Smart Structures
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Week 07
Lecture No: 40
Analysis of composite laminate with piezoelectric patches – computer programming Part 05

In this lecture, we will discuss about solving the actuation problem in composites with the help of Piezos using MATLAB. So, we will write computer codes and we will see how this analysis can be done. We have seen that this kind of analysis involves a lot of computation. So, this is not something which is to be done manually for even slightly complicated case. So, for that we need to develop a computer code and solve the problem and that is what, we are going to discuss today.

So, this is the example structure that is going that we are going to take.

It is a composite with a set of layers, and it has two piezoelectric patches and at the top and bottom. So, the top piezoelectric patch and the bottom piezoelectric patch are throughout the surface of it. And in this example, there are six layers of composites 1, 2, 3, 4, 5, 6 and at the two sides of it, there are two piezoelectric patches. So, there are total eight layers. Among those eight layers, six are composite plies, and at the at the two opposite surfaces these are piezoelectric patches.

These are axis: x axis, this is y axis. From the x axis, let us denote the dimension as a and along the y axis, let us denote the dimension as b. And then, the piezoelectric patches would be excited with voltage and we will see the response.

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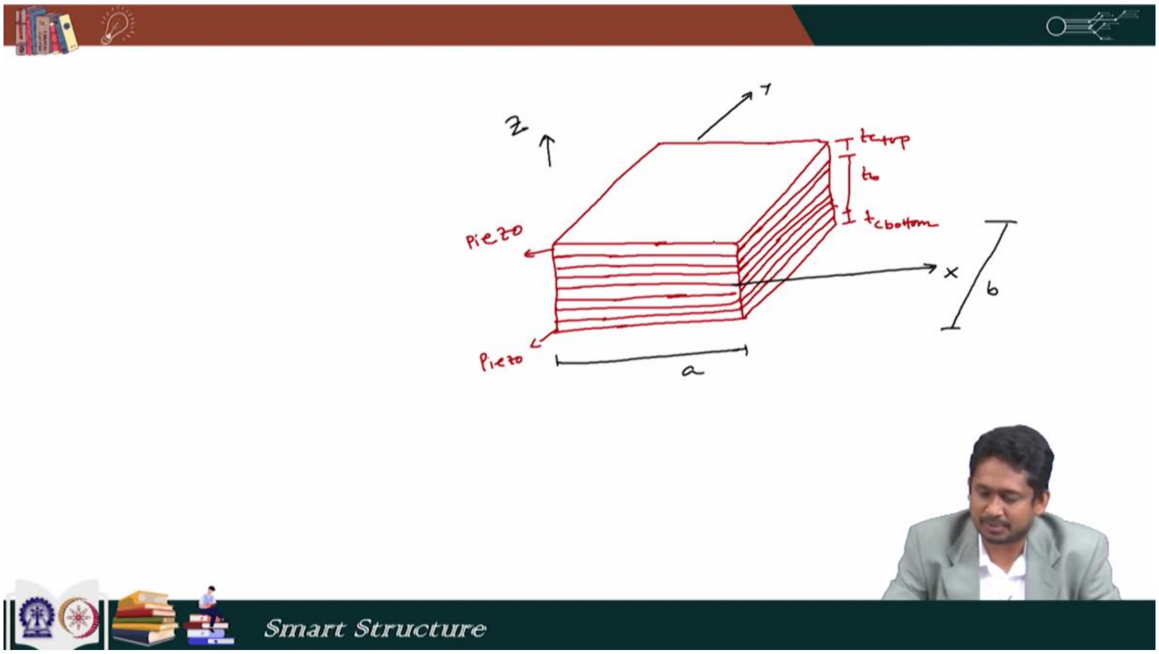

So, for that we have license from MATLAB for doing this exercise and the license is available in the course portal here. So, there is MATLAB menu and under that there is a MATLAB access and MATLAB training.

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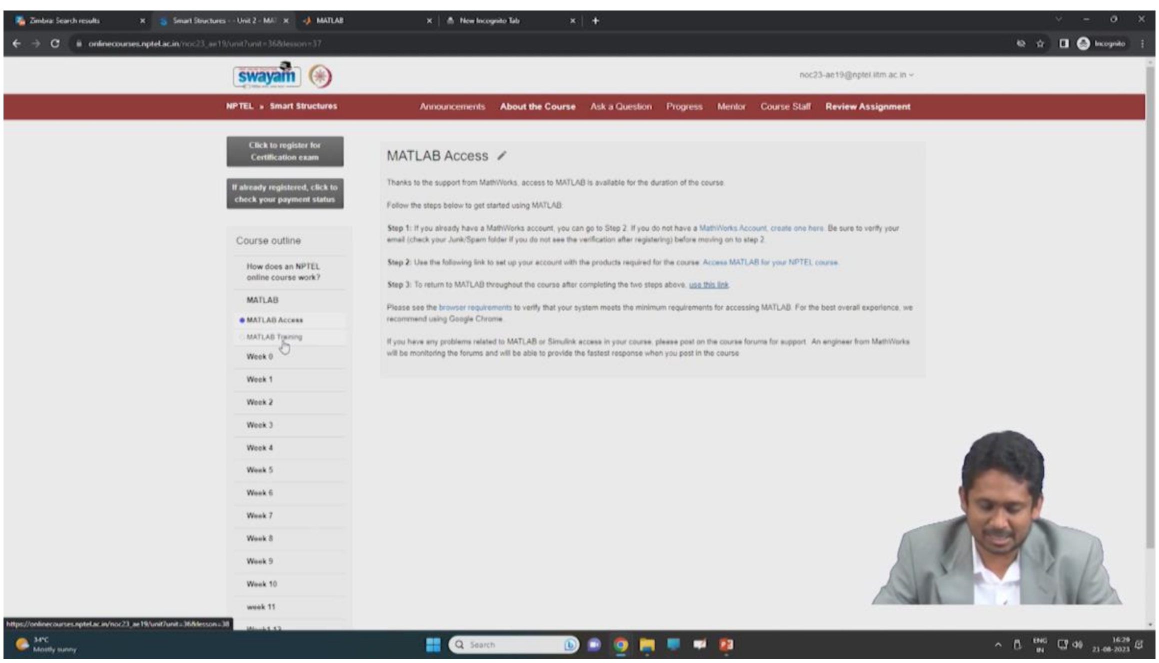

So, those who do not have much experience above with MATLAB they can check this training and however, we are not going use very advanced features of MATLAB. Basic features would do. Now to access MATLAB we can go to MATLAB here and the first two links through those, the sign up can be done, and after signing up we can access the MATLAB. So, after signing up we can come to this use link and that will take us to MATLAB.

So, if I go to this use link, then it will give us two options: one is open MATLAB online, one is installing MATLAB by going to the installation option, we can install it and use it. And by using this option open MATLAB online, we can do it through the browser. So, here I am doing it through the browser. So, if I open it here, it shows me this. So, the entire MATLAB front end is in our browser now. And these are the course which has been written. Now I am going to explain that.

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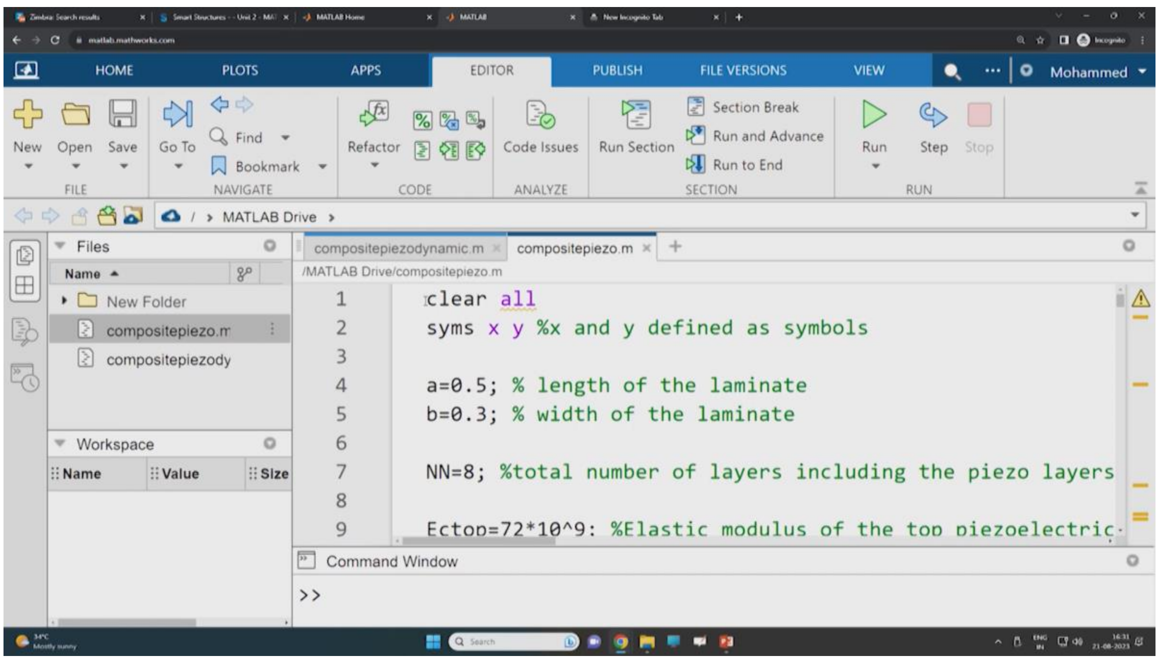

So, here we are going to use a symbolic toolbox. So, we will use the symbolic features. Initially we should do clear all, so in case there is something in any variable predefined that gets deleted otherwise, that can create some problem. And then x and y are our symbolic variables, which we are defining here. Now let us define the set of inputs.

So, here we have to first input the geometrical features. So, as we have seen that the plate has length of a and width of b, length means dimension along x axis and width means dimension along y axis. So, a is 0.5 and b is 0.3, now there are total 8 layers including the pieces and composites. So, that we are defining here as a variable NN. So, it is NN equal to 8.

Now, here we are writing the code in a more generalized way. So, we are considering the fact that the top piezo and the bottom piezo can be may not be identical, in terms of their thickness or in terms of their material properties. So, we define the variables Ectop that means, elastic modulus of the top piezo.

So, anything with a subscript top means that it denotes the top piezoelectric patch. So, Ectop means elastic modulus of the top piezoelectric patch, neutop means the Poisson's ratio of the top piezoelectric patch. rhoctop means the density of the top piezoelectric patch, and d31top means the d31 constant of the top piezoelectric patch. Similarly, we have Ecbottom, neubottom rhocbottom and d31bottom, they are the same quantities for the bottom piezoelectric patches. Now here, the quantities that we have given are same, but we can keep it different and that is fine.

Now, comes the properties of the composite plies. Now in here, we consider the fact that each ply has same material property and same thickness, but they can differ by – in fact, the thickness not need not be same, but we are considering same material property and their difference is in the orientation of the fibers. So, E1 E2 are the elastic modulus of each composite ply along two directions. E1 is in one direction which means the fiber direction and E2 is in the perpendicular direction. These are all unidirectional plies. neu12 is the major Poisson's ratio and then if I apply the formula, neu21 is equal to E2 multiplied by neu12 by E1, that gives me the minor Poisson's ratio. G12 is the shear modulus; rho is the composite ply density and theta means the orientation of fiber in each of the plies. So, there are 8 plies. So, the it goes on like this: 0, 90, 0, 0, 90, 0.

Again, if we go to the figure there is the first piezoelectric layer. So, here the orientation is 0, then 90 then 0 then again 0, 90, 0. So, it is a it is an example of cross ply which is symmetric, but again that is not a requirement, any kind of lamination sequence that we give symmetric anti symmetric or whatever, it would work even it did not be cross ply, it can be angle ply also.

Now, let us define the z's. So, z here means the z coordinate of each of these two sides of the layers. So, at the bottom most surface we have z1. So, here z1 means distance of the bottom most surface from the x y plane with the sign taken care of. So, bottom surface means we are going along the negative z direction. So, that is a negative quantity. The highest negative quantity in magnitude and then the next junction that is z2, then the next one z3, z4, z5, it goes all the way up to z9 because, there are total 8 layers. So, there are total 9 z quantities. So, if I subtract z1 from z2 that gives me the thickness of the bottom piezoelectric patch. Similarly, if I subtract zNN plus 1 that means, the last z to the 1, but last z. So, zNN plus 1, minus zNN, that gives me the thickness of the top piezoelectric patch and then, if I subtract zNN, I mean, if I subtract z2 from zNN that gives me the total thickness of the composite plies, that laminar thickness is – thickness of the entire composite laminate is that.

So, these are z's are arranged. So, we go from bottom to top. Now, let us give the force input. So, here the force can be distributed forces along the x y or z direction over the surface or there can be voltage actuation. So, qx means if there is any distributed force along the x direction that is q. So, here we are assuming it to be 0, we can give a constant value here or we can write it as a function of x and y also. Similarly, qy and qz vtop means the voltage that is applied to the top and vbottom means voltage that is applied at the bottom.

Now, after analysis we might be interested in finding out stress, strain, or any such quantity in any particular point and that is defined here. xp yp means the coordinate of that particular point and Lp means at which layer. Now, here the layer means we are not considering the piezo layer, here the layer is which composite layer. Again, if we go to the figure. So, xp yp denotes the location in the x y plane.

So, x means the value of the x coordinate and y is the y coordinate and then comes Lp, Lp means layer. So, while defining Lp, we call as the first layer and this we call as the second layer. So, that count of the Lp layer starts from the first ply layer not from the – So, the count for, Lp layer starts from the first ply, not the first piezo layer. So, it is 1 2 3 4, it is goes up like that.

So, in our case Lp means, 2 means, it is this layer. First there is a piezo layer, then ply number 1, then ply number 2. After that we have to go to define the Q matrix. Now, Q for the piezo layers is defined considering them to be transversely isotropic. So, in the x y plane, they are isotropic. So, if I know the Ec, I just divide Ec by 1 minus nu square, and then I multiply the matrix 1 nu 0; nu 1 0; 0 0 1 minus nu by 2, that gives me the Q for the piezo layer. So, Q 1 is for the first piezo layer and QNN is for the last piezolayer.

So, here we can see that we have defined Q as a 3 dimensional vector, the reason is that the first 2 indices are for the Q itself, I mean it is Q is a matrix. So, the element goes here that is why, we need the first 2 index and the last one, the last index is for denoting which layer it is. So, this denotes the Q matrix for the first layer. Similarly, this denotes the Q matrix for the last layer.

Now, we have to find out the Q matrix for the plies.

Now, while finding out the Q matrix for the plies because we are starting from E1 E2 nu12 nu21 and the G. So, first we need to find out the Q matrix with respect to the material coordinate and then we have to transform it to the x y coordinate. Now, because the properties of each ply are same. So, their Q matrix with respect to the 1 2 system is same. So, that is why we just define it once and we get it. This is the method for that. This is the equation for that, that we have discussed. And then we go through a loop which goes from 2 to NN minus 1, because the layer number 2 means the first ply layer, layer number NN minus 1 means the last ply layer. So, that is how we traverse. And while doing that, we define the transformation matrix. Here theta is the variable. So, theta denotes the orientation of the fiber. Here it is i minus 1 because i starts from 2. So, when i is equal to 2, I am in the first ply layer. So, that is why i minus 1, it denotes 1.

So, then we apply the same formula that we discussed T inverse Q into T and that gives us Q_{xy}. And then, we store that Q_{xy} in the 3 dimensional vector for Q. And accordingly, we finally get a 3 dimensional vector that contains the Q matrices for each layer. Now, that we have got the Q matrices, we have to find out the A B and D matrix. We know, the relations for A B D matrix and that is what is done here. And again, it has to be added over all the loops. So, we initialize A B and D to 0 and then, we write the formula for the A B and D matrix. And finally, at the end of the loop the A B D matrix is fully calculated.

Now, we have to define the basis functions, that we are going to use for this analysis. So, let us go back to the figure once again.

Here, the consideration that we are doing in this example, it can be changed also that is up to the user. Here, the consideration is that all the 4 edges are simply supported and that tells us that at x equal to 0 and a, and y is equal to 0 and b, which means at all the 4 edges my w is 0. And the bending moment may be 0, bending moment should be 0. So, at x equal to 0 and a, M_x is 0 and that tells us that D_{xx} is equal to d 2 w by dx 2, plus D_{xy} plus d 2 w by dy 2, is 0. And similarly, at y equal to 0 and y equal to b, M_y equal to D_{xy} equal to d 2 w by dx 2 plus D_{yy} multiplied by del 2 w by del y 2 equal to 0.

So, our approximation should satisfy at least this boundary conditions. If I can satisfy this, that is good, otherwise, even satisfaction of this, this would also serve the purpose. And then we have at x equal to 0 and a, we have, V_0 equal to 0. And we have N_x which means A_{xx} multiplied by del u_0 by del x, plus A_{xy} which means del V_0 by del y, equal to 0 and at y equal to 0 and y equal to b, we have u_0 equal to 0. And the natural boundary condition is N_y equal to A_{xy} multiplied by del u_0 by del x, plus A_{yy} del v_0 by del y, equal to 0. So, these are the set of essential and natural boundary conditions, if we can satisfy this in natural boundary condition also that is good. So, in this problem the function that we have chosen satisfy all the boundary conditions.

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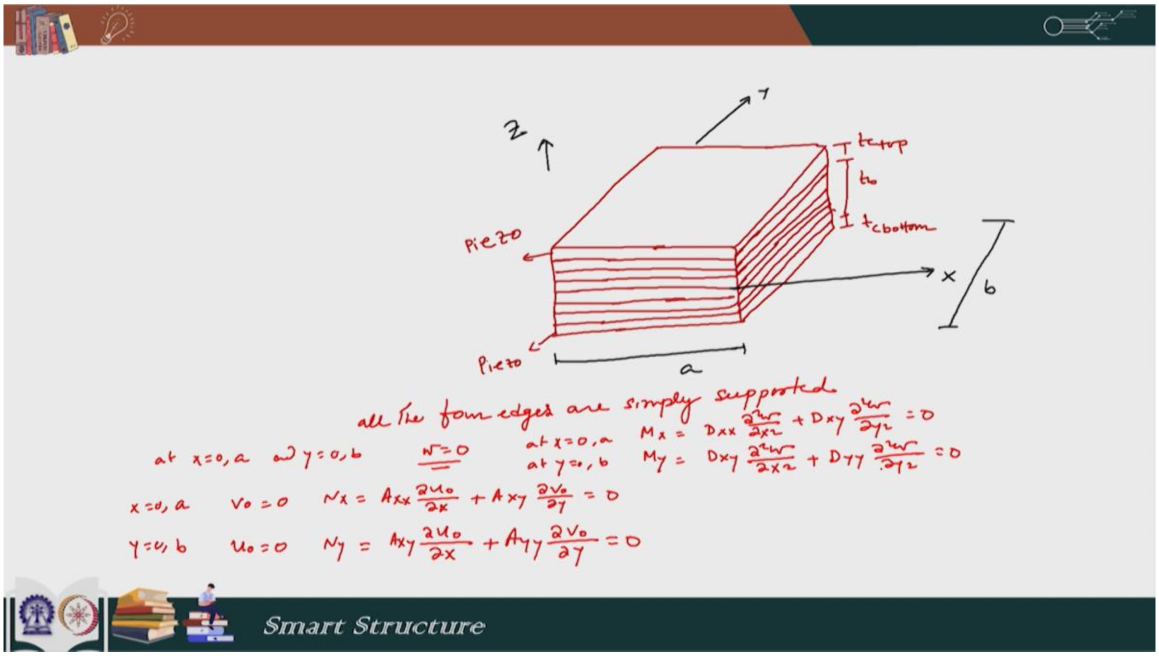

So, this these are what we have as phiu, phiv and phiw.

So, phi u is sin pi x by a, cosine pi y by b, and sin 2 pi x by a, cosine 2 pi y by b. Now, here we have just chosen 2 terms, we could have chosen multiple terms. In fact, while doing a problem in its entirety, we should keep increasing the number of terms until, the convergence is achieved. Here, for the sake of simplicity just for the demonstration purpose we are showing it with 2 terms. We could have chosen terms like sin pi x by a, multiplied by cosine 2 pi y by b, and also sin 2 pi x by a, multiplied by cosine pi y b, and so on.

Then there is a variable Nu, Nu is the length of this vector phiu. So, this basis functions are put in this array phiu and that is the length of it and that is Nu. Now, while discussing the theory we denoted this variable as m, but here this m might be used in for mass matrix also. So, that is why to avoid any conflict of variable names, we are using Nu here. Similarly, phiu is here, phiv is here. So, we are using this functions to define phiv and Nv is the length of it. And phiw, these are the functions used to define phi w and these are the length of it. So, it can be easily verified that this basis function satisfies all the boundary conditions.

Now, here we are defining the derivatives, because we are using the symbolic calculation. So, we can easily do the derivatives differentiation symbolically. And in MATLAB, the function that is used to do the differentiation is diff, d i f f.

So, diff phi u means which variable I am trying to differentiate and x means with respect to which variable. So, phiux means derivative of phiu with respect to x. So, phiu has two constituents, phiu1 and phiu2, this is my phiu1 and this is phiu2. So, it differentiates both and puts in an array phiux. Then we are finding out the derivative of phiu with respect to y. And then here we are differentiating phiv with respect to x. And then here we are differentiating phiv with respect to y. Now, here we are differentiating phiw with respect to x, and putting it in a variable in an array phiwx. Here we are differentiating phiw with respect to y.

Next, we need to calculate the second order derivatives of phiw. So, that is what we are doing here. Again, under differentiation phiw is what we are differentiating x is with respect to what and 2 is the order of differentiation. So, when we find out the derivative of first order, we did not need to put the order, but if we are doing it for second order or any higher order, we need to put the order and that is what we are doing here. So, this is derivative of w with respect to x done twice. This is second order derivative of w with respect to y. And it is a mixed derivative, derivative of w with respect to x and y. So, this is one derivative. And again, this entire thing is put under another derivative. So, that is how the mixed derivative is done.

So, after having defined these derivatives of the basic functions, now we define the quantities like NI, NW, BI, BW because if we go back to the formulation, we saw that our final equation contains these matrices like M_{II}, M_{IW}, K_{II}, K_{IW} and so on.

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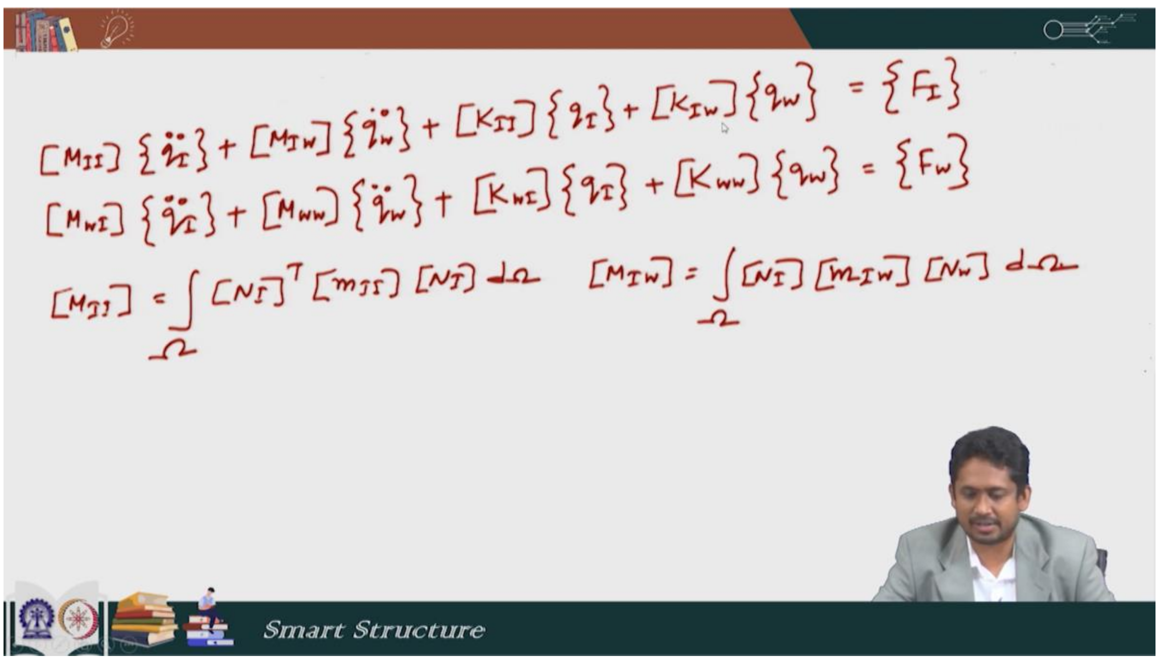

And to find out these, we need these quantities N_I, m_{II}, N_W, B_I, B_W and so on. And that is what we are doing here.

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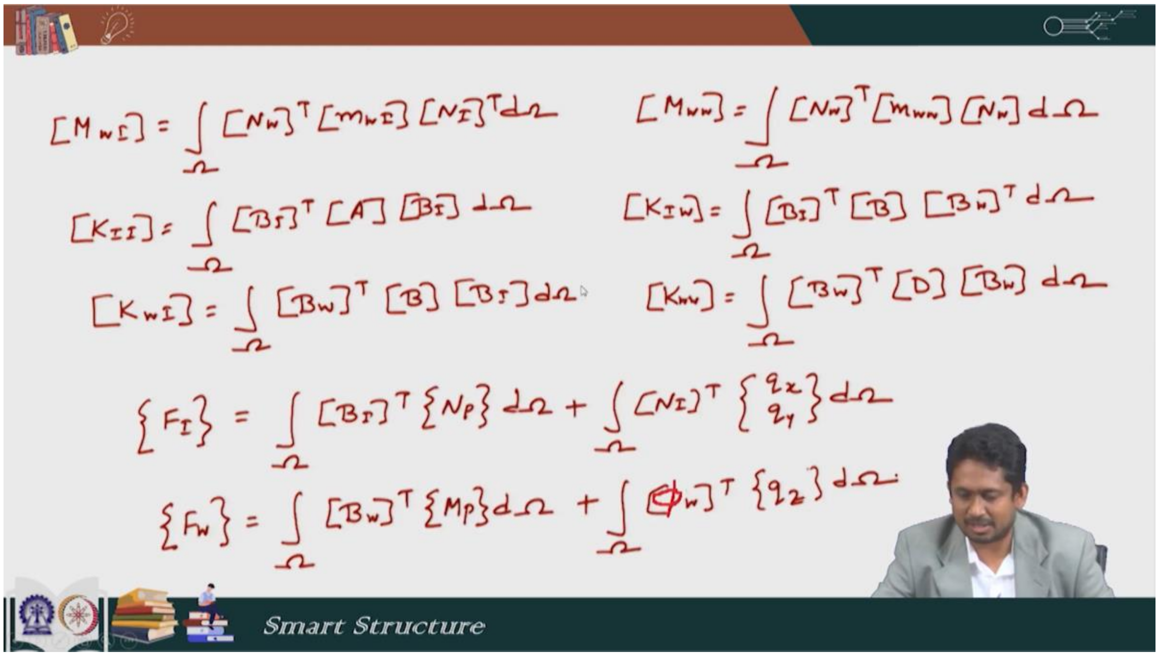

So, this is NI. So, as per the definition that we discussed while discussing the theory we have put it. So, in the definition of NI, first is phiu then 0s. Then in the next we put 0s, then phiv. And this is the definition of N_W that we discussed. Now, we define BI which involves derivatives of the basis functions, and that is how it is defined. So, again it is a row and below it is another row and below it is another row and it is Bw. It has 3 rows, the derivatives of phi is of different order with respect to different variables.

Now, we need to find out the thickness of the composite layers. So, total thickness of all the composite layers. So, we have seen that – if we again go back to the image, the total thickness of the composite layer is basically the difference of z at this point, minus at this point. So, at this point, the z is zNN minus 1, and here z is z2. So, if I subtract z2 from zNN minus 1, that gives me the total thickness of all the composite layers, and that is what is done here. And then we have found out the thickness of that bottom piezoelectric patch which is z2 minus z1, and the top piezoelectric patch which is zNN plus 1, minus zNN.

Next, we find out the quantities that are related to inertia. So, we have defined m, S and I, which are the mass, the first moment of inertia, and second moment of inertia and accordingly they are put in mII, mIW, mWI and mWW matrix.

Next, we find the components of the mass matrix. So, again if we go to the final equation that we use – here we have seen that, there are 4 matrices M_{II}, M_{IW}, M_{WI} and M_{WW}, and the formulas are like this. So, applying those formula, we found out the MII, MIW MWI and MWW matrix. Now, here one thing to note here, we are doing it symbolically. So, we can do symbolic integration and put the limits using the MATLAB symbolic toolbox, but that is not a general practice. In general, these integrations are done using numerical techniques.

For example, while using the finite element method, these integrations are done generally using the Gauss quadrature formula.

So, here symbolically the integrations are done. Now these are double integrals because the domain is 2 dimensional. So, we do that integral along x from 0 to a, once and then again, we evaluate the integral from y equal to 0 to b. And finally, this has been converted to a double variable because it is being done symbolically it might show it as a fraction. So, a double comment converts this entire result as a result to a double variable. So, we find out the constituents of the mass matrix.

Here using the same process, we find out the constituents of the stiffness matrix. We already know the formulas; we can just apply it and do it.

Now, comes N_p and M_p the block force and block moments. Now these are the relations that we used for block force and block moments. Now this relation for the block force was for one piezoelectric patch. If I want to do it for 2 piezoelectric patches, we can do it separately and add it up and that will give us the total block force for all the piezoelectric patches. So, here it was considering one patch and here it again it was considering 2 patches, 2 identical patches. So, this considers 2 identical patches. If the patches are not identical, then the formula would be little changed. So, it would be t_c, suppose we are doing it for t_c bottom. So, it would be t_c bottom. And here I would have t_b by 2, plus t_c bottom by 2, and same thing here. And accordingly, if I am doing it for the top piezoelectric patch. So, it is t_c top. Here it is t_b by 2 plus t_c top by 2. And then, we will do it for 2 patches and add them up that gives us the total block moment.

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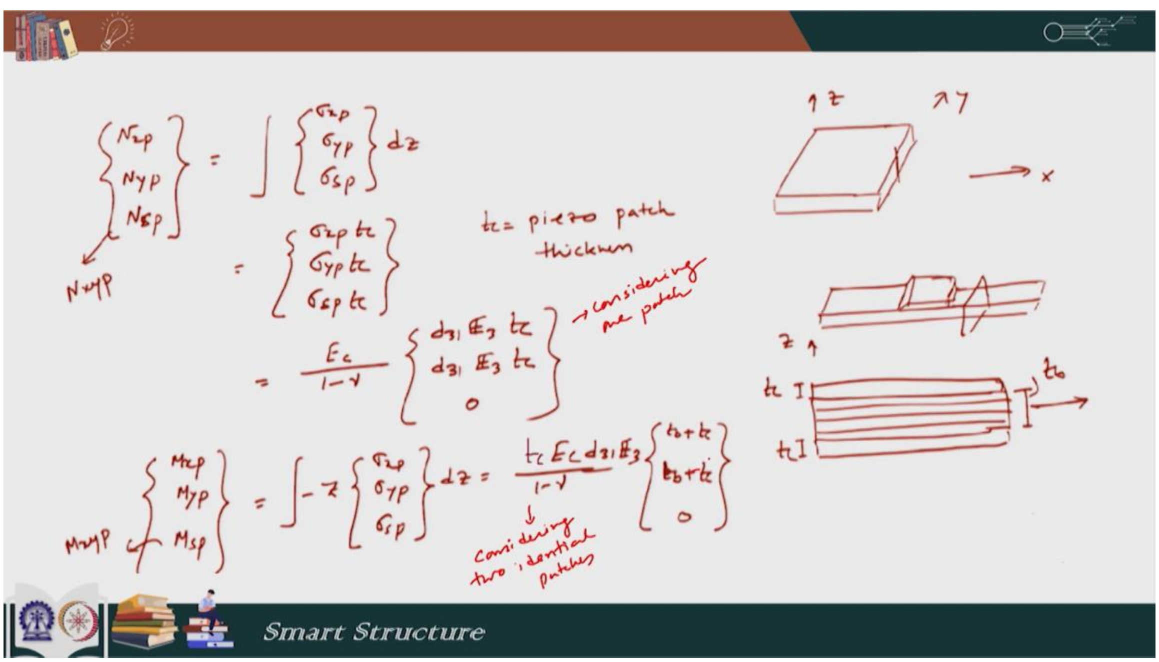

So, here it has been done in the generic way considering the piezos can be separate, their voltage can be separate, their properties can be separate. So, you find out the N_p and M_p. So, we do it for 2 patches and add it up and the added quantity is this. And similarly, the added block moment is this. Having found out N_p and M_p, we can find out the force vector. Force vector has 2 parts, FI is the in plane force vector and FW is the out of plane force vector and we are already familiar with the formula for the in plane and out of force vectors. Considering, the block force, block moment and the distributed in plane and out of plane forces and those also involves evaluating integrals in 2 dimensions and that is what we have done here. Just we have applied the same integral formula symbolically.

And finally, after calculating the M, K and F matrix, we are assembling everything here. So, that gives us one particular set of equations. So, here for example, if I combine everything and write it in this way M_{II}, K_{IW}, M_{IW}. So, combining everything it gives me full mass matrix, full stiffness matrix, and full force vector. So, then that gives me equation of the form M_q double dot, plus K_q equal to F. By solving, which we can find the entire q vector which has qu, qv and qw.

Now, here we are solving it as a static problem. So, the mass matrix we do not need. So, here we are just solving K_q equal to F. And there is a command in MATLAB that is a linsolve. So, that solves the system of equations K_q equal to F, and the returns the variable here. So, q is our solution.

Now, once we get our q, that means, we know all the unknown coefficients associated with each of these basis functions. After that, we can find out our quantities of interest like strain components and the stress components, at our point of interest and we defined our point of interest as a point with coordinate xp yp and composite layer number Lp. So, epsilon0p that means, epsilon 0 evaluated at that point p that is independent of the z coordinate. So, we find out the value of epsilon 0 at that point by this. We know that if I multiply the BI matrix with q, that would give me the epsilon 0. Now here, if I just multiply, it will give me the symbolic variable. And then, if I want to put the value of xp and yp at x and y, we have to use this comment subs under this everything. So, this expression and in the expression, x y should be substitute by xp and yp. And accordingly, kappa is defined. And after defining kappa, we have to find out the zp that means, the z coordinate of the point.

And we are finding out at the mid location of the Lp th layer. So, for that we can just use the formula and that will give me the z coordinate of the mid location of our layer of interest. Then, we can apply this equation, epsilon equal to epsilon 0 minus z into kappa, and that is going to give us the epsilon vector, the strain vector. And if, I multiply the strain vector with the corresponding q vector because if the Lpth layer. So, Lpth composite layer. So, absolutely it is 1 plus Lpth layer.

So, if I multiply the corresponding q matrix with that epsilon vector, that is giving us the sigma, the stress at that layer. So, we have found out the stresses at that particular point. So, accordingly we can be interested in find out stress strains at multiple points. Then, we have to define multiple points and we have to put this entire thing under a loop.

Now, we might be interested to look at the deflected shape for that what we can do is we can divide the entire surface into a grid point. And let us divide that into a 20 by 20 grid. So, Ndivx means the number of divisions along the x, Ndivy means number of divisions along y.

So, here we define a variable xn and yn. So, that will contain the x and y coordinate of the of our grid points. So, if we draw it it looks like this. So, here I have the grid and by grid points I mean these points. So, we are finding out the x and y coordinate of all of these points.

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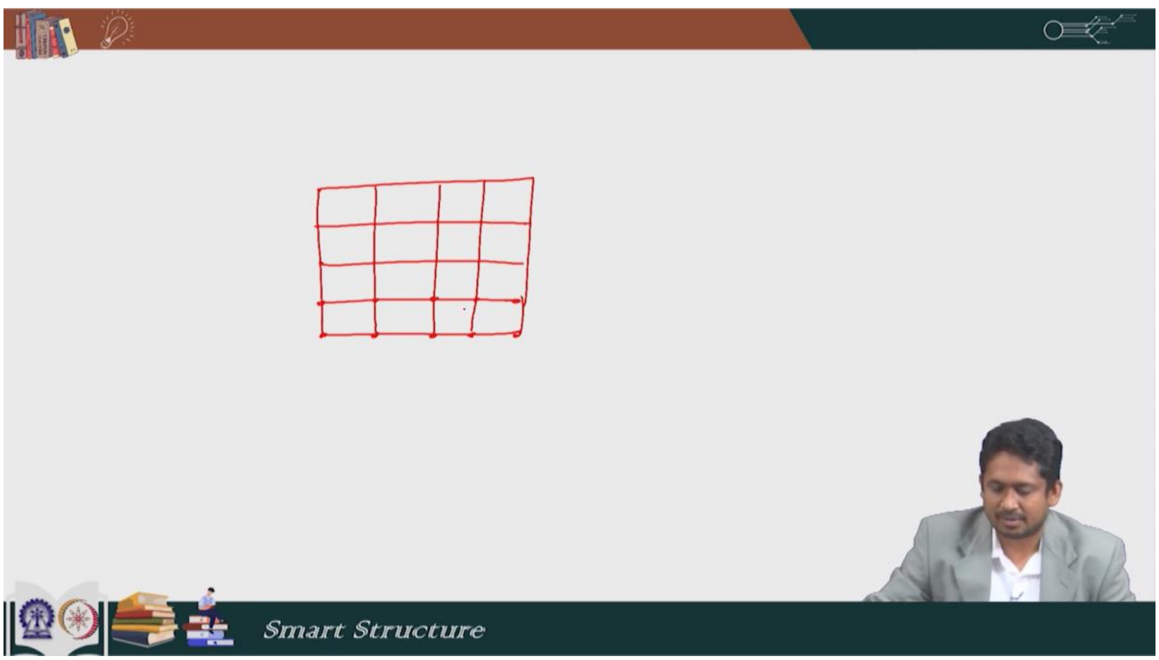

And that is what we are doing by that piece of the code. So, here we are finding out the coordinate of the grid points. And then at those grid points, we find out u, u_0 v_0 and w. Again, we use the substitute command because we have found out our q.

So, if I multiply the first any elements of q with the components of phiu that gives me u_0. And here, it will give me as a symbol. So, on on that if I substitute x and y by this grid points are the coordinates, numerical values that would give me the actual value of u_0 at that point, and same thing we do for v_0 and w. So, un means the value of u_0 at the grid points, v means the value of the v_0 at the grid points, and wn means the value of the w at the grid points. It is a 2 dimensional vector because i denotes the grid point number along the along the x direction, j denotes the grid point number along the y direction. So, finally, after that we can do a surface plot and we can visualize the displacement.

So, now after looking at the code if we run it. So, it is running now. If I want to stop it in between, we can just tap the button it would stop the code. So, now, it has run and it is showing how w is varying along x and y. Now, we have not put the x and y level that we can put easily. So, if I type x level and then it is X the unit in meter.

So, it puts the xlevel here. Similarly, ylevel also can be put. So, that is the y level. So, it is y direction in meter and then z level is the displacement w, that we were we are visualizing that is zlevel. So, it is w.

So, now, accordingly we can plot u_0, v_0 also. So, if I say un and if I just evaluate that particular line itself, that would plot u_0 for me. If I say v here, if I evaluate that that would plot v_0 for me.

And also, we were interested in finding out the stress which has already been calculated. So, if i write sigmap because, that is the variable that we used for the stress at that pth point, point p. So, if just type sigmap that is going to give me the sigma p as an array, the first quantity is our normal stress along x direction, second quantity is normal stress along y direction, this is the shear stress in the x y plane. So, this was about the code considering it is a static problem.

If, we want to consider it is a dynamic problem then, the entire code almost remains same. Then, the entire code almost remains same, only thing is that at the end here mass matrix would be used.

And at the end, we have to do the time marching using the beta new mark method. So, again if we just quickly go to the beta new mark method that we discussed. So, this was the beta new mark method considering a system M_q double dot plus C_q dot plus K_q equal to F. And this is the time marching scheme that we discussed. So, at first, we find out q. So, here n plus 1, means n plus 1th time step in our code that is I. So, if we want to follow our code then in the beta new mark part n plus 1, is i and n, is i minus 1.

So, at the i th time step which is n plus 1 here. So, q dot is this, entire quantity which is a function of the qn double dot, qn dot and qn at the previous time step which is known to us. Now here, we have not considered damping. So, these quantities associated with C matrix would be 0. And after finding out qi double dot, we can find out qi and qi dot using these two relations.

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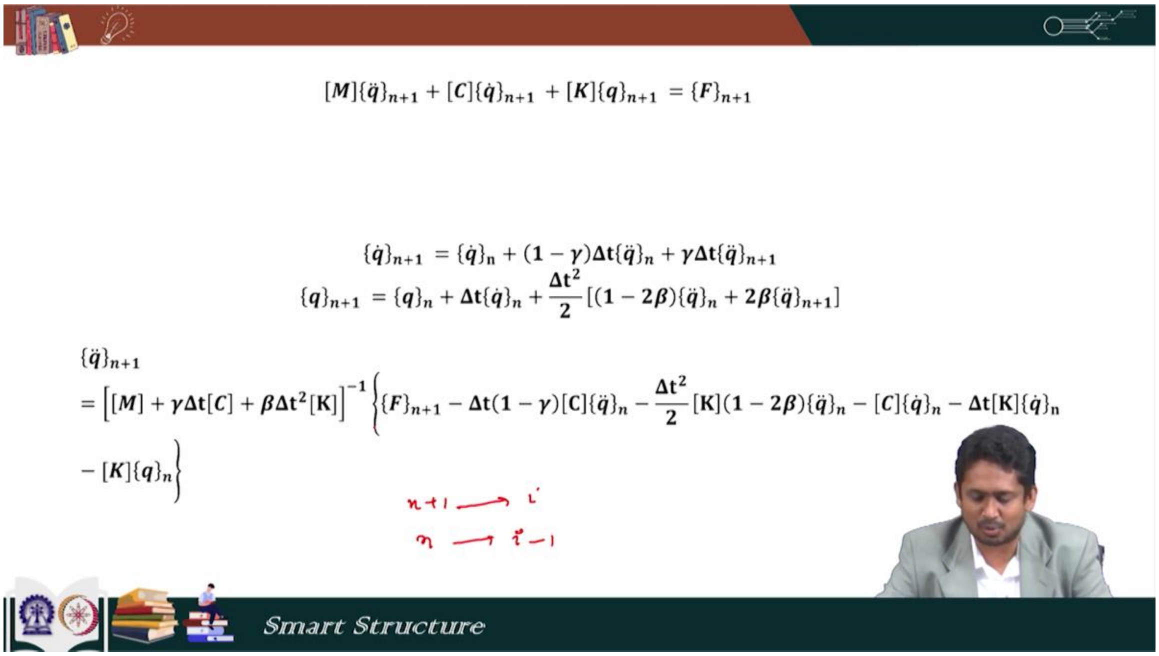

So, that is what has been implemented in the code here. So, we divided the entire time domain here, the entire time domain T was divided into set of time steps.

So, here the deltat is 0.005, that we have chosen it can be different depending on the problem. And then, suppose we are running the analysis till 0.8 second. So, we write an array like this that contains values like 0, 0.005, 0.01, 0.015 and so on till 0.8. That means, it contains the values of all the time instance and that is put in an array ti and Nts is the number of time steps, which is in this case that is going to be 161 because our deltat is 0.005 and our ti is 0.8. And also, the voltages can be a function of time. Similarly, the qx, qy, qz also can be function of time. Here, we have assumed that they are function of time of the form omega t. So, the voltage can be V sin omega t, qx can be q sin omega t, and so on. So, for that we have defined the corresponding omegas. So, omegax means the frequency corresponding to qx, omegay means frequency corresponding to qy, and omegaz means frequency corresponding to q, sorry. So, here omegax means frequency corresponding to px, omegay means frequency corresponding to py and omegaz means frequency corresponding to pz and omegaqdd is the frequency corresponding to the voltage. So, this voltage also can be a function of time. So, if it is sin omega t, the corresponding omega is this.

So, what we do here is – we run a loop. So, initially we initialize the quantities q, qd and qdd to 0, because at the initial time step, time t equal to 0, everything is 0. And, in beta new mark method, we have some parameters beta and gamma, we have set the parameters here. And then, we are running a loop from 2 to Nts number of time steps.

Now, here the force matrix has to be calculated at each and every time step, because that is a function of time. So, we have multiplied the expression of the force matrix into a quantity sin omega t, and the omegas whether it is omega x omega y and omega z that depends on which component we are evaluating. So, here we are evaluating the it corresponding to the voltage. So, it is omegaqdd and here it is corresponding to the force along x and y. So, it is omega x and omega y.

Now, we after evaluating this FI, we have got FI and FW as a function of time. We get our force vector at each and every time step. And then from that using the technique that we described, we find out q double dot, qdd means second derivative of q with respect to time, qd means first derivative of q with respect to time and q is q. So, evaluate that at each and every time step and we do the time matching. And, each after evaluating at each time step, we might be interested to find out u, v and w or quantities like epsilon and sigma, that we discussed before at our points of interest at each and every time step. So, here we are evaluating up, I mean, u_0, v_0 and w. So, here we are not evaluating u_0, v_0, w in the entire grid, rather here we are evaluating only one point. We could have found out u_0, v_0, w in the entire grid also as a function of time, but we are not doing that here to keep it simple. So, we are evaluating u_0, v_0 and w at point p at every time step and that is what is done here the same substitute command.

And here, we are finding out sigma and the time loop ends. So, if I run this code now. So, here we are printing this i. So, if I look at the command prompt, I can know at which time step the code is running. So, it is soon going to be at 160 first time step and come out of it.

Now, if I want to find out w as a function of time. So, we have seen that w is a one dimensional and now because we are not evaluating w at each and every point, we are evaluating at only one point. So, the only index that it needs is the time index I. So, if I evaluate w as a function of time. So, ti is the time and corresponding w. Ok, it was wp not w the variable. So, we can see w has been evaluated here. So, w is varying as a function of time in this fashion.

Accordingly, we can plot the variation of sigma, variation of epsilon and variation of other displacement quantities. And we can put the x y level also by using the command that we saw or, by using these buttons xlevel, ylevel and so on.

So, that was an overview of how we can do these calculations by writing a simple code in MATLAB. And again, the code has been written in a more generic way. So, the piezoelectric patches can be different and the basis functions, that we have chosen, here are sin functions depending on the boundary conditions, they can be different and accordingly proper basis function has to be put.

So, with that I would like to conclude this demonstration here.

Thank you.