Smart Structures
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Week - 11
Lecture No - 55
Analysis of Electro and Magneto Rheological Fluid Flow

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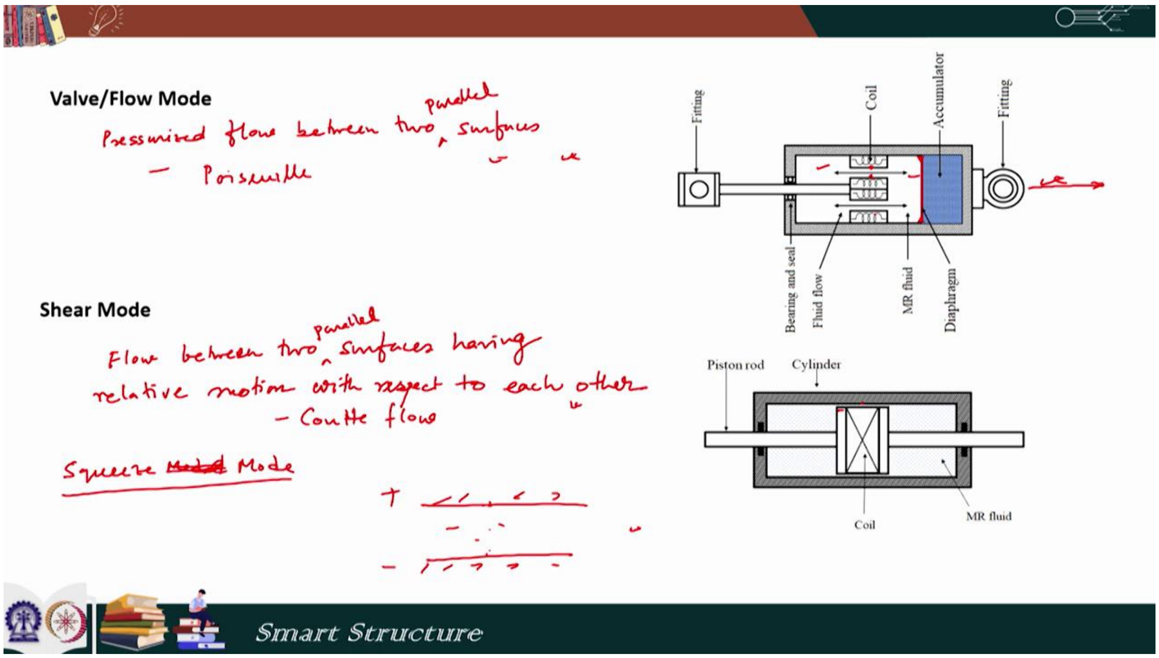

Today, we will continue our discussion on ER and MR fluids. So, ER and MR fluids are used in two main different modes, one is valve mode and another is shear mode. So, valve mode involves. It is a pressurized flow between two surfaces we can say. So, here is a device that uses the fluid in valve mode. Now, the pressure at these two sides can be different and that pressure gradient causes a flow in between these two surfaces and that is what valve mode is.

So, here we can say that these two surfaces do not have any relative motion, where the fluid has a relative motion with respect to these two surfaces. And shear mode is a different mode. Here we can see that this piston moves and accordingly this surface moves and this surface remains constant. So, it is a flow between two surfaces having relative motion with respect to each other.

Now, these surfaces are first of all they are parallel surfaces. So, we must use the word parallel. They are all parallel surfaces. Now, this corresponds to a flow named Poiseuille Flow and this corresponds to Couette flow. So, here it is a pressure mode, here it is a shear mode and apart from that there is also another kind of mode possible that is called squeeze mode, where the flow happens between two plates which have movement along each other.

So, there is a squeeze mode also. In squeeze mode again we have two surfaces and if the field is along this direction the the plate also have relative motion along this direction. So, maybe this plate keeps coming closer to this. So, the fluid layer between them is squeezed. However, in this mode also the lateral flow is often considered to be similar to the first case and this mode is less common as compared to the first two modes.

Now, we will discuss these two modes in more details. So, we will consider a rectangular flow passage between two plates and analyze the flow. Now, in most of the applications, the flow passage is annular. For example, if I look at this then in 3D, the actual view looks like the rotated version of it with respect to this axis. So, it is a cylindrical and there is a cylindrical annulars space between these two plates and that is where the flow takes place.

Same thing happens here as well as here also. So, these are all annular places where the flow takes place. However, we will do it for rectangular flow passage because that is simpler to understand and once we understand the rectangular case, its extension to the cylindrical annular case is much straightforward.

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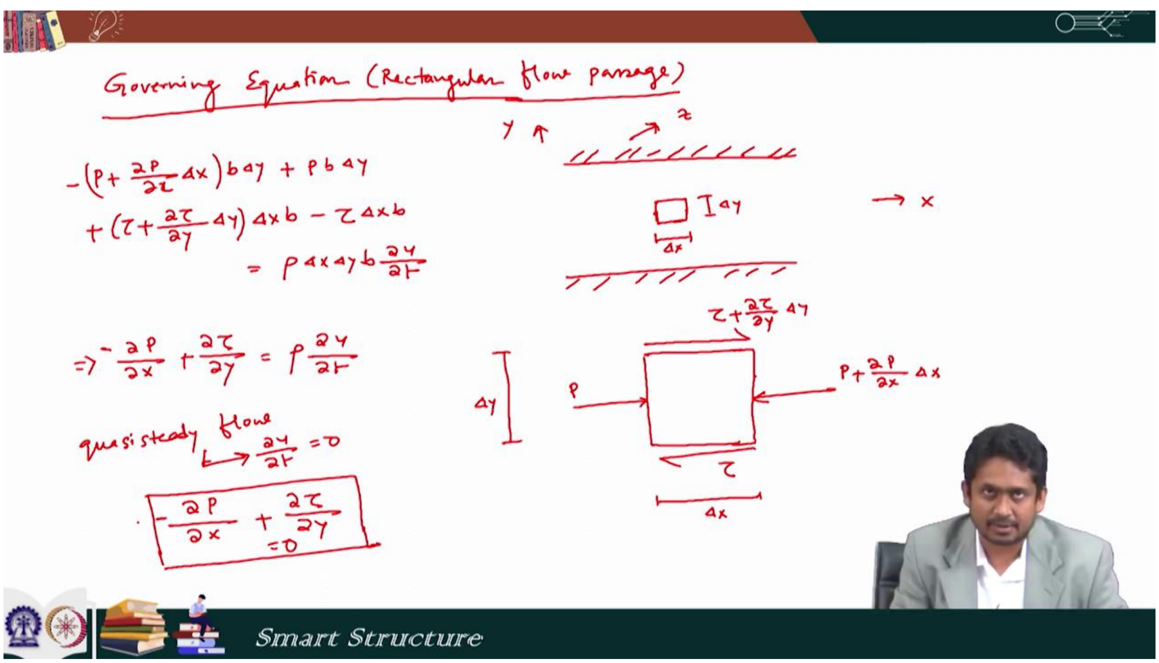

So, let us now understand the flow. Before going to any of these modes, let us define the governing differential equations for that because these two modes generally differ by the corresponding boundary conditions.

So, let us define the governing equations. So, it is a rectangular flow area. We can consider here that we have two parallel plates between which the fluid is flowing and let us consider a small area from it. So, we can consider this as x axis, this as y and this as z. Now we are looking at the x y plane. The dimension along z can be considered to be b. So, this entire flow passage has dimension b along the z axis. Now let us look a small part of it of size del x del y. So, this is del x, this is del y. So, let us call it P.

The pressure here is del P plus del P by del x multiplied by del x and there is velocity gradient along the y axis and that gives rise to shear stress. So, the shear stress here we can write as tau plus del tau by del y multiplied by delta of y and here it is delta. Now all we do is, we write the equilibrium equation of it and let us say that this small fluid element has a velocity u and it is as an acceleration del u by del t. So, if I write the equation of equilibrium if I balance all the forces and equate that to mass into acceleration it looks like this. P plus del P by del x multiplied by delta x.

So, that is the pressure here and the pressure multiplied by the area is the force. The area of this surface is this height which is delta y multiplied by b that is the dimension along the z axis. So, it is b multiplied by delta y then minus here we have P multiplied by the area. So, it is P multiplied by b multiplied by delta y. The force corresponding to this stress is this stress multiplied by this dimension which is delta x multiplied by b.

So, it is tau plus del tau by del y multiplied by delta tau y into delta x into b and here we have tau multiplied by delta x into b and that is equal to mass into acceleration. So, if the density of the fluid is rho, the entire volume is delta x delta y into b that is the mass of it and into acceleration means del u by del t. So, that is the entire balance equation and then if we cancel the common terms, now here this has to be a positive sign because this pressure has a direction in the positive x axis. So, after cancelling the equal terms what we get is del P by del x plus del tau by del y is equal to rho into del u by del t. Now we can say that this flow is a steady flow or say quasi steady flow.

In that case, del u by del t is 0. So, quasi steady flow this tells me that del u by del t is 0 if I make that approximation finally, the governing differential equation that we get is del P by del x plus del tau by del y equal to 0. So, that is our governing differential equation. Now here depending on the flow type whether it is a valve mode or shear mode we have to put the appropriate boundary condition and depending on whether there is an electric field applied or not accordingly, this fluid constitutive relation will change and the solution would differ. Now let us look into the specific cases.

Before going there, one thing to note is this has a negative sign and this has a positive sign. So, finally, this is a negative quantity and the constitutive relation is modified as this. So, I have to put a negative sign here. So, this is our governing differential equation.

$$-\frac{\partial P}{\partial x}+\frac{\partial \tau}{\partial y}=\rho\frac{\partial u}{\partial t}$$

$$-\frac{\partial P}{\partial x}+\frac{\partial \tau}{\partial y}=0$$

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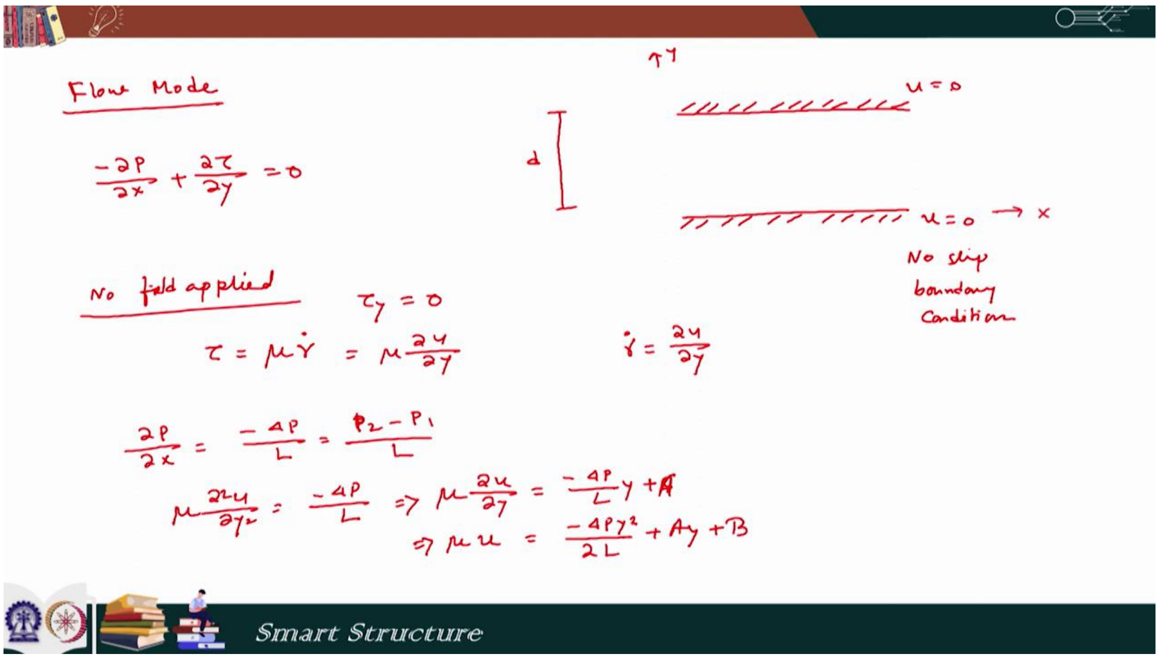

Now let us go to special cases. So, there is something called a flow mode or the valve mode. In the flow mode, our consideration is this. We have two plates which are separated by a distance d these are x axis, this is y and again the dimension along z axis is b and in between these two domains, the fluid flow follows the governing differential equation that we derived minus del P by del x plus del tau by del y equal to 0. Now let us come to the boundary condition. Here these two plates are fixed.

So, here the boundary condition is u equal to 0 and here is also u equal to 0 and that is called a no slip boundary condition. Now first we would analyze it for a case when there is no actuation is there which means there is no yield. There is no question of any yield happening because tau y is 0. There is no electric field applied here. Now in this case, our tau is mu multiplied by gamma dot and we are representing the velocity along x direction as u.

So, if the velocity along x direction is u, the gamma dot is del u by del y. Now in the solid case, we are denoting generally the displacement as u. So, del u by del y would have been a gamma just the shear strain. Here u is our velocity which is the time derivative of displacement. So, del u by del y is gamma dot which is time derivative of gamma and that can be written as del u by mu into del u by del y.

So, tau is equal to mu into del u by del y. If I put this constitutive relation here and also if we consider the fact that there is difference of pressure at two sides P 1 and P 2 which are separated by a length L then del P by del x is minus del P by L equal to P 2 minus P 1 by L. So, after we put everything in the approximation, the equation looks like this. Then if we integrate it once, it becomes mu into del u by del y equal to minus del P by L y plus A, maybe a constant and again if we further integrate it becomes mu u equal to minus del P y square by 2 L plus A y plus B. Now our job is to evaluate these two constants.

If I want to evaluate these two constants we just have to apply the boundary condition that at y equal to 0, u is 0 and at y equal to d, u is 0.

$$\mu u = -\frac{\Delta P y^2}{2L} + Ay + B$$

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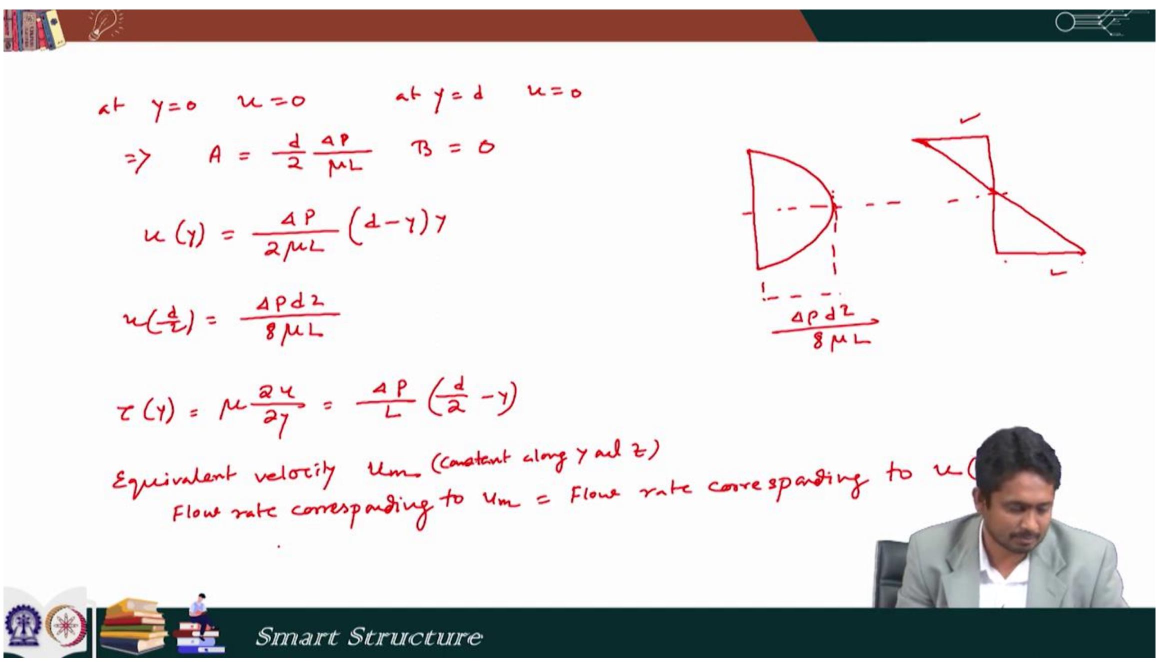

So, at y equal to 0, u is 0 and at y equal to d, u is 0 and that gives us the values of A and B and A becomes equal to d by 2 del P by mu L and B becomes equal to 0. So, after putting everything, mu of y becomes del P by 2 mu L multiplied by d minus y into y. Now if I want to find out the value of u at d by 2 by putting y equal to d by 2, the expression comes to be delta P d square by 8 mu L. Now if I plot it, the variation of the u with respect to y it looks like this.

So, here it is 0. Here it is 0 and here the value is delta P d square by 8 mu L. Now, that we have found out the velocity profile. Our next job is to find out the strain rate and stress. Now we know that tau as a function of y is mu multiplied by del u by del y and if I differentiate this by y and multiplied by mu finally, the expression comes to be del P by L d by 2 minus y. So, the velocity profile looks like this if you have the right side of the velocity profile.

If we draw the stress profile it looks like this. So, here I have maximum negative stress. Here I have maximum positive stress and these two values are same. These value and these value are same. Whatever it is here the same thing is here.

Only they differ by sign. So, we have the highest stress magnitude at the top and bottom surface. At the mid zone, the stress is 0 and that we can see here also. If I draw a tangent to this curve, the slope is 0 and that is what we can see del u by del y is 0 here. So, the stress is 0 at the middle. Now, let us consider a quantity which is equivalent velocity u m.

So, we define this quantity as a velocity which is constant throughout the depth and throughout the width, but that gives me the same flow rate. So, the flow rate corresponding to u m which is constant along y and z. So, flow rate corresponding to this u m is equal to flow rate corresponding to u as a function of y. So, for that, we have to find out the actual flow rate which is corresponding to the actual velocity distribution and then we will find out what can be the flow rate as a function of u m if I assume it to be constant and then you can equate those 2 quantities and that will give me something called u m.

$$u(y) = \frac{\Delta P}{2\mu L}(d-y)y$$

$$u\left(\frac{d}{2}\right) = \frac{\Delta P d^2}{8\mu L}$$

$$\tau(y) = \mu\frac{\partial u}{\partial y} = \frac{\Delta P}{\mathrm{L}}(\frac{d}{2} - y)$$

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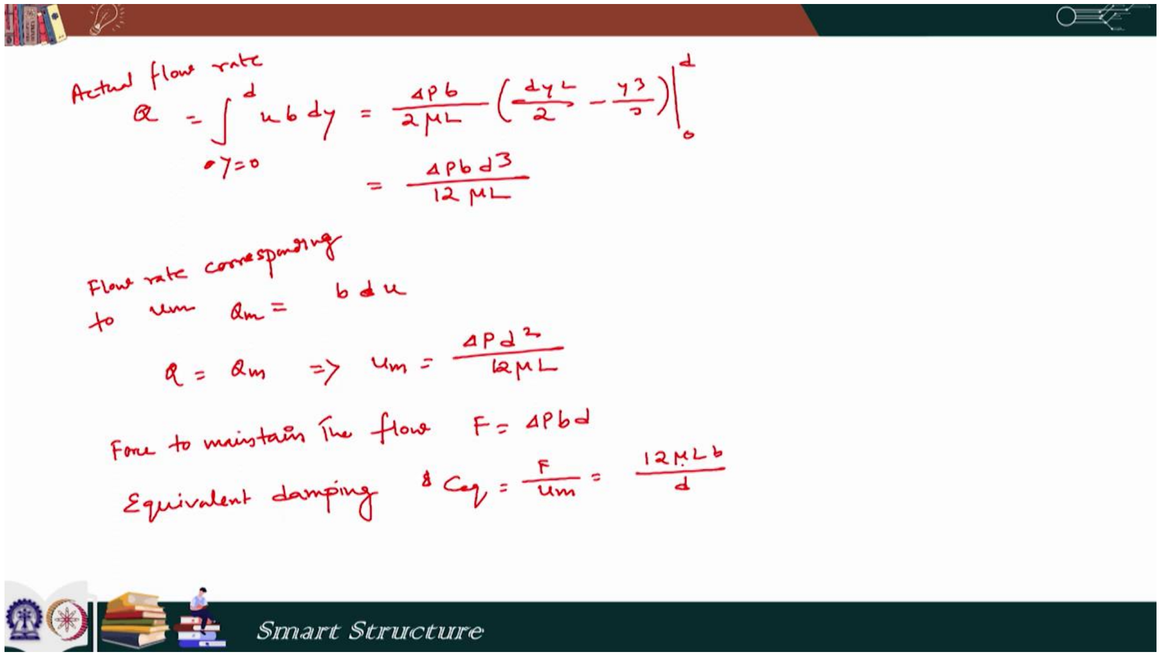

Now, Q the actual flow rate is equal to from y equal to 0 to d and that we can get by integrating this quantity and that gives us the flow rate as del P b by 2 mu L multiplied by d square by 2 minus y cube by 3 which is evaluated between at 0 and d are the limits.

So, finally, it comes to be del P multiplied by b d cube by 12 mu L and then we have flow rate corresponding to u m and that is equal to b d multiplied by u and that let us call it Q m. So, if we equate Q equal to Q m that gives us u m as delta P d square by 12 mu L. Now, we will try to define something called equivalent damping for the system. So, that equivalent damping would be defined as a force which is required to maintain this flow divided by this velocity u m and that is our equivalent damping. So, for that let us find out the force first.

Now, the force to maintain the flow which we can find out as a function of the pressure and the dimensions b d because this pressure del P is applied across a section which has depth d and width b. So, the total force is del P minus b d. Then if I divide this force F by u m that would give me the equivalent damping. So, equivalent damping is C equivalent equal to F by u m and finally, the quantity comes to be 12 mu L b by d. So, that is the equivalent damping when there is no electric or magnetic field applied.

$$Q = \int_{y=0}^{d} ubdy = \frac{\Delta Pb}{2\mu L}\left(\frac{dy^2}{2} - \frac{y^3}{3}\right)\Bigg|_0^d = \frac{\Delta Pbd^3}{12\mu L}$$

$$Q_m = bdu$$

$$u_m = \frac{\Delta Pd^2}{12\mu L}$$

$$C_{eq} = \frac{F}{u_m} = \frac{12\mu Lb}{d}$$

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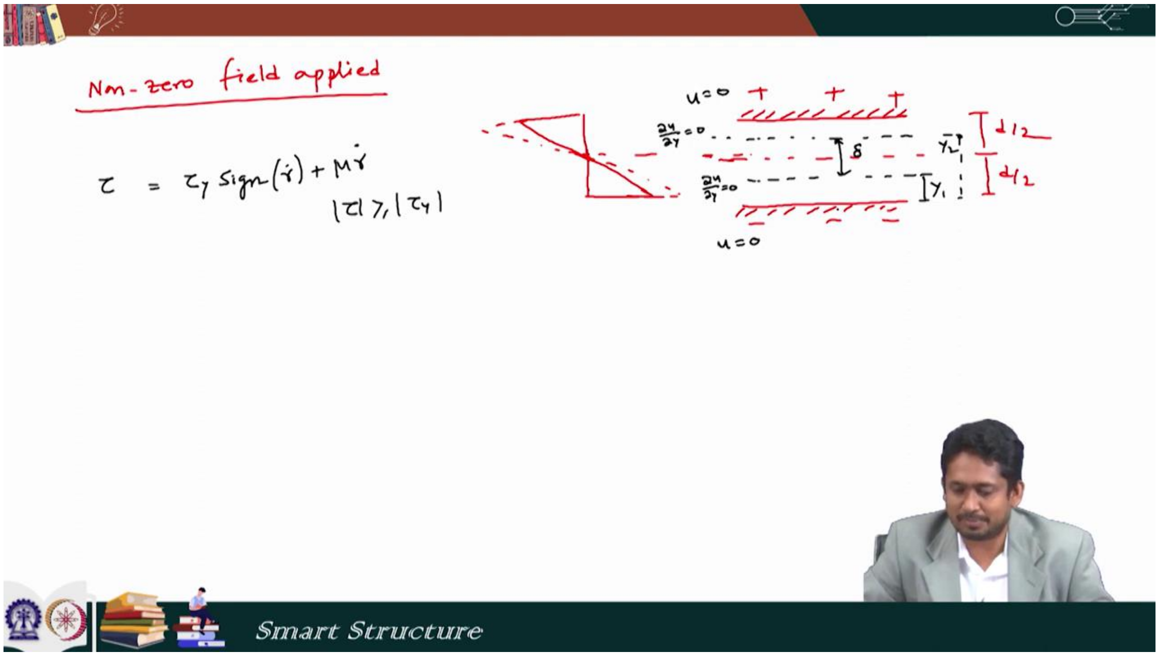

Now, we will consider case where there is non-zero field applied. Now, to understand this non-zero case, we have to understand few things first. Again if we draw the entire flow area, it looks like this and under the zero applied field, the stress distribution took this form. So, when there is a field applied suppose plus minus plus minus plus minus, there would be some finite value of the yield stress. Now, no matter how much pressure we apply, the stress at the mid part of it at d by 2 is going to be 0.

If I apply higher pressure, only this diagram is just going to change like this, but it would always cross the zero line at the middle. So, no matter what pressure we apply at the middle, the stress is going to be 0. So, this is our d by 2. This is d by 2. So, around the midline there would be an area where the stress is always less than the yield stress.

So, that part would behave like a solid. Now, if the applied pressure is more than the thickness of that pre yield region is going to be less. If the applied pressure is less then that pre yield region is going to be more for a given electrical or electrical mechanical field. Now, initially we do not know the size of that pre yield region. The region that behaves like a solid. So, let us assume that the thickness of that region is delta.

Now, we can denote a region like this which has a thickness delta and that region is the pre yield region and from the bottom plate to the starting of the pre yield region, let us denote that dimension as y_1 and again the other dimension to the end of the pre yield region let us denote as y_2. So, now while applying the boundary condition, we have to apply it accordingly. Here we have the no slip boundary condition, u equal to 0, u equal to 0. This parts behave like a solid. So, in this part there is no flow which means gamma dot is 0 here which also means that del u by del y is 0 in this mid block which has a pre yield region.

So, that means, that at this line and at this line, del u by del y is 0 and that gamma dot should be continuous throughout. There should not be any jump in between. So, the boundary condition that we apply here is del u by del y is 0. Here also the boundary condition is del u by del y is 0.

So, again at y equal to 0, u is 0. At y equal to y_1, del u by del y is 0. At y equal to y_2, del u by del y is 0 and at y equal to d, u is 0. So, these are the boundary conditions that we have to deal with and the fluid model that we would be dealing with is the Bingham plastic model. So, tau is again tau y multiplied by sin of gamma dot plus mu of gamma dot when the stress is more than the yield stress.

$$\tau = \tau_y \sin(\dot{\gamma}) + \mu\dot{\gamma}$$

So, this is the material model that we will be using. The fluid model and these are the boundary conditions. The governing differential equation remains the same.

So, with this set of condition we have to solve the problem and we will do that in the next lecture.

Thank you.