Smart Structures
Professor Mohammed Rabius Sunny
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Week 08
Lecture No: 43
Temperature and Stress Dependent Phase Transformation Modeling (Continued)
Part 03

In the previous lecture, we discussed about the variation of martensite fraction with temperature. And then, we saw that this martensite fraction is dependent on stress also. And, we looked at a diagram like this which says that, the characteristic temperatures, the start and finish temperatures for martensite and austenite, they vary linearly with stress. So, with increasing stress this temperature shifts towards right side of this curve. Now, in this 2D plot. So, here we have we are not showing martensite volume fraction here, we are only showing the characteristic temperatures, because it is a 2D diagram.

Now, in this plot, this region is martensite region because at any given stress this region is below the martensite finish temperature. So, this is a martensite region and this region is austenite. And this region can be M or A depending on the history. So, for example, if previously the temperature was beyond the austenite finish temperature, if the material was totally austenite and then, if you reduce the temperature then, in this region the material is in. So, in this region the material is in austenite phase. Similarly, if the temperature was below martensite finish temperature, if the initial phase was martensite, then, we can see within this region is martensite and then, in between in this region it starts transforming.

Now, we will incorporate sigma, the stress in the equations that we developed in the last lecture. So, we can say that this is the study of effect of stress on xi. So, at any stress sigma, we denote M_f star, M_f as M_f star. So, we denote characteristic temperatures as M_f star, A_s star, and A_f star, and, there is M_f star also. So, this is not in the sequence of increasing temperature but, I have just listed it here. So, all these are characteristic temperatures. Star denotes that, it is for a non zero value of stress. And we saw that we discussed in the last lecture that, these variations are linear and that is experimentally observed. So, we can always define a slope. So, let us define a slope alpha here. And we can always define a slope beta. Now, alpha is equal to beta in general. But again, it is not a requirement they can be different also. So, we can write, this can be different as well depending on the material. So, we denote a quantity C_M equal to tan alpha. So, tan of this angle alpha. So, there is one small mistake here, this is alpha not beta. So, C_M equal to tan alpha. And similarly, we define another quantity C_A and that is equal to tan beta.

$$C_M = \tan\alpha$$

$$C_A = \tan\beta$$

Now, we can say that at a stress sigma, our martensite finish temperature is M_f star. So, M_f star, for a certain sigma. So, this quantity shows the difference between M_f star, and M_f. So, this is M_f star, minus M_f. So, we can write that and this is our sigma the stress. This is sigma. So, we can write sigma by M_f star, minus M_f is equal to tan alpha is equal to C_M. And similarly, we can also write sigma divided by A_f star, minus A_f is equal to tan beta and, that is equal to C_A.

$$\frac{\sigma}{M_f^* - M_f} = \tan\alpha = C_M$$

$$\frac{\sigma}{A_f^* - A_f} = \tan\beta = C_A$$

Now, we will incorporate these effects in the transformation equation that we derived in the last lecture. So, the equation that we wrote for martensite to austenite transformation, and that was xi equal to xi_0 by 2 multiplied by cosine alpha A, T minus A_s, plus xi_0 by 2 for sigma equal 0. Now, this equation we modified little bit and add one more term to incorporate the effect of sigma, and write this as the same thing with b_A multiplied by sigma added to it. And then, we have xi_0 by 2, and this is for sigma not equal to 0. Now, the question is what is b_A, how do I find it out.

$M \rightarrow A$

$$\xi = \frac{\xi_0}{2}\cos\alpha_A\,(T - A_s) + \frac{\xi_0}{2} \quad \text{for } \sigma = 0$$

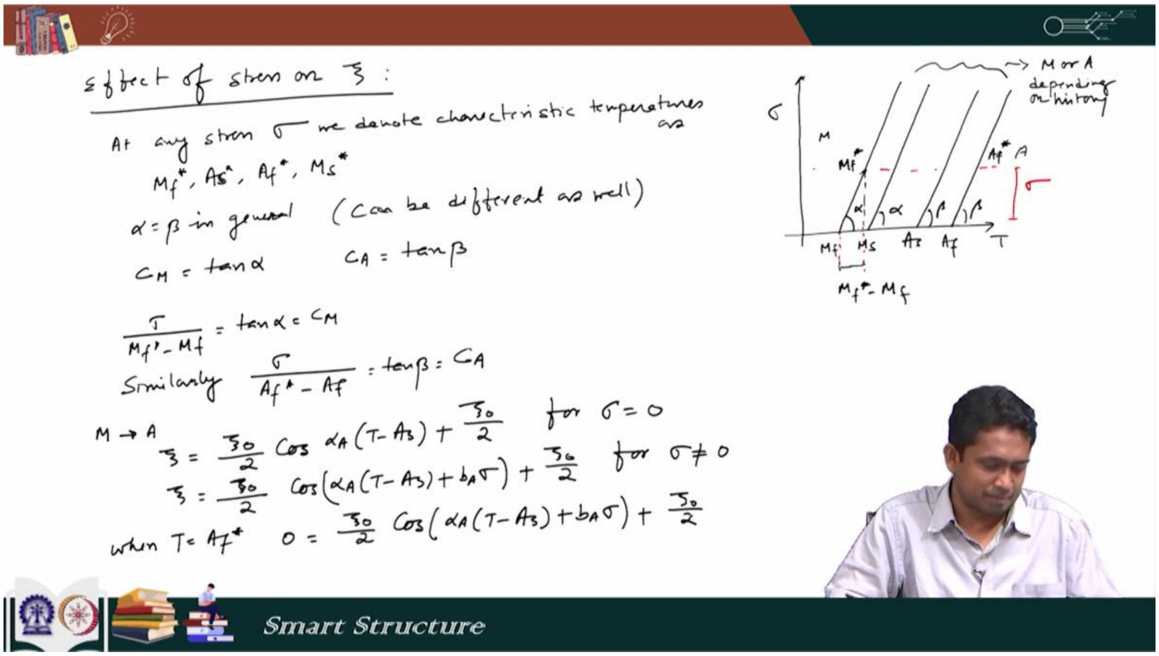
$$\xi = \frac{\xi_0}{2}\cos(\alpha_A(T - A_s) + b_A\sigma) + \frac{\xi_0}{2} \quad \text{for } \sigma \neq 0$$

Now, to find this out all we can do is, when our temperature is A_f star. So, for a non zero value of sigma the transformation completes at a temperature A_f star. So, when T is equal to A_f star the transformation completes and, completion of transformation means xi becomes 0. So, we have 0 equal to xi by 2 cosine alpha A, multiplied by T minus A_s, plus b_A sigma. And then we have this, xi_0 by 2.

$T = A_f^*$

$$0 = \frac{\xi_0}{2}\cos(\alpha_A(T - A_s) + b_A\sigma) + \frac{\xi_0}{2}$$

(Refer Slide Time: 9:35)



Now, we can cancel ξ_0 by 2 from both the terms and finally, this gives me equal to minus 1. And if, this quantity is minus 1, it simply means that, alpha A, T minus A_s, plus b_A sigma equal to pi.

$$\cos\left(\alpha_A\left(A_f^* - A_s\right) + b_A\sigma\right) = -1$$

$$\Longrightarrow \alpha_A\left(A_f^* - A_s\right) + b_A\sigma = \pi$$

Now, it is not T, it should be A_f star, because the transformation is happening when the temperature is A_f star. Similarly, this is also A_f star. Now, we can again do some mathematical manipulations on that and we know that, alpha is equal to pi by A_f minus A_s; that is how, we defined alpha A in the last lecture. And then, this we can write A_f star, minus A_f, plus A_f minus A_s. And then, we have b_A sigma is equal to pi. So, if we multiply this quantity with this it gives me pi and at the right hand side, we have pi. So, they cancel. And this quantity A_f star, minus A_f is nothing, but sigma by C_A, as we saw just now. So, this again we will write as alpha A. So, this is sigma by C_A plus b_A sigma equal to 0. And finally, we are left with b_A equal to minus of alpha A by C_A.

$$\frac{\pi}{A_f - A_s}\left(A_f^* - A_f + A_f - A_s\right) + b_A\sigma = \pi$$

$$A_f^* - A_f = \frac{\sigma}{C_A}$$

$$\alpha_A \frac{\sigma}{C_A} + b_A\sigma = 0$$

$$b_A = -\frac{\sigma_A}{C_A}$$

So, we wrote the modified equation for the variation of xi, when transformation from martensite to austenite takes place. In that modified equation we had a term b_A, and this is what our b_A is in terms of alpha A and C_A.

Now, we will do it do the same thing for austenite to martensite transformation. So, for that the equation that we wrote in the last lecture was 1 minus xi_0 by 2 multiplied by cosine of alpha A multiplied by T minus M_f plus 1 plus xi_0 by 2, and that is when sigma is equal to 0. Now, this we again write it in a modified form as 1 minus xi_0 by 2 cosine alpha A multiplied by T minus M_f plus b_M sigma. So, we add a term b_M multiplied by sigma to incorporate the effect of stress sigma, plus 1 plus xi_0 by 2, when sigma is not equal to 0.

$A \rightarrow M$

$$\xi = \frac{1-\xi_0}{2}\cos\alpha_M\left(T - M_f\right) + \frac{1+\xi_0}{2} \quad \text{for } \sigma = 0$$

$$\xi = \frac{1-\xi_0}{2}\cos\left(\alpha_M\left(T - M_f\right) + b_A\sigma\right) + \frac{1+\xi_0}{2} \quad \text{for } \sigma \neq 0$$

Now, again we do the same thing when the transformation ends, we have the temperature T is equal to M_f star and at that stage xi becomes 1. So, when T is equal to M_f star, we have xi is equal to 1, 1 minus xi_0 cosine alpha M, M_f star, minus M_f, plus b_M sigma plus 1 plus xi_0 by 2. Then we can take this 1 plus xi_0 by 2 in the left hand side, that gives us 1 minus xi_0 by 2. And then, both right hand side and left hand side has 1 minus xi_0 by 2. So, cancelling that we get this expression alpha M multiplied by M_f star, minus M_f plus b_M sigma cosine of that and that is equal to 1. And that means, this quantity is equal to 0. Now, we know that M_f star, minus M_f that is equal to sigma by C_M. And then, we have plus b_M sigma equal to 0. And on cancelling sigma we get b_M equal to minus alpha M by C_M.

$T = M_f^*$

$$1 = \frac{1-\xi_0}{2}\cos\left(\alpha_M\left(M_f^* - M_f\right) + b_M\sigma\right) + \frac{1+\xi_0}{2}$$

$$\cos\left(\alpha_M\left(M_f^* - M_f\right) + b_M\sigma\right) = 1$$

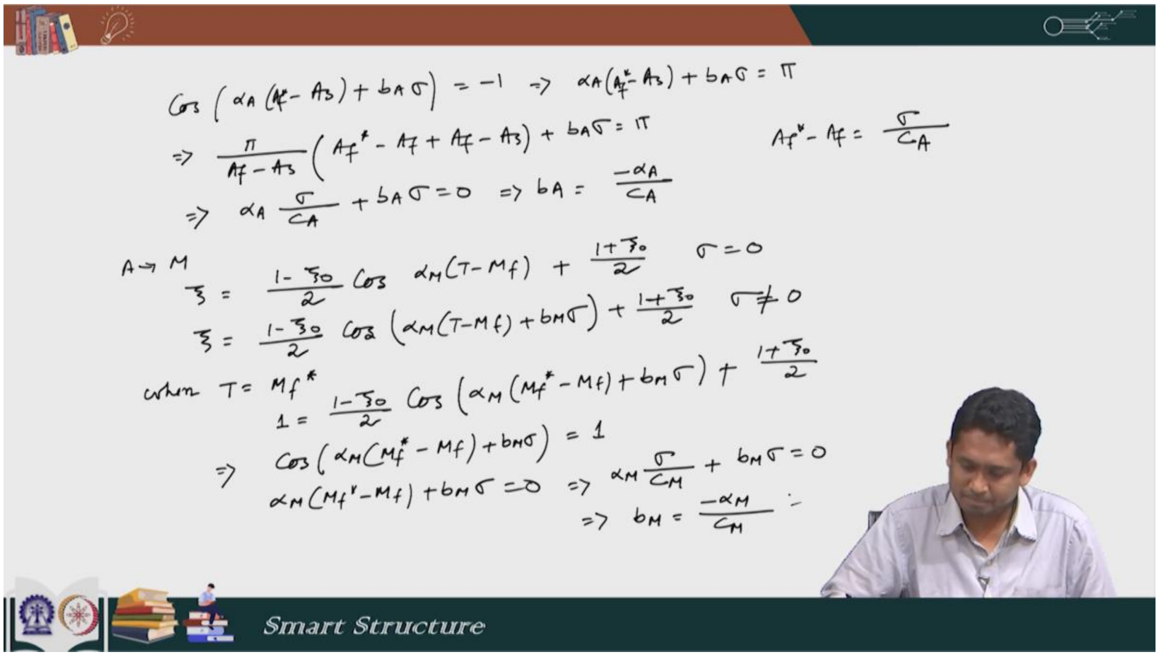
$$\alpha_M\left(M_f^* - M_f\right) + b_M\sigma = 0$$

$$\alpha_M\frac{\sigma}{C_M} + b_M\sigma = 0$$

$$b_M = -\frac{\sigma_M}{C_M}$$

So, this was our modified equation for transformation from austenite to martensite, with effect to incorporate the effect of sigma we had quantity b_M and this is our b_M. So, the transformation equations are now derived with sigma incorporated.

(Refer Slide Time: 15:47)



Now, let us look into this diagram once again. So, again this is M_f, M_s, A_s and A_f. Now, we have seen that both temperature and sigma have effect on the martensite fraction. So, for a constant stress, if I increase the temperature the transformation takes place. So, if at this side, we have xi is equal to 1, if we keep increasing the temperature in between A_s to A_f the transformation takes place, and it becomes austenite, xi becomes 0. Similarly, if we start from this side and reduce the temperature, then initially xi is equal to 0, its austenite and then, between M_s and M_f the transformation takes place. And it becomes martensite, xi becomes 1.

Now, similarly if I keep the stress constant, it can be 0 or non 0. Sorry, if we can keep the temperature constant and if we increase the value of sigma then, on increasing the stress in between these two lines in between the M_s and M_f lines, the transformation takes place. And it becomes from austenite to martensite. Similarly, here, if we reduce the stress suppose we are we are in this region the material is fully martensite. And if we reduce the stress then when it processes these lines A_s and A_f, it becomes it transforms and it becomes austenite. So, increase in stress is equivalent to reduction in temperature. Because reduction in temperature helps in transformation from austenite to martensite. Similarly, increase in stress helps in transformation from austenite to martensite. Similarly, reduction in stress is equivalent to increase in temperature, because increase in temperature transforms from martensite to austenite, and same thing is done by a reduction in stress.

Now, when martensite is formed because of the effect of sigma, then generally we are at the we are comparatively higher stress, because we cannot see a martensite induced by stress at 0 stress. So, we have to start from somewhere and we have to increase the stress and we will be at somewhere non zero stress region to see martensite which has been induced by increasing stress. And at that stress at than high stress, generally martensite remains in detwinned phase. So, that is why stress induced martensite is often associated with detwinned martensite. So, when we get martensite by stress induction, we generally get detwinned martensite.

Now, we look into a range of stress within which this transformation takes place. So, range for sigma to produce stress induced martensite. So, we have this as the equation. So, first let us look into the austenite transformation. So, T minus A_s, plus b_A sigma plus xi_0 is equal to 0. Now, for this equation to hold this term ranges from 0 to pi. So, we have 0 alpha A, T minus A_s, plus b_A sigma, this term between 0 to pi.

$$\xi = \frac{\xi_0}{2}\cos(\alpha_A(T - A_s) + b_A\sigma) + \frac{\xi_0}{2}$$

$$0 \le \alpha_A(T - A_s) + b_A\sigma \le \pi$$

So, when it is 0, it is in one side, when it is pi, it is in the other side. So, when it is pi, then it is totally austenite, when it is 0, then it is at the beginning of the transformation. Now, from this inequality, we can write sigma is less than C_A, T minus A_s, and also, we can write alpha A, T minus A_s, plus b_A sigma, less than pi, and this comes to be alpha A, T minus A_s, minus pi by b_A is less than sigma.

$$\sigma \le C_A(T - A_s)$$

$$\alpha(T - A_s) + b_A\sigma \le \pi$$

$$\Rightarrow C_A(T - A_s) - \frac{\pi}{|b_A|} \le \sigma$$

So, this is the sigma this is the upper limit of sigma, and this is the lower limit of sigma. So, we have C_A, T minus A_s minus pi by b_A, C_A, T minus A_s.

$$C_A(T - A_s) - \frac{\pi}{|b_A|} \le \sigma \le C_A(T - A_s)$$

So, what it means is that when our sigma for a given temperature is in this region. So, for if our temperature is this much. So, when the sigma is in this region, the transformation takes place, this is the upper limit of sigma, this is the lower limit of sigma and in between them, the transformation takes place.

Similarly, there can be a range for M_f also. So, range for sigma to produce stress induced trans martensite. And accordingly, we have alpha M, T minus M_f, plus b_M sigma is equal to pi. b_M sigma should be less than equal to pi and greater than equal to 0, and that is the range within which the transformation takes place. And finally, it gives us a limit, and the limit is C_M, T minus $M_{f,}$ minus pi by b_M. Here sigma and then we have C_M, T minus M_f.

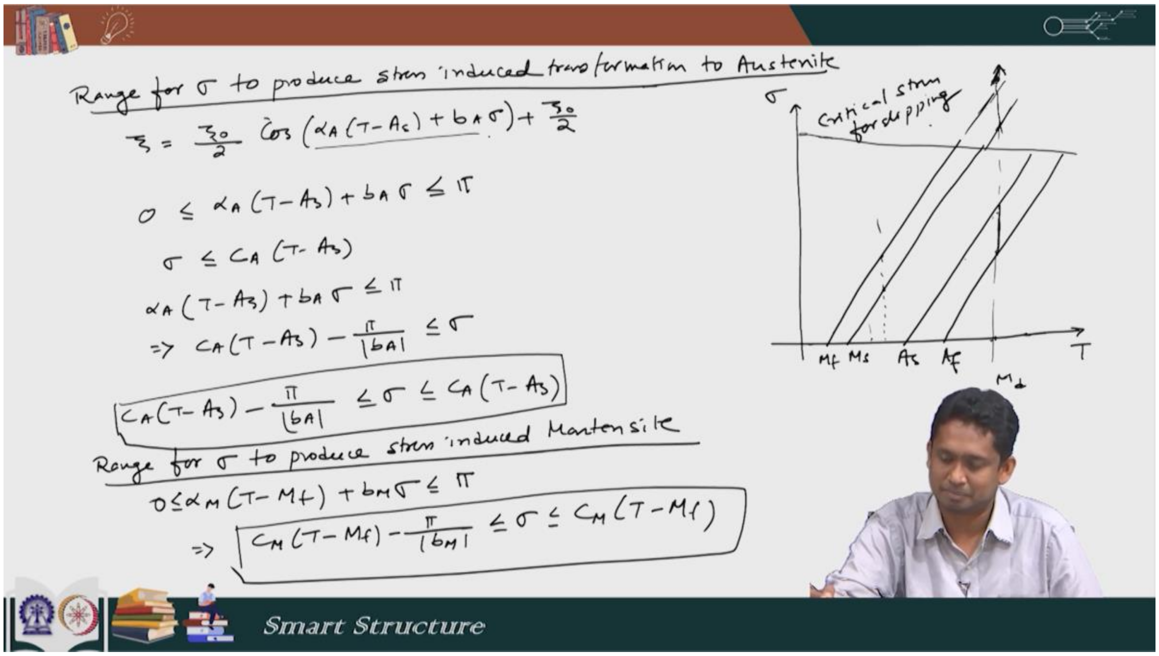
$$0 \leq \alpha_A\left(T - M_f\right) + b_M\sigma \leq \pi$$

$$C_M\left(T - M_f\right) - \frac{\pi}{|b_M|} \leq \sigma \leq C_M(T - M_f)$$

So, this is the range within which the transformation to austenite happens for a constant temperature. This is the range of sigma within which transformation to martensite happens for a given temperature. So, this means that our temperature is increasing and while it increases here, this is the lower limit of sigma, here the here the transformation starts. This is the upper limit of sigma the transformation ends. So, those are the ranges.

Now, few more things to note. There is a certain value of sigma, beyond which slipping takes place and that is certain value of sigma, that certain critical value of sigma that can be a function of temperature also. So, suppose the certain critical value of sigma varies like this. So, now if we take our temperature here and from here if we start increasing the sigma, then ideally for martensite transformation to takes place, we should have crossed these A_f lines, here. So, when we reach, here the martensite transformation should have taken place, but what this upper limit for sigma is saying that, before that when we are crossing this line, this sigma critical line, here itself our material starts slipping, which means there is a certain temperature beyond which, if we start increasing the stress, we cannot reach the martensite level, because sigma critical puts a limit on that. So, it is critical stress for slipping and generally, this critical temperature we denote as M_d. So, again just to explain because there is a limit set by the critical stress for slipping, now if we start from a very high temperature, before we can cross the M_f lines for the martensite to form, we are already crossing the critical stress line for slipping. So, which means at those high temperature, we cannot have a martensite transformation.

(Refer Slide Time: 27:50)



Now, if we look at the limit once again for sigma. So, from this we can say that sigma is equal to C_M multiplied by T minus M_f, minus pi by b_M.

$$C_M(T - M_f) - \frac{\pi}{|b_M|} \leq \sigma$$

$$\sigma = C_M(T - M_f) - \frac{\pi}{|b_M|}$$

So, this is a lower limit beyond which austenite starts transforming to stress induced martensite. Now, when the material is austenite, we have its stress strain behavior linear, but when the material is stress into martensite, which is generally detwinned, the stress strain behavior is not linear. So, when the stress starts crossing this limit, the linearity of the stress strain behavior ends and the material starts behaving non-linearly. So, we can write this lower limit as sigma linear and this can also be written as T minus M_f by, So, we know that our b_M is minus alpha M by C_M, but here we have mod of b_M. So, finally, we have a negative sign here and then we can simplify this further, we can write this as C_M multiplied by T minus M_f. Then we have pi C_M. And alpha M is pi by M_f minus M_s and this finally, can be written as C_M multiplied by T minus M_s.

$$\bar{\sigma}_{lin} = C_M\left(T - M_f\right) - \frac{\pi}{|b_M|}$$

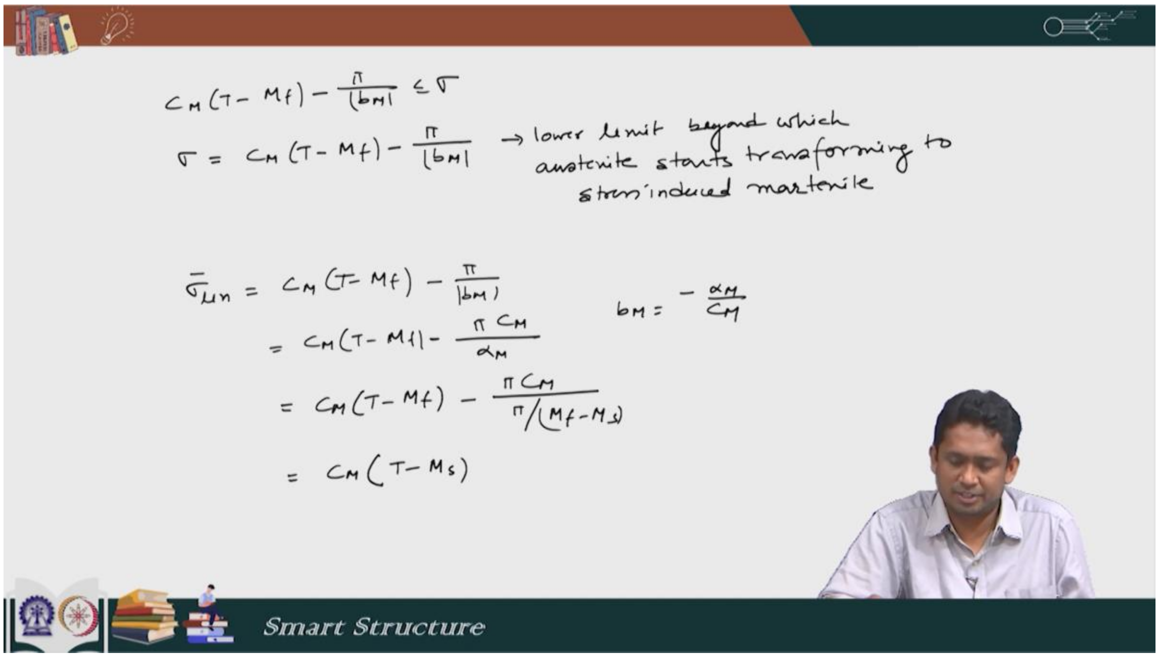
$$= C_M\left(T - M_f\right) - \frac{\pi C_M}{\alpha_M}$$

$$= C_M\left(T - M_f\right) - \frac{\pi C_M}{\pi / \left(M_f - M_s\right)}$$

$$= C_M(T - M_s)$$

So, this sets a limit for a certain stress beyond which the material starts becoming stress induced martensite from austenite and the linear behavior stops and the material starts becoming behaving like behaving like a non-linear material, I mean the stress strain behaviors starts becoming non-linear.

(Refer Slide Time: 31:05)



So, this was about the effect of stress on this austenite and martensite transformation, and we have seen that, martensite and austenite can be formed with the effect of stress or with the effect of temperature, or by a combine effect and we have seen various limits.

Now, with this I would like to conclude this lecture.

Thank you.