$$u_m = \frac{\Delta pd^2}{12\mu L}(1-\bar{\delta})^2\left(1+\frac{\bar{\delta}}{2}\right)$$

So, that is the equivalent flow throughout the passage.

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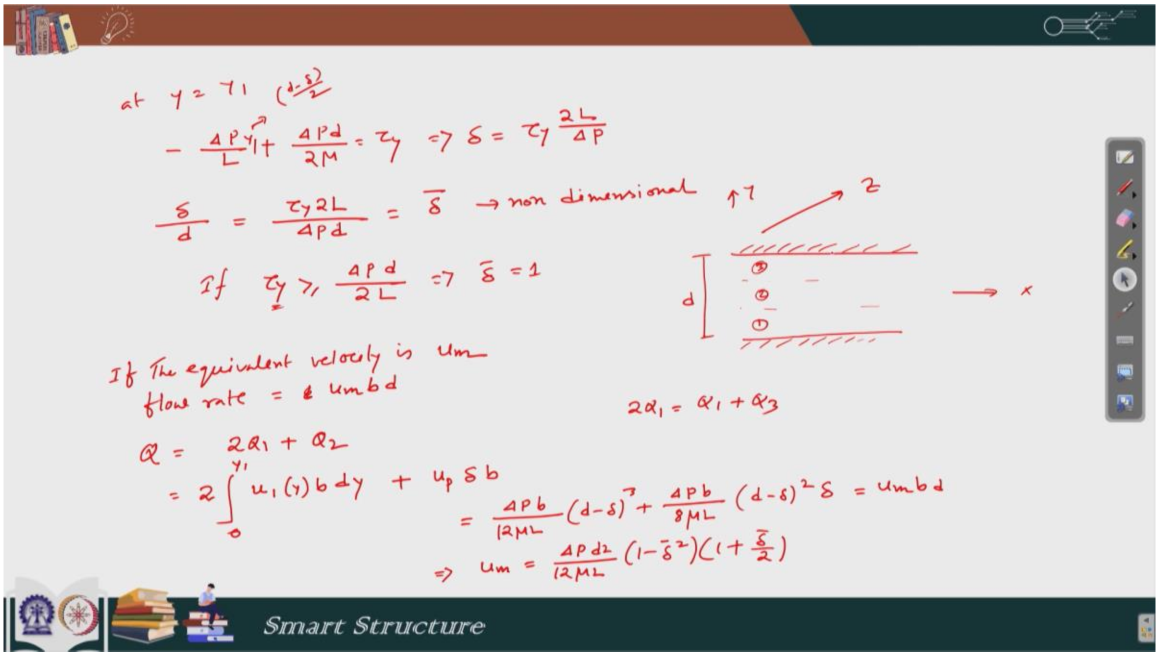

Now, that we have got the equivalent flow we can find out the damping coefficient. So, the total force is delta p b d, pressure multiplied by the area and here we are using the suffix a to denote that it is in active mode.

$$F_a = \Delta pbd$$

So, it is under electrical mechanical field and then if we divide this, F_a by the equivalent velocity u_m, that gives me the corresponding damping. So, by doing that the final expression of C equivalent comes as 12 mu L b d divided by d square multiplied by 1 minus delta bar multiplied by 1 plus delta bar by 2.

$$C_{eq}^a = \frac{F_a}{u_m} = \frac{12\mu Lbd}{d^2\left(1-\bar{\delta}\right)^2\left(1+\frac{\bar{\delta}}{2}\right)}$$

Now, we know that C equivalent for the inactive case that means, when we did not have the electric field applied, was C equivalent 0 equal to 12 mu L b by d.

$$C_{eq}^0 = \frac{12\mu Lb}{d}$$

So, this is the damping coefficient under no field, and this is damping coefficient under field, which we can call as active mode. So, here we can see that when we have this delta bar equal to 1, that means, our yield stress is sufficiently high to block the flow. In that case, our active damping coefficient is going towards an infinite value. So, with those we can represent C_{eq} active in terms of C_{eq} 0 and the expression is C equivalent 0 divided by 1 minus delta bar whole square multiplied by 1 plus delta bar by 2.

$$C_{eq}^{a} = \frac{C_{eq}^{0}}{\left(1-\bar{\delta}\right)^{2}\left(1+\frac{\bar{\delta}}{2}\right)}$$

Now, let us define a number called Bingham number, and the Bingham number is defined as Bi, which is equal to tau_y multiplied by d divided by mu u_m. So, tau_y is the yield stress and we can see that this quantity mu u_m by d that is also the dimension of stress and that is viscous stress. So, it is basically a ratio of yield stress and viscous stress and that quantity finally, comes to be tau_y d by mu 12 mu L divided by delta p d square multiplied by 1 minus delta bar whole square multiplied by 1 plus delta bar divided by 2. And that can be written as 6 into delta bar divided by 1 minus delta bar whole square multiplied by 1 plus delta bar by 2.

$$Bi = \frac{\tau_y d}{\mu u_m} = \frac{\tau_y d}{\mu} \frac{12\mu L}{\Delta p d^2 \left(1-\bar{\delta}\right)^{2}\left(1+\frac{\bar{\delta}}{2}\right)}$$

And the ratio of the active damping coefficient divided by the damping coefficient under no field can also be written in terms of Bingham number as Bi divided by 6 delta bar.

$$\frac{C_{eq}^{a}}{C_{eq}^{0}} = \frac{Bi}{6\bar{\delta}}$$

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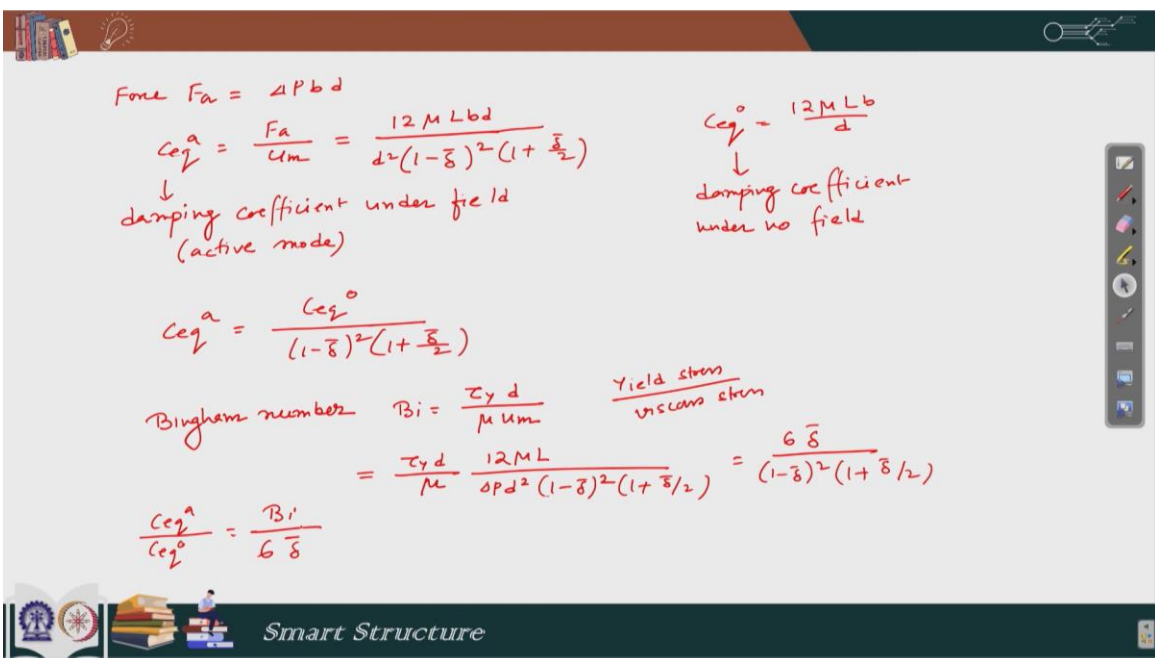

So, this was about the flow in the valve mode or flow mode. So, we have seen three different regions. We have to treat them separately and they have some conditions in between them which we have to satisfy. And finally, after satisfying the condition we get the velocity profile. And then, after getting the velocity profile we apply the same approach. We find out a equivalent velocity. And from here, we found out equivalent damping. And then we see how the equivalent damping coefficient changes when there is some electric field applied as compared to when there is no electric field.

So, with that I would like to conclude this lecture here.

Thank you.