Smart Structures
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Week 06
Lecture No: 35
Mechanics of FRP Composite Laminate Numerical Examples (continued)
Part 06

In this lecture, we will look into some numerical examples related to composites. So, here, there is one problem which is related to a composite ply and the ply is as usual unidirectional for our case. It contains 60 percent by volume of carbon fibers. The properties of carbon fiber is given and there is epoxy matrix. The properties of epoxy is given. We have to find out the longitudinal elastic modulus and tensile strength.

So, the given data is - we have V_f is equal to 0.6, 60 percent as given. So, V_m which is 1 minus V_f is equal to 0.4 and our goal is to find out the elastic modulus E_1 of the composite. So, as we know, the equation is E_{1f} multiplied by V_f plus E_m multiplied by V_m, E_{1f} is given to us. So, 294 multiplied by 0.6 and E_m is given to us, 3.6 multiplied by 0.4 and the entire thing in GPa and finally, the value comes to be 177.84 GPa.

Now, we have to find out the tensile strength, which means, what is the stress this ply can bear. Now, here if we keep increasing the stress. So, as we looked at before. Suppose there is fiber and matrix, if we keep increasing the stress, our strain remains same for both fiber and matrix and accordingly, based on their elastic modulus, they are stressed. So, when we keep loading it, either the matrix or fiber will fail first and whatever fails first, accordingly that determines the tensile strength of this material or the tensile stress of the composite during failure.

Now, we know that, we know the ultimate tensile stress in the fiber and its corresponding Young's modulus. So, the ultimate tensile strain is - for the fiber, ultimate tensile strain is $sigma_{1fu}$ divided by E_{1f}.

Now, $sigma_{1fu}$ we know 5.6 GPa and E_{1f} is 29.4 GPa, and that gives the value to be 0.01905. So, that is the amount of strain it can bear, the fiber. Similarly, the ultimate tensile strain of the matrix is $sigma_{mu}$ divided by E_m and $sigma_{mu}$, we know it is 105 Mega Pascal and the elastic modulus is 3.6 Giga Pascal. So, if we divide it, the value that we get is 0.0292.

So, we can see that the fiber material reaches its ultimate strain first. So, if we keep loading it, the fiber would fail first. So, during failure, the strain in fiber would be 0.01905. So, in the strain in the matrix would also be 0.01905. Now, when the strain in the fiber is 0.01905, the stress is 5.6 GPa, but the stress in the matrix is not in the ultimate stress, rather it is 0.1905 multiplied by the elastic modulus. So, if we want to find out the stress in matrix during failure of fiber. So, you may want to call it epsilon$_m$, let us we call it epsilon$_m$ star and at that time the stress is 0.01905 multiplied by 3.6 and that in Giga Pascal. And stress in fiber during failure is epsilon$_{1f}$, if we call it epsilon$_1$f star that is equal to epsilon$_{1fu}$ and that is equal to 5.6 GPa. So, finally, the tensile strength of the ply is - if the stress in the matrix is sigma$_m$ star. So, its contribution to the tensile strength, it is just sigma$_m$ star multiplied by 0.4 and then for the fiber part, we multiply that with 0.6 and the final value that we get is 3.3874 Giga Pascal.

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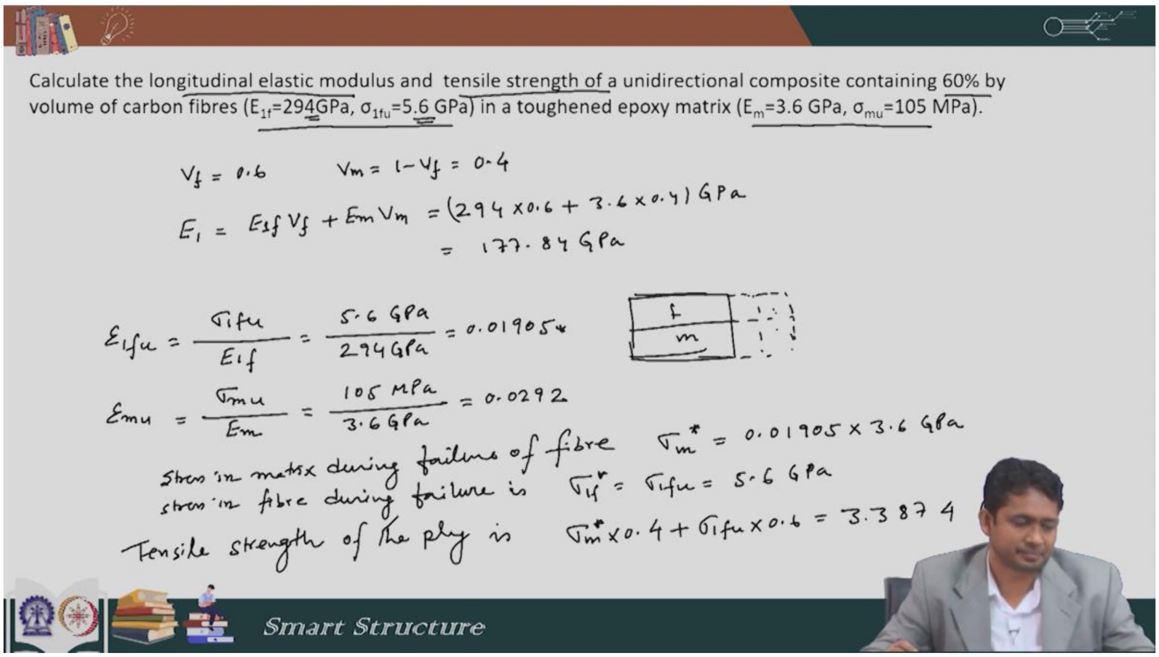

Now, let us look into another problem.

Here we are supposed to evaluate the transverse modulus, E_2 of a composite ply. If the properties given are - E_{2f}, the transverse elastic modulus of the fiber is 14.8 GPa, E_m is 3.45 GPa, nu$_m$ is 0.36 and V_f is 0.65.

Now to do this, first we need to find out E_m star and E_m star is E_m by 1 minus nu$_m$ square. So, 3.45 GPa by 1 minus 0.36 square and that gives E_m star as 3.964 GPa. And then E_2, we can have as it is and V_f is given to be 0.65. So, V_m is 1 minus V_f, which is 0.35. Then we can find out our elastic modulus in the transverse direction, which we get from this relation, 1 by E_2 is equal to V_m by E_m prime plus V_f by E_{2f} and then, after solving this equation, E_2 is found to be 7.56 GPa.

Now we need to find the same thing using the Holpensie relationship. To use the Holpensie relationship, we need a factor called eta and eta$_1$ is found as E_{2f} minus E_m by E_{2f} plus xi$_m$ multiplied by E_m. Now let us use the xi$_1$ to be 1. So, if we put this numbers here, if we put

xi_1 is equal to 1 finally, the value of eta comes to be 0.622. Now, if we use the Holpensie relationship, where eta is there, then E_2 comes to be E_m multiplied by 1 plus xi_1 eta_1 V_f divided by 1 minus xi_1, sorry, 1 minus eta_1 into V_f. And then, if we put all these numbers finally, the value comes to be 8.133 Giga Pascal. So, we can see that there is some difference between the experimentally obtained value and the sorry, the value that we obtained from the rule of mixture based equation and the Holpensie equation.

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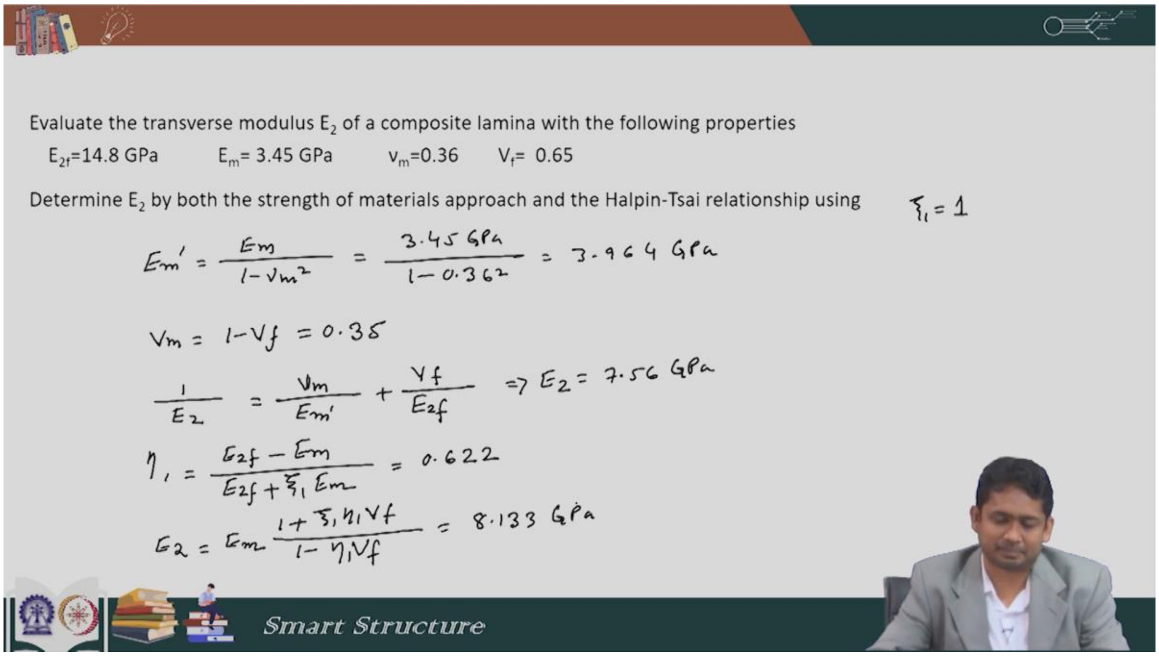

The next problem is - there is a glass epoxy specimen, the weight of which is 0.98 gram and it was burned and after burning, the matrix burns out and what remains is the fiber, the weight of the remaining fiber was found to be 0.49 gram. Now the densities of glass and epoxy are given, we have to find out the density of the composite in the absence of void and then it says that if the density of the composite is little different and is given 1.5 gram per milliliter which is obtained experimentally, then what is the void fraction?

So, if the experimentally obtained density is less than what we get analytically, that means, there is some void which is causing the less density and we have to find out the void fraction.

Now, here we are given the weight of the fiber. So, if we divide the weight of the fiber by the density, we will get the volume of the fiber which we can denote as small v_f. So, volume of the fiber is 0.49 divided by 2.4 and that would give us: that is in milliliter and that would give us the volume of the fiber.

Now, we know the total weight of that composite which is 0.98 gram and we know the volume of the fiber. So, if we multiply the volume of the fiber with the density of the fiber that gives us the weight of the fiber that is fine and that we already know. Now if we take the volume of the matrix which is unknown to us and if we keep that as an unknown. So, if we take that volume and multiply it by the density of the matrix and then we get the weight of the matrix. And the weight of the matrix and weight of the fiber gives the total weight of the composite. So, in this entire equation our only unknown is the volume of the matrix which we can solve by find by solving this equation.

So, total weight of the specimen is equal to 0.98 gram and that we can find out by - we already know the weight of the fiber which is 0.49 gram plus if we take the volume of the matrix in milliliter and if we multiply the volume of the matrix with the density of the matrix. So, that gives us the total weight of the specimen which is this. So, we can solve it and find out V_m. So, V_m is 0.98 minus 0.49 and the entire thing divided by 1.2 and that will come in milliliter. So, we know the total volume of the fiber, we know the volume of the matrix now. So, the total volume of the specimen, which we can call as V and that is equal to V_f plus V_m, which is now known to me and from here density of the composite can be found out which we can call rho. So, it is total weight of the composite which is 0.98 and divided by the total volume of the composite and that will come in gram per milliliter because these volumes are in milliliter and this weight is in gram. So, from there we can find out the final density of the composite and here, this comes to be 1.6 gram per milliliter.

Now, the question is our theoretically obtained density is little higher than what is experimentally obtained and the reason behind this difference is the presence of voids. So, the void volume fraction can be found as rho_{ct}, which means theoretically obtained density minus rho_{ce} which is the experimentally obtained density divided by rho_{ct} and then it comes to be 0.0625. So, rho_{ct} is our 1.6 gram per milliliter, and rho_{ce} the experimentally obtained density is 1.4 gram per milliliter.

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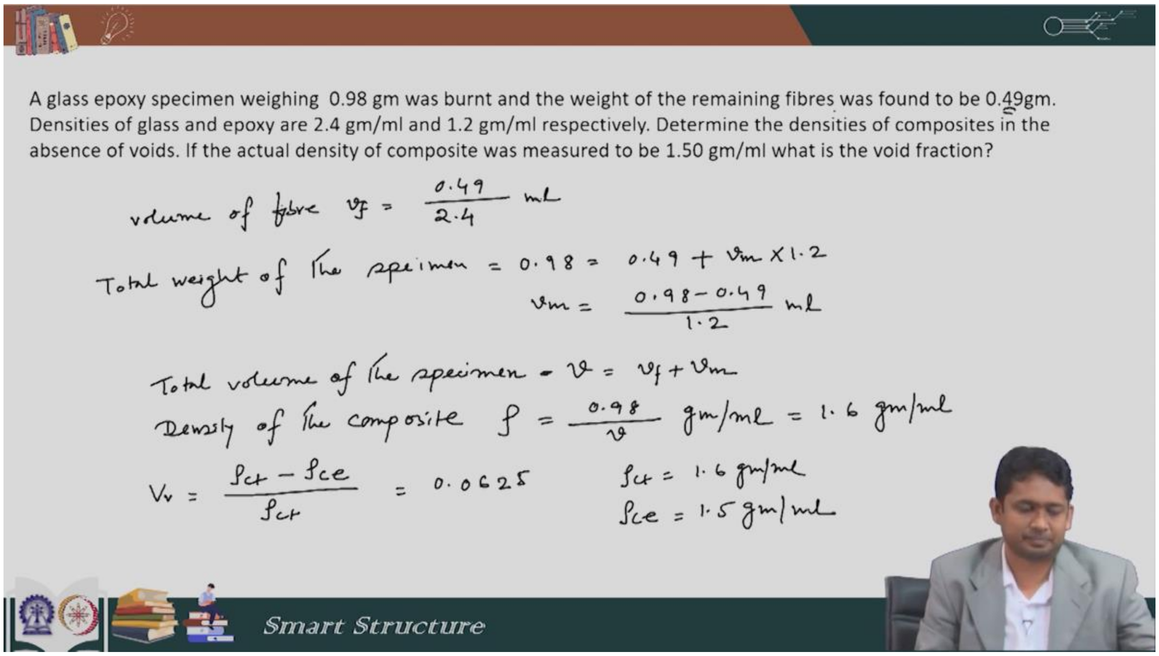

Now, this problem involves finding out the reduced stiffness matrix in both the principal material direction, and 1 2 directions. We have been given the properties like E_1, E_2, E_6, nu_{12} and we know the ply angles: 0 degree, 90 degree, 45 degree and minus 45 degree. So, we have to find out the Q matrix in the 1 2 system and in the x y system if the triangles are this.

Now, to do this, we need to first find out nu_{21}. So, we already have nu_{12} and we know the relation between nu_{21} and nu_{12}, and that is E_2 multiplied by nu_{12} divided by E_1. So, if we do that, the value that comes is 0.0207. So, we can see that there is significant difference between nu_{21} and nu_{12}. Now, we have to find out the constituents of the Q matrix in the material direction. So, we have Q_{11}, and Q_{11} is E_1 minus nu_{12} nu_{21} and that gives us the value to be 145.75 GPa. Then we have Q_{22}, and Q_{22} is E_2 by 1 minus nu_{12} nu_{21} and the value is 12.06 GPa. And then we have Q_{12} is equal to Q_{21} and this we can write as nu_{12} E_2 by 1 minus nu_{12} nu_{21} or also we can write nu_{21} E_1 by 1 minus nu_{12} nu_{21}. And the value comes to be 3.016 GPa. And then we have Q_{66} and this is E_6 and that is - as given it is 6 GPa. So, the terms like E_{16} sorry, Q_{16} Q_{26} are absent here. So, there is no coupling between the shear and the normal components. So, these are Q matrix in the principal material direction.

Now, we have to find out this Q matrix for different fibre orientations. So, if you talk about a 0 degree ply. So, in a 0 degree ply would look like this. So, here the fibre as oriented in the global x direction only. So, here system 1 2 axis and x y axis, they are same here. So, there is no difference between the Q matrix with respect to x y and Q matrix with respect to 1 2. So, we can write here Q with respect to x y system for 0 degree orientation is just the Q matrix that we get for the material direction. So, it is 145.75 3.016 0 and then, here we have 3.016 12.06 0, 0 0 6 and that is in GPa. And then, if the fibres are oriented suppose in 90 degrees. So, in this case these are x axis as well as our axis 2 is in this direction. So, as well as it is same as axis 2 just in opposite sense and here, we have y axis and axis 1. So, here we can say that, the properties can be found out easily from here. Now, because our fibres are oriented in y direction here, so, Q_{11} would be just $Q_{\gamma\gamma}$ and similarly, Q_{22} would be Q_{xx}. So, without even having to do any transformation, we can find out the properties here. So, 12.06 now goes for Q_{xx}, then 3.016, 3.016 and then 145.75, then 0 0 6 in GPa. So, that is for 90 degree.

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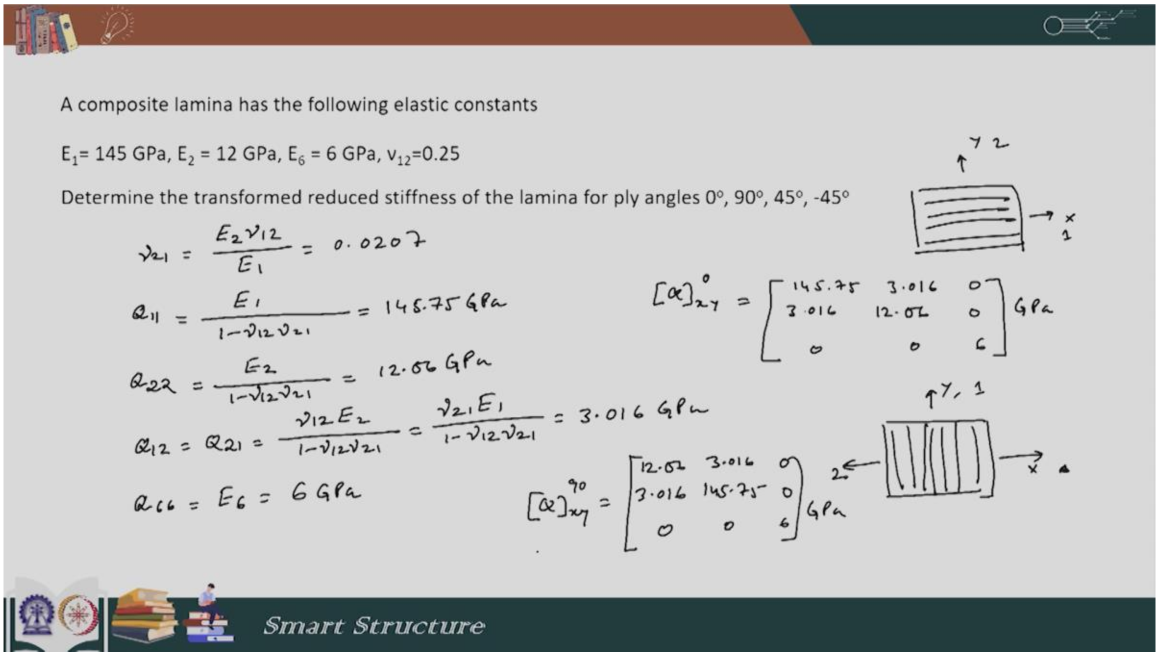

Now, we have to find it out for the 45 degree orientation. So, we know, for orientation theta, that can be found out by applying some transformation. So, to do that the equation looks like this. So, we can put all 2s here and the expression is T inverse multiplied by Q in the 1 2 system multiplied by T. Where our T is the transformation matrix, which is m square, n square, 2mn; n square, m square, minus 2mn; and then we have minus mn, mn, m square minus n square, where m is equal to cosine theta, and n is equal to sine theta. So, we can find it out for any theta. So, this is the equation. So, we can do the transformation and all these elements are found out. Then finally, in the Q matrix, we have Q_{xx}, $Q_{x\gamma}$ and Q_{xs}. So, this factors of putting 2 and then getting it removed, this was all because of we are defining 2 kinds of shears, one is engineering shear, one is tensorial shear and then we can apply this formula, and get our Qs. So, if we apply theta is equal to 45, the matrix comes to be 46.96 34.94 33.40 and then we have 34.94 46.96 and again 33.40, and the matrix is symmetric. So, we get the other properties.

Now, here this angle of orientation is 45 degree. So, which is just between 0 and 90. So, that is why you are getting Q_{xx} and $Q_{\gamma\gamma}$ to be same, Q_{xs} and $Q_{\gamma s}$ to be same, but for any other orientation like 30 40 60, we would not get this, we can get these properties different, but because it is 45 degree, we are getting this to be same. If we apply the similar transformation, considering it to be minus 45, then it is this. Our Q_{ss} changes. So, these are the values of Q matrix that we get for different types of, I mean different fiber orientations.

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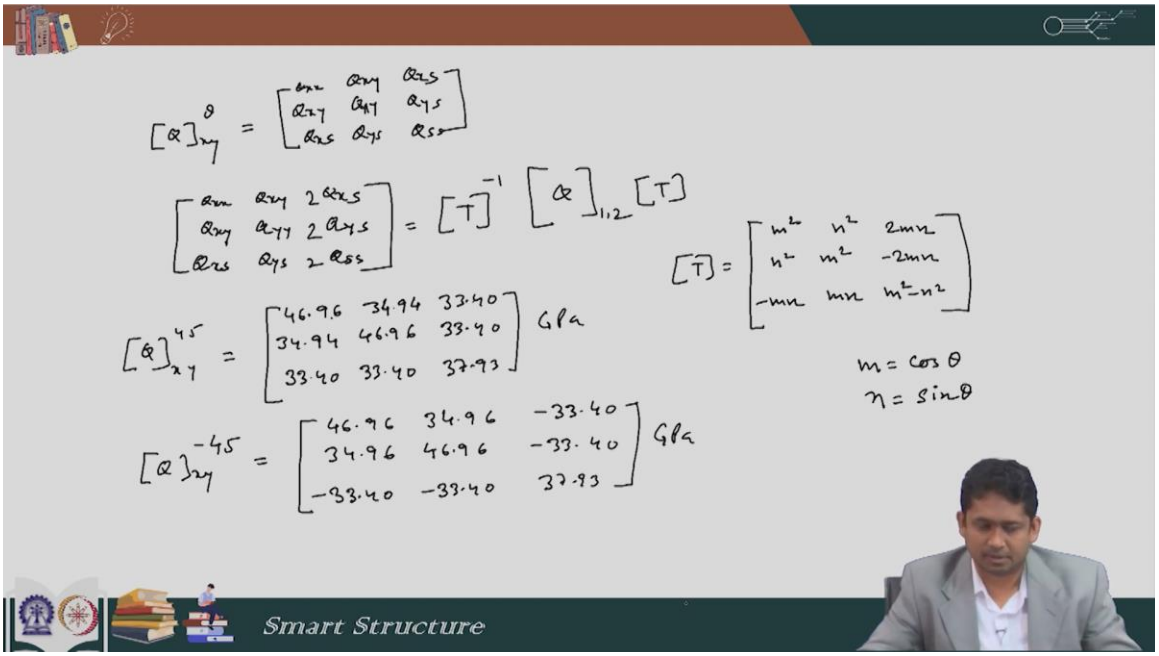

Now let us see, if we make a laminate using these plies with different fibers of orientation, how the laminate properties look like, how the A B D matrix look like?

So, let us assume that our fiber laminate orientation is this: 45, minus 45, minus 45, 45. So, if you want to find out the A matrix. So, it would look like this first of all. So, there are 4 layers and let us assume that each layer has a thickness of 0.5 millimeter. So, this is z is equal to 0, this is z is equal to minus 1, this is z is equal to minus 0.5, and all are in millimeter. This is z is equal to 0.5 millimeter, this is z is equal to 1 millimeter.

Now, we want to find out the A matrix, we know that the A matrix is k is equal to 1 to n, number of layers. And then, Q matrix in the x y system for the kth layer multiplied by z_k minus $z_{k \text{ minus } 1}$. So, what we are doing is, we are just taking Q and integrating over each layer and adding it up. So, we can call this as z_0, this as z_1, z_2, z_3, z_4 and n in our case is equal to 4. So, finally, if we add this up. So, in this case because thicknesses are same, it is nothing but adding up all the Q matrix and finally, multiplied by the thickness of each ply. So, finally, after doing it, after putting all the numerical values, the A matrix comes to be this, that is in GPa millimeter. Now, we want to find out the B matrix. The B matrix in this case would be 0, because it is symmetric. We can do it also and see it.

Now, let us find out the D matrix. And to find out D matrix what we do is: we take k is equal to 1 to n and Q matrix of the kth layer multiplied by z_k cube minus $z_{k \text{ minus } 1}$ cube by 3. And then, after putting all the values, what we get is this 31.307, 23.293, 16.7; 23.293, 31.307, 16.7 then, we have 16.7, 16.7, 25.287 and that is in GPa millimeter cube. So, here this matrix is symmetric. And because it is symmetric, we have B matrix as 0, and apart from that this matrix is balanced also. So, we can see that number of 45 is two and number of minus 45 is two and that makes A_{xs} and $A_{\gamma s}$ as 0, but this laminate is not antisymmetric and because it is not antisymmetric, we do not have these two components as 0.

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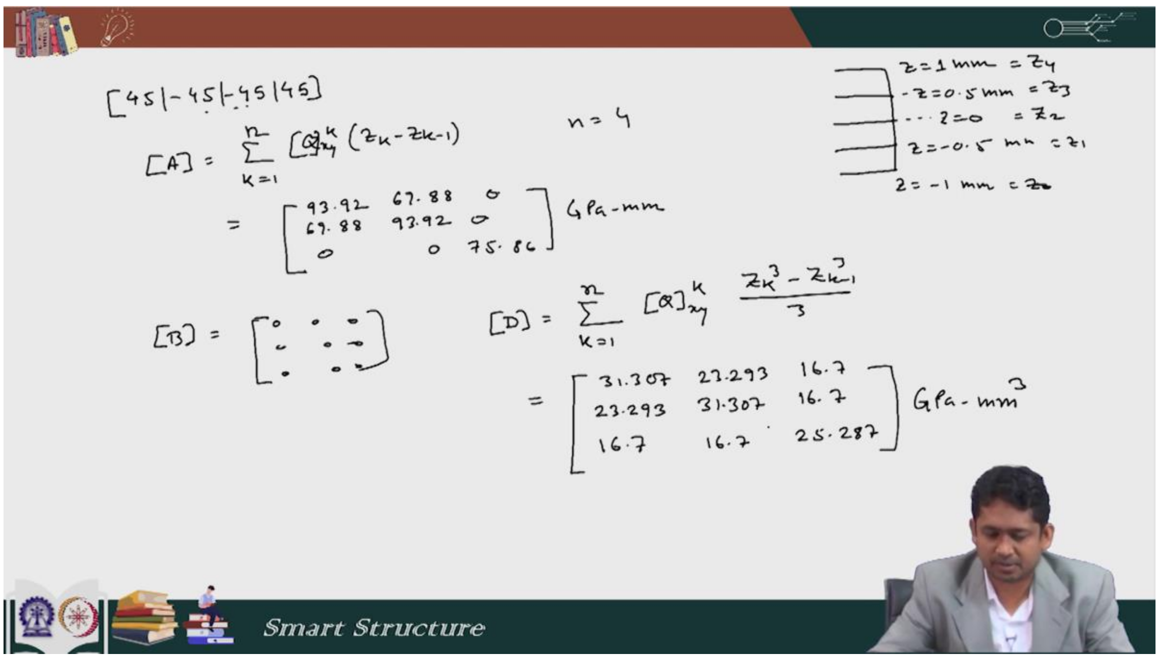

So, with this I would conclude this lecture here.

Thank you.