**Smart Structures**
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**Week - 08**
**Lecture No – 42**
**Temperature and Stress Dependent Phase Transformation Modeling**

In the last lecture we saw a basic introduction to shape memory alloys. We saw the different phases austenite and martensite and we saw how this phase transformation leads to shape memory effect. Now, today we look into the phase transition temperatures and the corresponding mathematical formulation. Now, to do this we define a variable called xi and this denotes martensite fraction. What it means is when xi is equal to 1 the material is fully martensite xi is equal to 1 means a full martensite material martensite and xi is equal to 0 means it is austenite and when xi is between 0 and 1 this means partially martensite and partially austenite. So, when phase transition takes place if it transits to an austenite phase xi becomes 0 it if it transits to a martensite phase xi becomes 1 and this entire variation of this xi can be shown by a graph like this.

So, all this phase transformation can be seen as a variation of xi and if we say that the transition is happening through temperature that means, how xi is varying with time and this kind of variation is shown by a graph like this. Now, here are some key points this we call M f this we call M s and here we call it A f and we call this as A s ok. So, there is one error in the diagram we have. So, this is xi and this is temperature xi is varying with temperature now here xi is equal to 1.

Now this diagram when the temperature increases follow a path follows this path and when the temperature decreases it follows this path it means suppose I have a material at temperature beyond A f. So, in this region maybe here the martensite fraction is 0 which means the material is entirely austenite. Now if I reduce the temperature then while reducing it will follow this arrow. So, it will follow this path. So, till it comes to M s it will maintain xi is equal to 0 which means there is still it is still fully austenite there is no phase transformation.

Now, when it crosses M s and if we further reduce the temperature xi starts growing up and gradually it becomes 1 when the temperature is M f. So, from M s to M f there is a variation of xi and starting from 0 it becomes 1 when it reaches M f. If I further cool it down it will maintain xi equal to 1 which means below M f the material is fully martensite. Now if we start increasing the temperature it will not follow this path it will follow this path which is shown by the arrow pointing towards the right side. And till A s which we called austenite start temperature xi will be maintained will be maintaining a value of 1 and after A s the value will reduce and it will become 0 when it reaches A f.

So, here we have now 4 quantities A s and that is called austenite start temperature and we have A f and that we called austenite finish temperature and then we have M s what is that is called martensite start temperature and then we have M f and that is called martensite finish temperature. Now this behavior as we can see that the when temperature increases it follows a certain path when temperature reduces it follows a certain path. So, it is a path dependent behavior. So, to know the amount of martensite quantity present the martensite fraction just knowing the temperature is not sufficient. For example, if somebody tells me that the value of the present temperature is between M s and A s then if it is here, it can xi can be 0 if it is here xi can be 1 and that depends on what the temperature was before.

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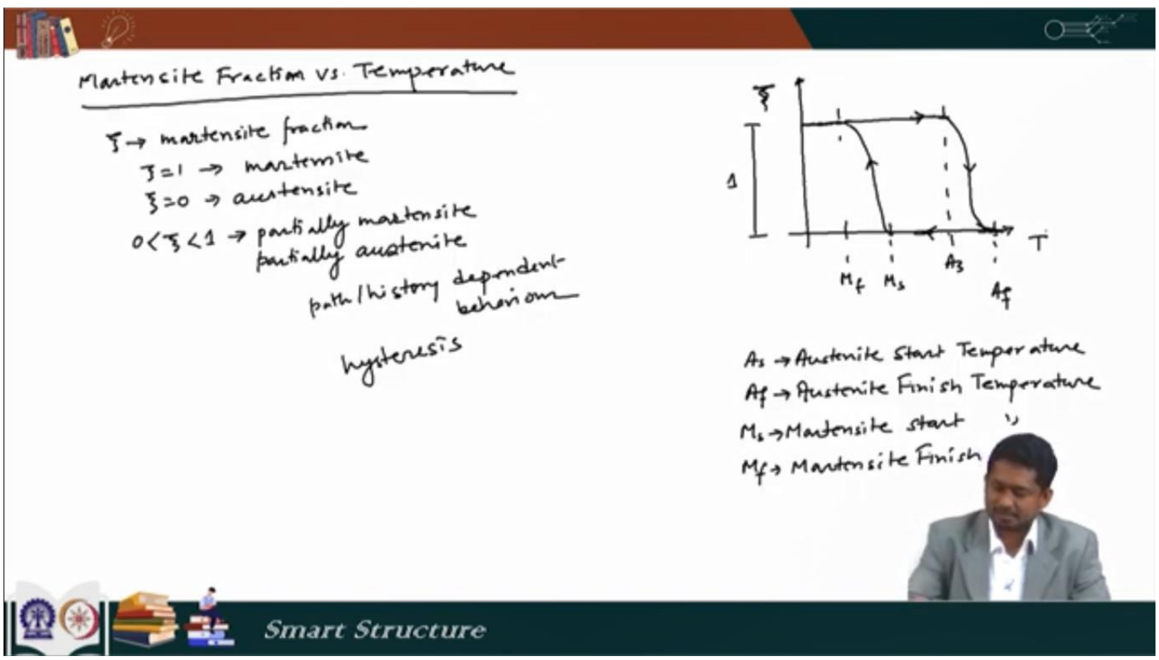


So, if the temperature was more than A f before and it has been cooled down to this temperature which is between M s and A s then we can say that xi is 0. On the other hand, if the initial temperature was if the initial temperature was less than M f and it is heated up to a temperature which is between M s and A s then xi is equal to 1. So, it is here. So, it is path dependent it is history dependent and it is historic. So, it is a path or history dependent behavior and it shows hysteresis and this is martensite fraction versus temperature behavior.

Now, we will try to fit some equation here and again we can see that this part of the graph it is just a constant. So, here xi is independent of temperature till A s in this part of the graph again it is constant xi is equal to 0 which is that is at M s and beyond whereas, this part between M s and A s and this part between M s and A s and A f. So, this part between M s and A M f and this part between A s and A f they show some curve variation and we need to mathematically model that. So, to do that so, there are different types of curves that has been fitted to it here we will talk about a curve that is that is based on cosine function. So, what we do is we consider xi to be a function like this.

$$\xi = C_1 Cos\alpha_M(T - M_f) + C_2$$

$$T = M_S$$

$$\Rightarrow 0 = C_1 Cos\alpha_M(M_s - M_f) + C_2$$

$$\alpha_M = \frac{\pi}{M_s - M_f}$$

$$T = M_f$$

$$\rightarrow 1 = C_1 + C_2$$

$$C_1 = C_2 = \frac{1}{2}$$

$$\xi = \frac{1}{2} Cos\alpha_M(T - M_f) + \frac{1}{2}$$

$$\xi = C_1 Cos\alpha_A(T - A_s) + C_2$$

$$T = A_f \Rightarrow 0 = -C_1 + C_2$$

$$1 = C_1 + C_2 \Rightarrow C_1 = C_2 = \frac{1}{2}$$

$$\xi = \frac{1}{2} Cos\alpha_A(T - A_s) + \frac{1}{2}$$

$$\alpha_A = \frac{\pi}{A_f - A_s}$$

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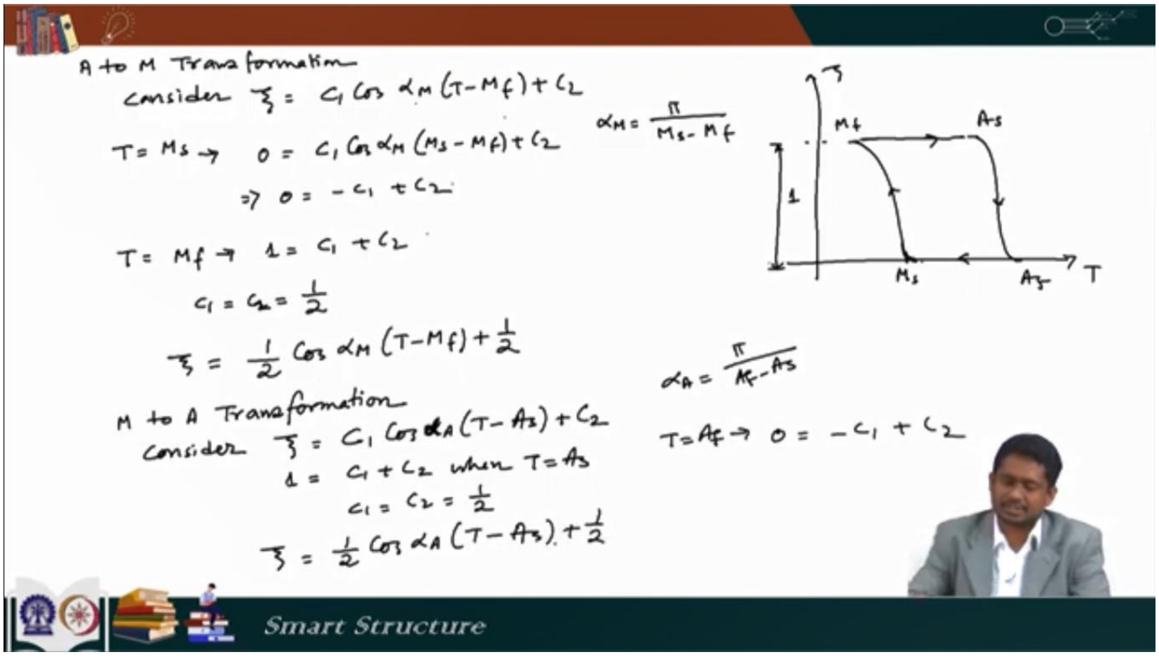


So, it is C 1 cosine alpha M t minus M f plus C 2 and this is for austenite to A to M transformation, austenite to martensite transformation. So, this is how we model the graph now all we need to know is the value of C 1 and the value of C 2. So, if we draw it once again quickly so, we have M f here we have M s here we have A s here we have A f here and this is the these are the parts. Now we are interested in modeling this graph this part of the graph and we are modeling it by cosine function. So, when the temperature is M s which means and we are considering austenite to martensite temperature that means, it is following this path.

So, when the temperature is M s the fraction of martensite is 0. So, T is equal to M s that tells me that xi is equal to 0 and then if we put the values C 1 cosine alpha M M s minus M f plus C 2 and also, we have one quantity alpha M here alpha M is pi by M s minus M f. So, alpha M is pi by M s minus M f. So, this quantity is just cosine of pi. So, this becomes zero is equal to minus C 1 plus C 2 because cosine of pi is minus 1 and then if you put the other condition which is T is equal to M f and at T is equal to M f with the material transform to martensite which means xi becomes 1 here.

So, xi is 1 here. So, we have 1 is equal to C 1 and then we put T is equal to M f when you put T is equal to M f this quantity becomes 0. So, it is cosine 0 which is 1. So, it is C 1 plus C 2. Now, we can solve these two equations and find out C 1 and C 2.

So, C 1 becomes equal to half and C 2 is also equal to half. So, finally, after putting the values the graph the variation is just this half into cosine alpha M multiplied by T minus M f plus half where alpha M is this. So, that is how we model this part of the graph again we can model this part of the graph also. So, for M 2 martensite to austenite transformation we can again consider xi to be equal to C 1 multiplied by cosine alpha a T minus austenite start temperature plus C 2 and then we can put the boundary conditions. So, when the temperature is increasing when it is at austenite start temperature is A s.

So, T is equal to a s at that time xi is equal to 1. So, 1 equal to and T is equal to A s means this quantity this cosine quantity is 1. So, it is C 1 plus C 2 when T is equal to A s and when T is equal to A f xi becomes equal to 0. So, T is equal to A f that tells me that. So, this is A f and we need to define alpha A that is pi by A f minus A s.

So, when T is equal to a f this quantity is just pi and cosine of pi means minus 1. So, at that time f equal to 0. So, that is 0 is equal to minus C 1 plus C 2 and again solving these two we get C 1 is equal to C 2 is equal to half and finally, after putting everything the equation comes to be half cosine alpha A T minus a s plus half. So, that is how this part of the graph this part of the curve can be model and this part of this part of the curve between m s and m f or m s and a f or between a s and a f they are just a constant their values are 0 or 1. Now sometimes it may happen that the transformation starts from a altogether different state.

For example, instead of from here or from here the transformation might come from start from here. So, it may happen that there is a temperature T 0 and at that temperature the material has martensite fraction as xi 0 as the martensite fraction where xi 0 is some non 0 value between 0 and 1. So, if the process starts at T is equal to T 0 and xi is equal to xi 0 and assume that is somewhere here. So, this point is we can assume to be T 0 and xi 0. Now from here if we reduce the temperature, it keeps reducing the, but the xi remains constant till a temperature of M s is reached.

So, here we have M s and here we have M f. So, till a temperature of M s xi 0 is maintained and after that it follows a curve and it becomes xi becomes 1 when M reaches when It reaches M f. Similarly, in this part in this part in this side if we increase the temperature, it maintains the same xi which is xi 0 till the temperature is A s and then if the temperature is further increased it does a transformation and xi becomes 0 when the temperature is A f. Now this part so, we can show it by a different color. So, this is the initial phase where we have temperature as T 0 and martensite fraction xi 0 and from here it is starting.

So, from here to M s it is just a constant and from here to A s it is just a constant, but from A s to M f we need to model it and from M s to M f we need to model it and from A s to A f we need to model it. Now in the previous case the transformation started from here the so, the initial condition was xi is equal to 0 at T is equal to M s, but here the transformation is starting from here. So, the initial condition is xi is equal to xi 0 at T is equal to M s. So, again if the transformation is starting at T is equal to T 0 and xi is equal to xi 0. In this case again we have the same assumption xi is equal to half multiplied by cosine alpha M T minus M f plus not half I mean half was in the previous set of conditions here we have to keep it free.

So, it is C 1 and plus C 2. Now we put the initial conditions. So, at T is equal to M s which is this point we have xi is equal to xi 0 C 1 cosine alpha M and T is now M s minus M f plus C 2 and we know that this quantity is pi because alpha M is pi by M s minus M f. So, this gives me xi 0 is equal to minus C 1 plus C 2 and then at the end when the temperature is a M f we have at T is equal to M f we have xi is equal to 1 and T is equal to M f means this quantity is 0. So, it is cosine 0 means 1.

So, it is C 1 plus C 2. So, now solving these two quantities we get C 2 is equal to 1 plus xi 0 by 2 and C 1 is equal to 1 minus xi 0 by 2. So, finally, the equation looks like 1 minus xi 0 by 2 cosine alpha M T minus M f plus 1 plus xi 0 by 2. See the process starts here and if the temperature is reduced till T is equal to m s xi remains constant xi 0 if it is further reduced then xi follows this equation. So, this is the path here. Now we will model the other part of it.

So, let us assume that we are increasing the temperature. So, while increasing till T is equal to A s xi remains xi 0 and after that it keeps reducing. So, we have M to A M to a transformation and starting at starting at T is equal to T 0 and xi is equal to xi 0. So, again we have the same consideration xi is equal to C 1 multiplied by cosine alpha A multiplied by T minus A s plus C 2 and now we will put the boundary conditions. So, that we have here we can see that when T is equal to A s when the temperature is austenite start temperature this quantity is 0.

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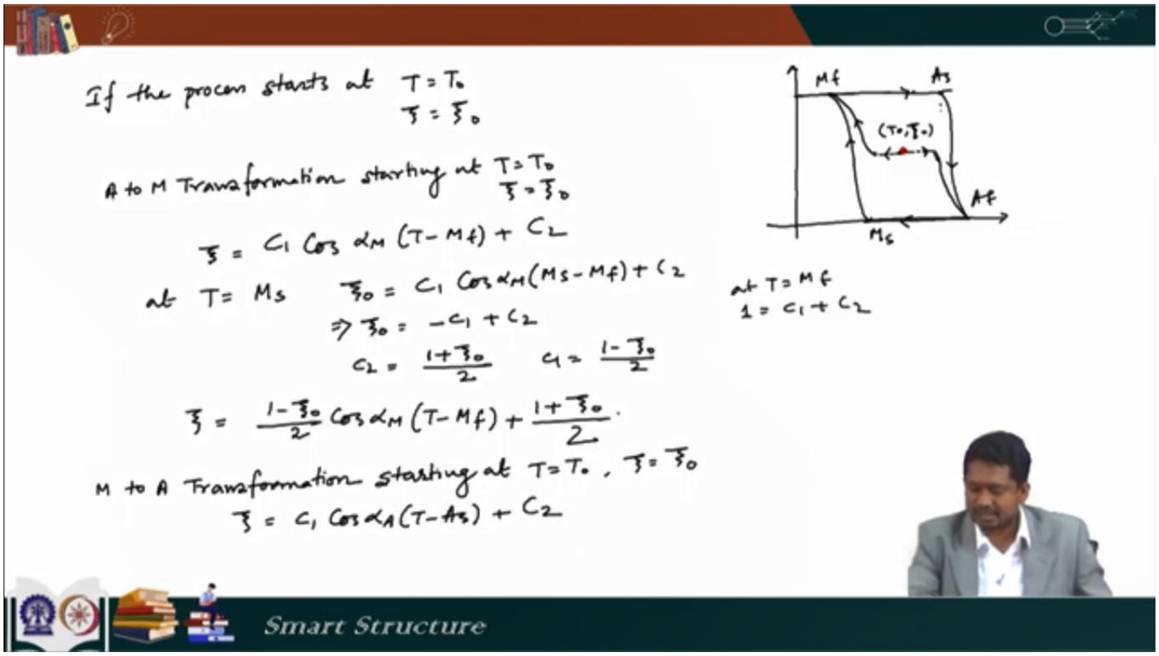


$$T = T_0, \xi = \xi_0$$

$$\xi = C_1 Cos\alpha_M (T - M_f) + C_2$$

$$T = M_s \Rightarrow \xi_0 = C_1 Cos\alpha_M (M_s - M_f) + C_2 \Rightarrow \xi_0 = -C_1 + C_2$$

$$C_1 + C_2 = 1$$

$$C_2 = \frac{1 + \xi_0}{2}$$

$$C_1 = \frac{1 - \xi_0}{2}$$

$$\xi = \frac{1 - \xi_0}{2} Cos\alpha_M (T - M_f) + \frac{1 + \xi_0}{2}$$

$$T = T_0, \xi = \xi_0$$

$$\xi = C_1 Cos\alpha_A (T - A_s) + C_2$$

So, it is 1. So, C 1 plus C 2 and at that temperature xi is equal to xi 0. So, we can write xi 0 is equal to C 1 plus C 2 and then when the temperature reaches A f at that time xi becomes 0. So, here we have 0 and A f means this quantity is pi. So, it is minus C 1. So, minus C 1 plus C 2 is equal to 0.

So, we have 0 is equal to minus C 1 plus C 2. Now, if we solve these two equations, we get C 1 is equal to C 2 is equal to xi 0 by 2 and then if we put it back our xi becomes xi 0 by 2 multiplied by cosine alpha A T minus a s plus xi 0 by 2. So, again when the temperature reduce as increases from T is equal to A s to A f and if the if at T is equal to A s our xi is xi 0 then xi follows this path. Now, so far, we have seen that these transformations are fully temperature driven. If we reduce the temperature the material converts to martensite, if we increase the temperature the material converts to, I mean transforms to austenite.

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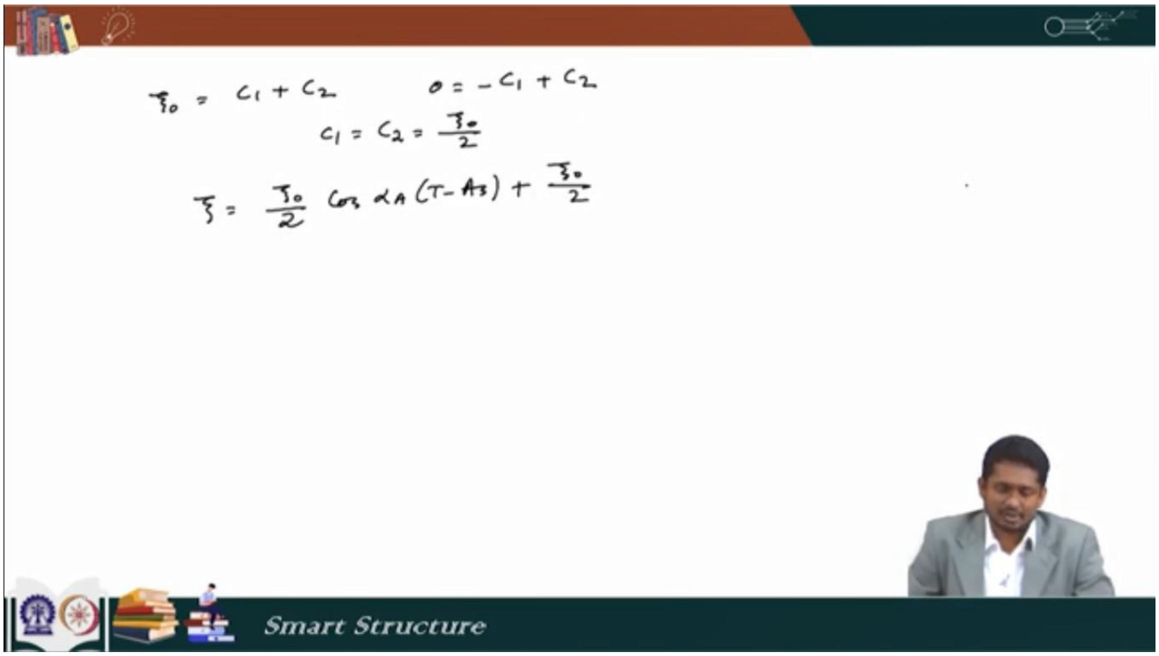


$$\xi_0 = C_1 + C_2, 0 = -C_1 + C_2$$

$$C_1 = C_2 = \frac{\xi_0}{2}$$

$$\xi = \frac{\xi_0}{2} Cos\alpha_A(T - A_s) + \frac{\xi_0}{2}$$

Now this kind of transformations can happen due to stress also. So, a transformation can be purely stress driven or it can be a combination of that. So, these characteristic temperatures M s A s M s A s M f A f they can be dependent on those stresses. So, this is when there is no stress under no stress these temperatures are this M s A s and M f A f. Now, under stress these characteristic temperatures can change.

So, if we look at this the if we look at a graph of how this characteristic temperature changes with stress it looks something like this. So, suppose that our stress is 0 and at 0 stress we have a martensite finish temperature and little higher than that we have martensite start temperature and then we have martensite austenite start temperature and we have austenite finish temperature. Now, if the material is under stress, then under the stress this temperature shifts towards right. So, at a higher stress the martensite start temperature will be somewhere here the martensite the martensite finish temperature will be somewhere here the martensite start temperature will be somewhere here accordingly the austenite start temperature and the austenite finish temperature. And if we join it if we do it for several values of stresses, we see that we get a straight line.

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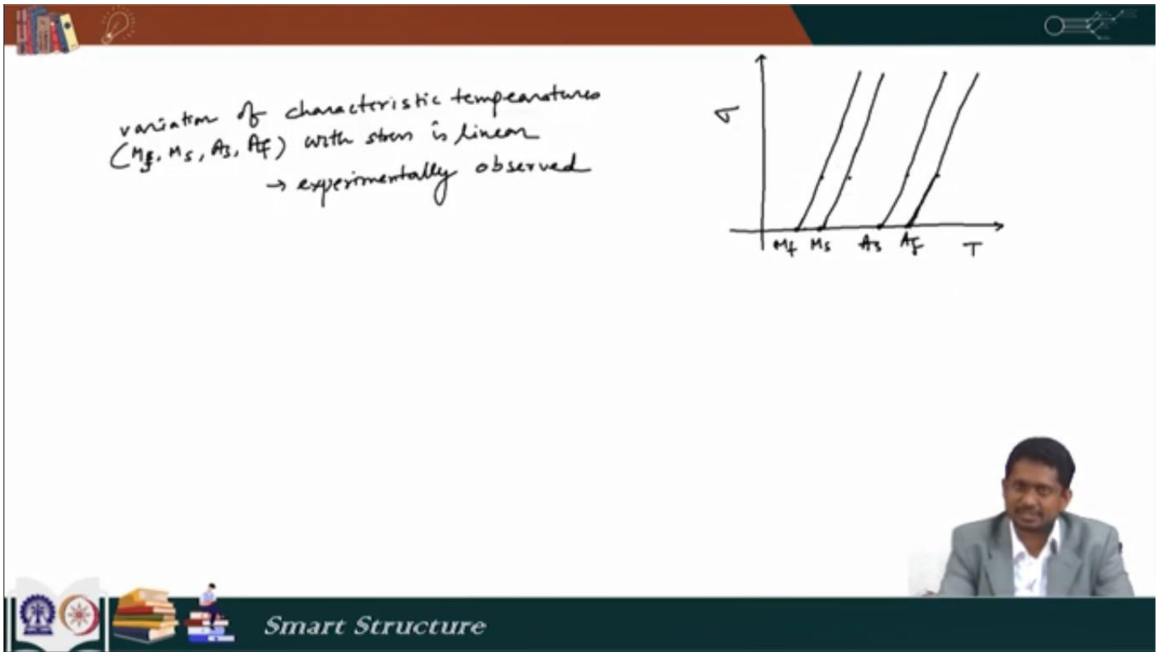


So, these are all are straight lines. So, this is we can say M f M s A s A f and as we keep increasing the stresses these characteristic temperatures keep increasing and this variation of characteristic temperatures which are our M s M f M s A s A f I am writing in the ascending order of the temperature of the value and this temperature the variation of this characteristic temperatures with stress is generally linear and this is experimentally observed. And also, if we look at these straight lines generally in most of the cases the slope of the straight lines corresponding to M s M f A s and A f are more or less same and these are basic experimental observation for most of the materials. Now the question is if the material is simultaneously is under some nonzero stress and then the temperature is varied then how does this transformation equation look like. So, that we will see in the next class let us stop this lecture here. In the next class we will bring in the effect of stresses in those transformation equations and we will see beyond that.

So, with that let me conclude the lecture here.

Thank you.