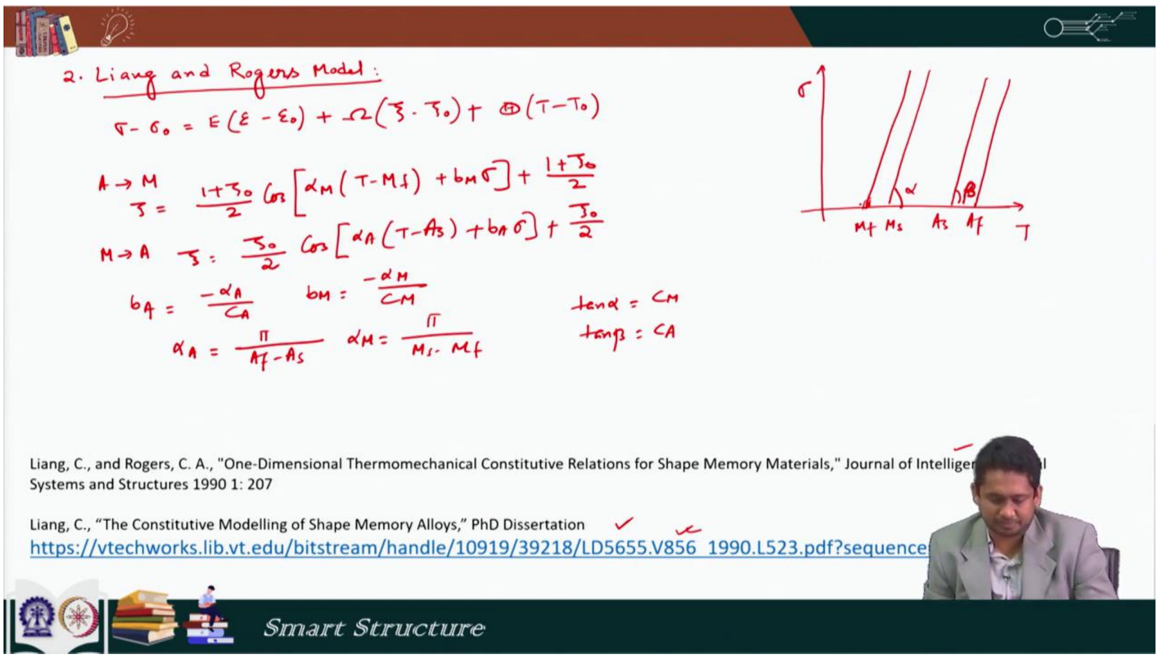
Smart Structures
Professor Mohammed Rabius Sunny
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Week - 09
Lecture No - 46
Constitutive Relations of Shape Memory Alloys – Continued

(Refer Slide Time: 04:59)



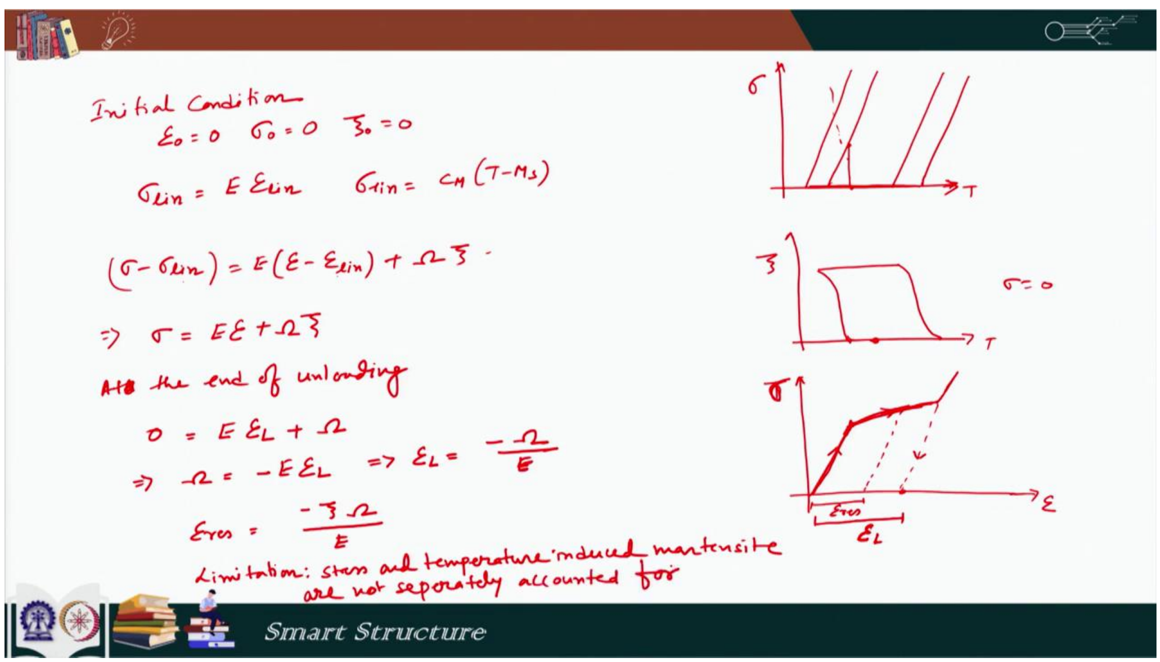
In the last lecture, we started with discussion on constitutive relations of shape memory alloys and we look into various approaches that are used for such constitutive modeling and then we looked into one of the models that is known as Tanaka model. Today we look into another model which is known as Liang and Rogers model. Now Liang and Rogers model can be found in sufficient detail in these two articles. The first one is a research paper or journal paper by Liang and Rogers and the second one is the PhD thesis of Liang at Virginia Tech and it was done under the supervision of Dr. Rogers CA. Now this thesis can be downloaded through this through this link.

It is freely available there. So, if you are interested to know more about this model, these two articles can be referred. Now Liang and Rogers model, it is quite similar to the Tanaka model. It also uses the same phase diagram. So, we have Mf Ms here and As and Af here. So, the same limitation applies here. We do not differentiate between the temperature induced and a stress induced martensite. So, accordingly it has the same constitutive relation which is sigma minus sigma 0 is equal to E multiplied by epsilon minus epsilon 0 plus omega multiplied by xi minus xi 0 plus capital phi multiplied by T minus T0. However, the main difference comes in the equation that defines the evolution of xi with temperature and stress.

In case of the Tanaka model, the equation was based on exponential function. However, in Liang's model, it is based on the cosine function. So, in the last week we discussed about the variation of xi with sigma and T using cosine function and that is exactly what Liang's and Rogers model is. So, we do not need to discuss here about that in details. However, for the completeness of this lecture, we would just very briefly go over it.

So, when it is from austenite to martensite, we derived that xi is equal to 1 plus xi 0 by 2 multiplied by cosine of alpha m T minus Mf plus bm sigma plus 1 plus xi 0 by 2 and from martensite to austenite transformation, it is xi equal to xi 0 by 2 multiplied by cosine of alpha A T minus As plus bA sigma plus xi 0 by 2 and we have this bA equal to minus alpha A by CA and bm equal to minus alpha M by CM where alpha A equal to pi by Af minus As and alpha M equal to pi by Ms minus Mf and we have the tan of that angle, tan alpha equal to CM and if we call it beta. So, tan beta equal to CA. So, these are the equations that are needed to model the shape memory alloy using Liang and Rogers model.

(Refer Slide Time: 13:22)



Now, let us look into a specific case. Let us assume that there is a transformation happening. So, again in that phase diagram, assume that we are here and from here we are increasing the stress. So, the initial condition is epsilon 0 equal to 0. Initially, it is not deformed sigma 0 equal to 0 and we have xi 0 equal to 0. So, we are assuming that it is in austenite condition. So, again if we draw the xi versus temperature diagram when sigma equal to 0, it just look looks like this.

So, we are somewhere here. So, for this diagram, sigma equal to 0. We are somewhere here and the transformation is starting because we are gradually increasing sigma. Now, we know that as it progresses once it starts crossing this line, transformation starts taking place. So, till this the entire relation is linear. So, we can say, sigma lin equal to E multiplied by epsilon lin. This quantity is sigma lin and we have already defined that as sigma lin equal to CM multiplied by T minus Ms. We saw that once it starts crossing this line, the transformation starts taking place. So, till here at this point here, the stress is CM into T minus Ms and that we wrote as the limit of linearity, sigma lin is equal to CM into T minus Ms. And after that if we increase the stress further.

So, here if we draw the stress versus strain diagram, the increase is linear till sigma lin and then after that because the transformation takes place, it goes to this region. So, when it is starting from here, the relation that we can write as sigma minus sigma lin equal to E multiplied by epsilon minus epsilon linear plus omega multiplied by xi and we saw this in the last lecture also at the end of it. So, this quantity is del sigma. This quantity is del epsilon and we saw that in the last lecture as well, but here we have just broken sigma and epsilon into the initial and the current value. Now, we already know that sigma line is equal to E into epsilon lin.

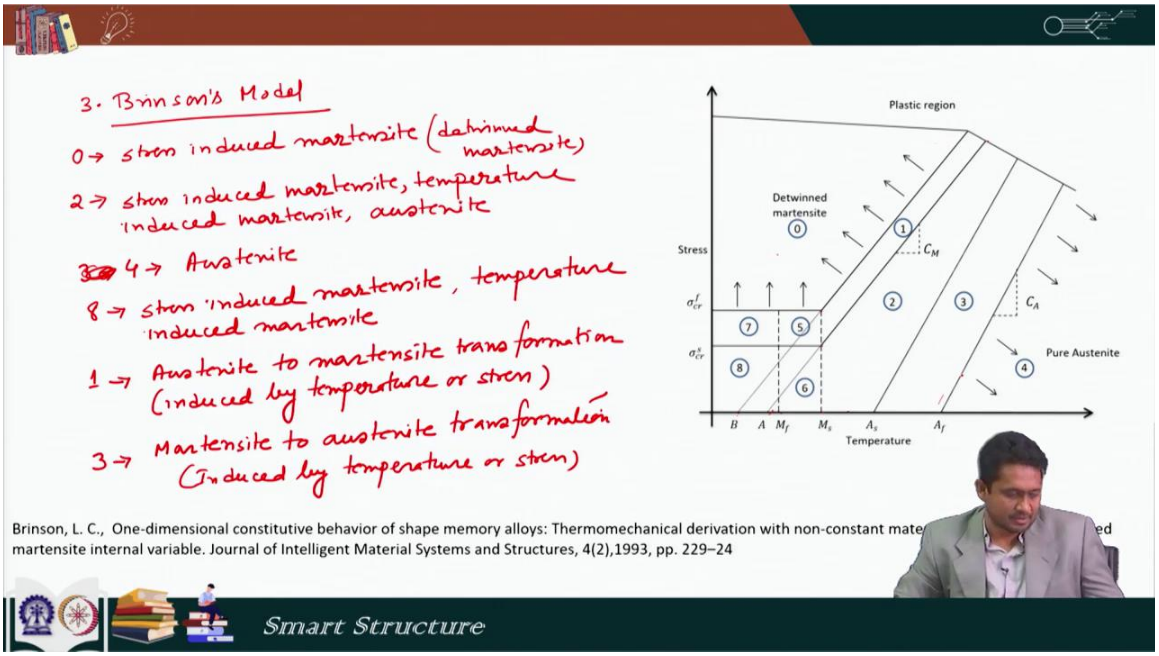
So, this sigma line cancels with this E into epsilon lin. So, we are left with sigma is equal to E epsilon plus omega xi. Now suppose we start unloading it. So, suppose we take it to the level till the detweening takes place, I mean, the stress induced martensite formation takes place and after that again the curve starts becoming linear. We have already seen that.

So, suppose we start unloading from here and then if we write the equation at the end of the unloading phase, that means, when we are here then the final condition is my sigma is equal to 0. My epsilon is equal to epsilon L because we know that there is a maximum recoverable strain epsilon L that comes here. So, that is here and then we know that our xi is equal to 1 because even if we unload it, it is still stress induced martensite. So, xi is equal to 1. So, taking that as final condition.

So, at the end of unloading we can write 0 because we started with 0 stress and ending with 0 stress equal to E multiplied by epsilon L because we started with 0 strain, but and the at the end we have some strain remaining epsilon L plus omega because we started with xi equal to 0 and here we have xi equal to 1. So, that gives us omega is equal to minus E into epsilon L and this we use quite often. So, we wrote this equation somewhere here at any xi, I mean, between this point and this point at any xi and we wrote this equation at the end of the unloading phase. Now, suppose at from any xi, xi equal to any value which is xi we unload it then the remaining strain, we may like to call at epsilon residual. Now, that quantity can be written as epsilon residual equal to minus of xi omega by E because here we could have written epsilon L equal to minus of omega by E.

So, this we are writing because here xi is equal to 1. So, we can generalize this and we can say that when xi is equal to xi this becomes minus of xi omega by E. So, this was about the Liang and Rogers model. Again the equations are same. The constitutive relations are same. Only the evolution of xi has a different form and if model has the same limitation as that of the Tanaka model, so, stress and temperature induced martensite are not separately accounted for.

(Refer Slide Time: 22:37)



Now, let us discuss another model that is called Brinson's model and Brinson's model accounts for stress induced and temperature induced martensite separately. So, it considers two different variables for those two different quantities. However, before that let us look into that phase diagram here and this phase diagram is significantly different from the phase diagram that we saw so far. Here we can see that there is one line sigma critical s that signifies the stress at which stress induced martensite formation takes place and there is another line that signifies the stress at which the stress induced martensite formation finishes.

Now, these two lines intersects with these two inclined lines and the point of intersection this point of intersection the first one and the second one and the the line corresponding to that when that line matches with the temperature line we are calling that as Ms and here we have another line and that is Mf. In the previous cases these inclined line was extended and if they are intersecting the temperature line at B and A, those were the corresponding Mf and Ma, but here Mf and Ma are slightly different. So, this divides the entire plane into several zones and each zone has its own significance. So, first we will look into those zones in which there is no transformation taking place and then there will look into some of the zones where some transformation takes place. So, the first one is 0 the zone 0.

This zone is entirely detwined martensite because it is above sigma CRF and this is the temperature is below the temperature shown by the Mf line, martensite finish temperature line. So, sigma 0 is stress induced martensite or we call it a detwined martensite. Then let us look into zone 2. Zone 2 means this. So, zone 2 can have austenite and zone 2 can have temperature induced martensite or stress induced martensite or austenite.

Now, it depends on what the initial condition is. So, suppose the material is entirely austenite. That means, we are our initial condition is somewhere here and if we reduce the temperature, then when it is in this zone it is fully austenite. Similarly, it can be temperature induced martensite. So, if we keep reducing the temperature and suppose it comes here, here it is fully temperature induced martensite and then if we increase the temperature again it comes back here and then it is fully temperature induced martensite.

Or from here if we increase the stress and it goes towards the detwined zone and then if we come back, it remains as detwined martensite or stress induced martensite. However, in this zone there is no transformation taking place. Now, not 3, let us look at 4, 4 is purely austenite that is beyond the temperature Af. So, it is fully austenite and then we have 8, 8 can be stress induced martensite and it can be temperature induced martensite as well. For example, if we start from here in the austenite zone and then if we increase the temperature, so, when it crosses these lines and it comes to the 8 zone, it is a temperature induced martensite or if I start from here and increase the stress then it becomes detwined and then when again we unload it and it comes back, it remains as detwined or stress induced martensite.

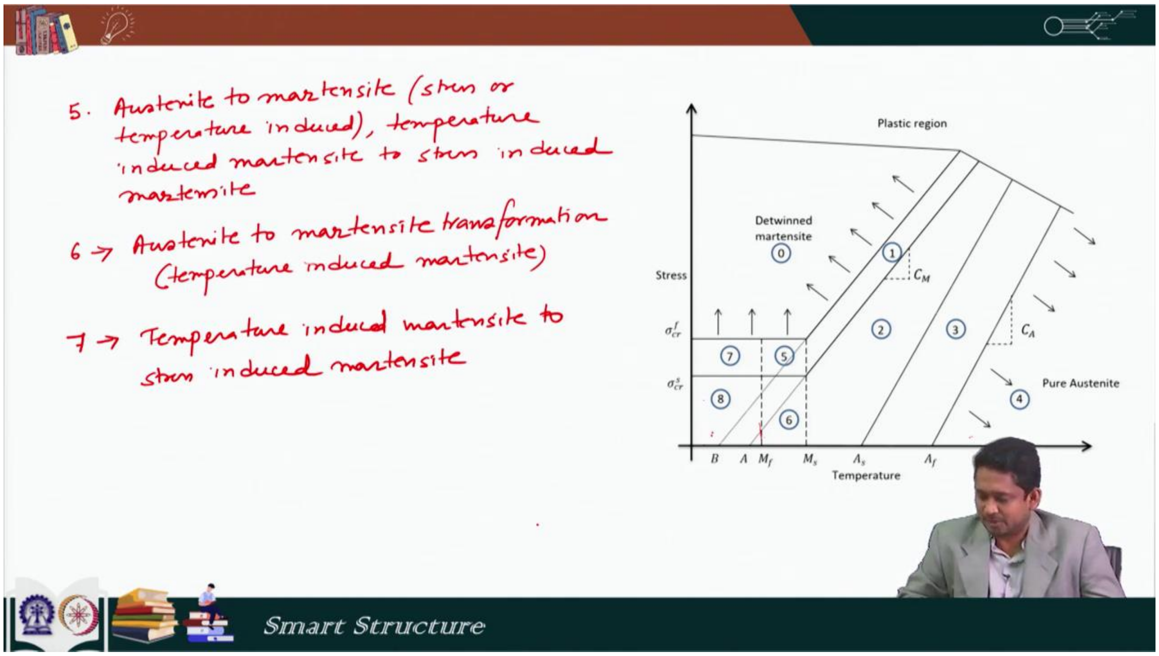
Now, let us look into the transformation zones. Zone 1 signifies austenite to martensite transformation and it can be induced by temperature or stress. For example, here if we reduce the temperature from the austenite side suppose we are here initially in the purely austenite zone and if we reduce the temperature then when it crosses these two lines the As and Af lines, so, when it is in zone 1 the transformation takes place due to temperature. After the transformation, it becomes a stress induced martensite that is fine, but that transformation is induced by temperature because we are reducing the temperature. On the other hand if I start from this side of 0 stress and then if you gradually increase the stress then also through this zone 1, martensite transformation takes place and that transformation is due to the action of stress not temperature.

So, it is austenite to martensite transformation. After the transformation, it is a detwined or stress induced martensite. However, that transformation can happen through stress or temperature. Then comes zone 3. Zone 3 is martensite to austenite transformation and that is, it can be induced by temperature or stress.

Again let us look into it. Suppose, we are we are at this side. We are in fully detwined martensite state and if I gradually increase the temperature then it crosses the As line it comes into zone 3 and austenite formation takes place and it continues till Af. So, in this zone 3 this transformation is a stress induced transformation. On the other hand, we may start somewhere here and gradually reduce the stress that means, go down vertically. In that case, this transformation is a transformation induced by stress.

So, in the previous case the transformation was induced by temperature because we started from here and we are going horizontally along the temperature axis. So, we are crossing this line horizontally, this zone horizontally. So, it is a transformation induced by temperature. In that case we are going down vertically. So, it is a transformation induced by stress.

(Refer Slide Time: 29:04)



Now zone 5 is austenite to martensite stress or temperature induced and temperature induced martensite to stress induced martensite. So, this is our zone 5. There are various things are possible here. So, again if we start from somewhere here in the austenite zone and gradually reduce the temperature then it is crossing this zone 5 horizontally which means along a line which is parallel to the temperature line. In that case, the transformation to martensite is induced by temperature or we can be somewhere here.

We can be somewhere here and gradually increase the stress. In that case the transformation is induced by stress. Similarly, there can be transformation from temperature induced martensite to stress induced martensite through the same line if the initial condition is such that we have martensite here and if we increase the stress when it crosses zone 5, the transformation to stress induced martensite takes place because of stress. So, zone 5 has several transformations possible and zone 5 imposes a limitation of this Brinson's model. So, we will treat zone 5 separately at the end of the discussion of Brinson's model.

Then comes zone 6 and zone 6 has austenite to martensite transformation and this is temperature induced martensite and this is a straight forward. We are just starting from the austenite phase, high temperature phase. We are reducing the temperature and as we are crossing zone 6, the transformation starts and ends at this line and at the end of it, we have a fully temperature induced martensite. Then we have zone 7 and zone 7 is temperature induced martensite to stress induced martensite. So, suppose we are here in zone 8 and we are at fully temperature induced martensite then if we if we increase the stress as we cross zone 7, the transformation from temperature induced to stress induced martensite takes place.

So, that is how zone 7 is a zone where temperature induced martensite to stress induced martensite takes place. Now please remember when we unload it, so, suppose zone 7 is crossed in the other direction that means, we started from stress induced martensite and we reduce the stress and when we are crossing zone 7 at that time, no reverse transformation takes place. So, no transformation takes place. Transformation takes place only in the one way direction. When we go from low stress to high stress at that time, temperature induced martensite to stress induced martensite transformation takes place.

So, these are the various zones that forms in this phase diagram and each zone has its own significance which we discussed. Now there is another zone that is a plastic zone. So, in the last week we discussed that there is a critical stress beyond which martensite slips. So, this line shows that critical stress and that critical stress also varies with temperature. So, in this zone the only possibility is a slipped martensite.

So, we do not talk about here. We do not go to this zone. Our entire dealing is within this plane. So, with this I would like to conclude the lecture here. In the next lecture we will look into the mathematical formulation of the Brinson model. We will see that we take care of xi as a combination of two part one is stress induced and one is temperature induced and accordingly the constitutive relation is modified and also our omega e will become a function of xi in this model.

So, we will talk about it in the next lecture.

Thank you.