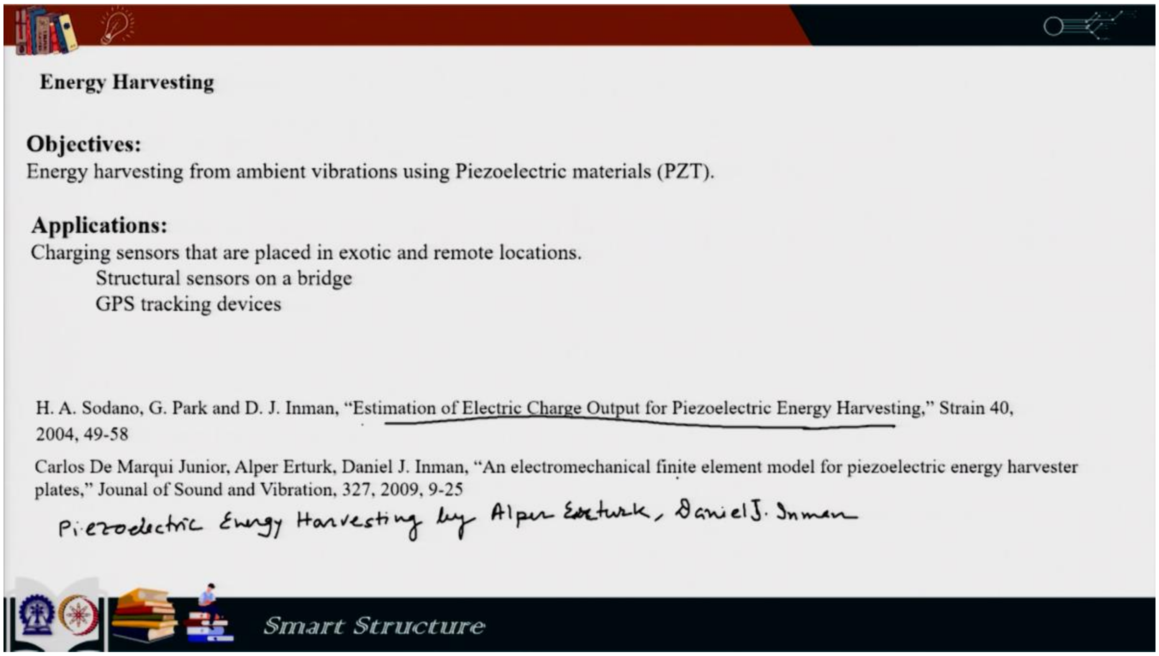
Smart Structures
Professor Mohammed Rabius Sunny
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur
Week 05
Lecture No: 27
Energy Harvesting and Vibration Control
Part 03

In this lecture, we will discuss the analysis of a beam with piezoelectric patches for energy harvesting applications. The objective is to harvest energy from ambient vibrations using piezoelectric material. The applications are charging sensors that are placed in remote environments. So, these sensors are generally requiring low power. So, the amount of power is not a problem, but the problem is changing the power supply for them. For example, if you think of a structural health monitoring applications of a bridge structure or any other structure.

Now, the sensors may be embedded somewhere and those locations may not be always accessible. So, to change the batteries that powers the sensors is not an easy job and doing it regularly is not something very desirable. Similarly, a GPS tracking device that also works at a remote location. So, if instead of having to change the batteries, if it is possible to harvest energy from the vibration of the structure on which it is mounted, then it becomes independent of any external power requirement.

So, that is the motivation of doing this kind of works. So, with this motivation several amount of research work has been done considering various type of structures. Here, we will very closely follow this research paper by Sodano, Park and Inman, Estimation of Electric Charge Output for Piezoelectric Energy Harvesting. And also, we will look at another paper from the same research group and Electro-mechanical finite element model for Piezoelectric Energy Harvester plates. And apart from that this book Piezoelectric Energy Harvesting by Atak and Inman is going to be helpful. So, we will follow the formulation that is laid out in this paper and also, we will see how the formulation looks when we look at the notations or the analysis methodology that we use.

(Refer Slide Time: 3:38)

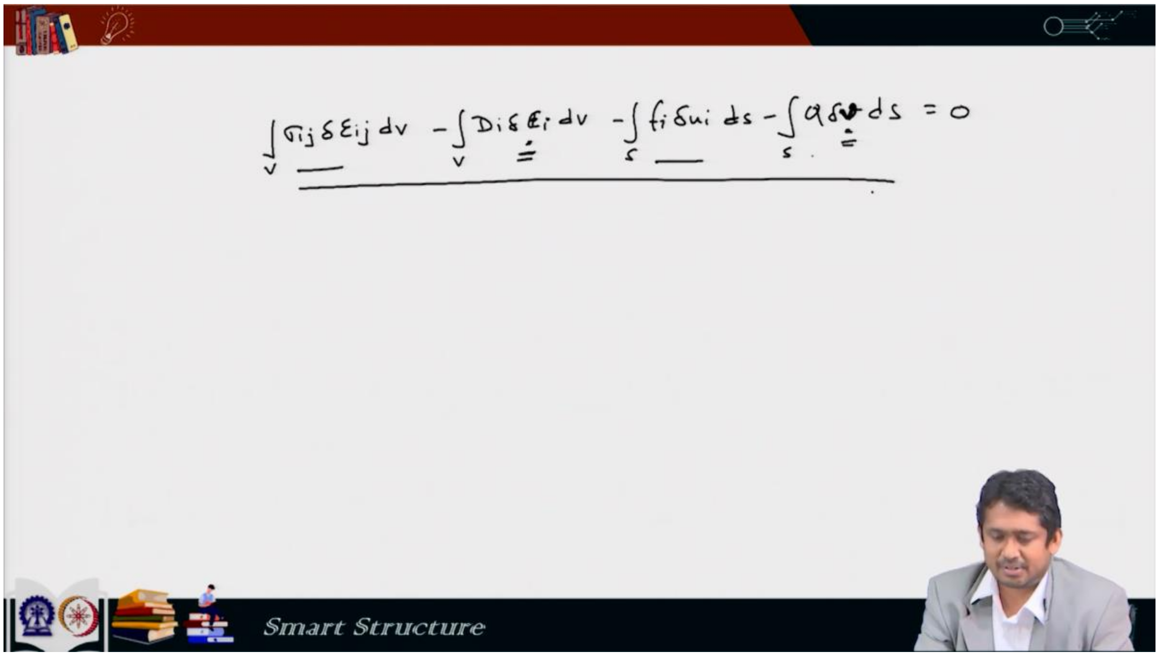


Now, we have seen that the virtual work equation looks like this considering the piezoelectric energy effect.

$$\int_V \sigma_{ij}\delta\varepsilon_{ij}dv - \int_V D_i\delta\varepsilon_i dv - \int_S f_i\delta u_i ds - \int_S Q\delta v ds = 0$$

So, this equation we derived and so far, we have solved only the induced strain actuation problems. In those kinds of problems, the electric field and the potential are known to us. So, their variation is 0. And also, in those problems because electric field potentials are known to us. So, the only unknowns that we need to solve are the mechanical response. So, that is why this minus this would suffice and we would not have this. But when we want to do energy harvesting application in that case the structure is vibrating because of external load and we need to see how is the electrical response. So, the electrical response is something that is not known to us. So, del of variation of electric field or the variation of voltage is non-zero there and they are unknowns. So, that is why in that case we need to have the entire equation in the formulation.

(Refer Slide Time: 5:32)



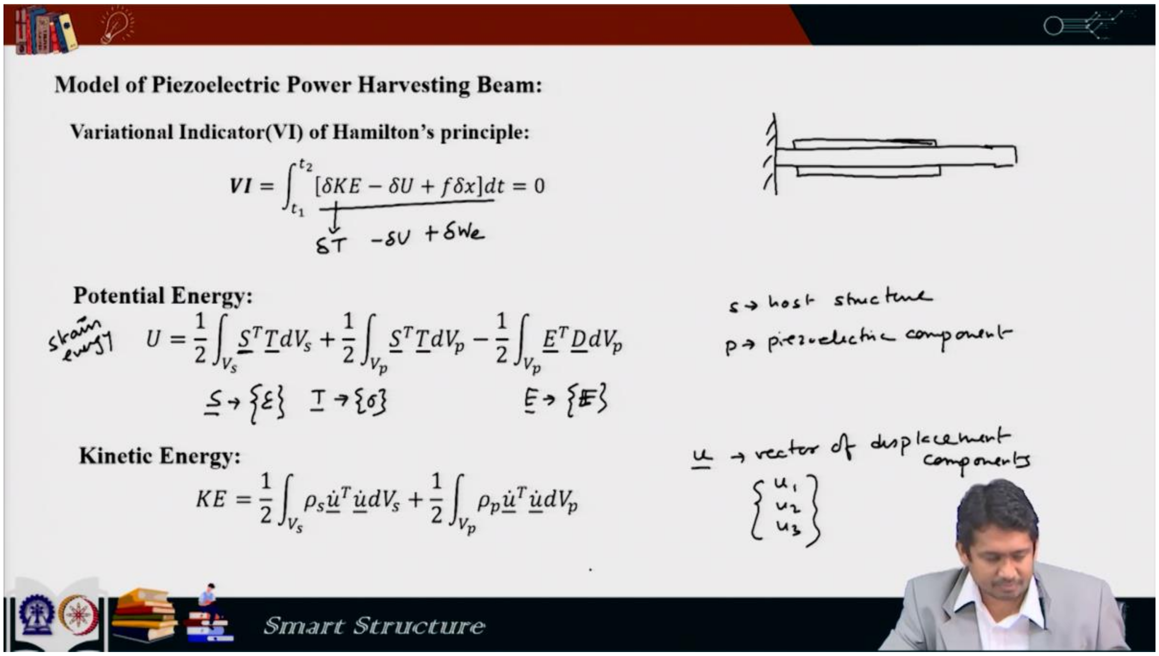
Now here the structure that is considered is similar to what we have been considering so far. So, there is a beam and there are piezoelectric patches at top and bottom and then it is under some kind of load. So, it is vibrating. Now for this case, it is a case of dynamic problem. So, the variation indicator comes from the Hamilton's principle and it looks like this. We have already discussed that.

Now, if we go by our notation then this is delta T, delta U remains same we have also use delta U and this is delta W_e. Now, potential energy or to be specific strain energy can be written as half integral over the volume of the host structure. So, here S denotes host structure and P denotes piezoelectric component. So, half into S^T into T integrated over the volume of the host structure. Now here S means strain vector.

Now, here vector has been denoted with this line at below the variable and T is stress vector. So, we have used epsilon and sigma here they are using S and T. And similarly, the same thing for the piezoelectric domain minus E^T D. So, E here is electric field. So, we are using double stroke E for that and D is the electrical displacement and that is valid only in the piezoelectric domain. So, here the ah integrals are written separately for the piezoelectric part and the host structure part.

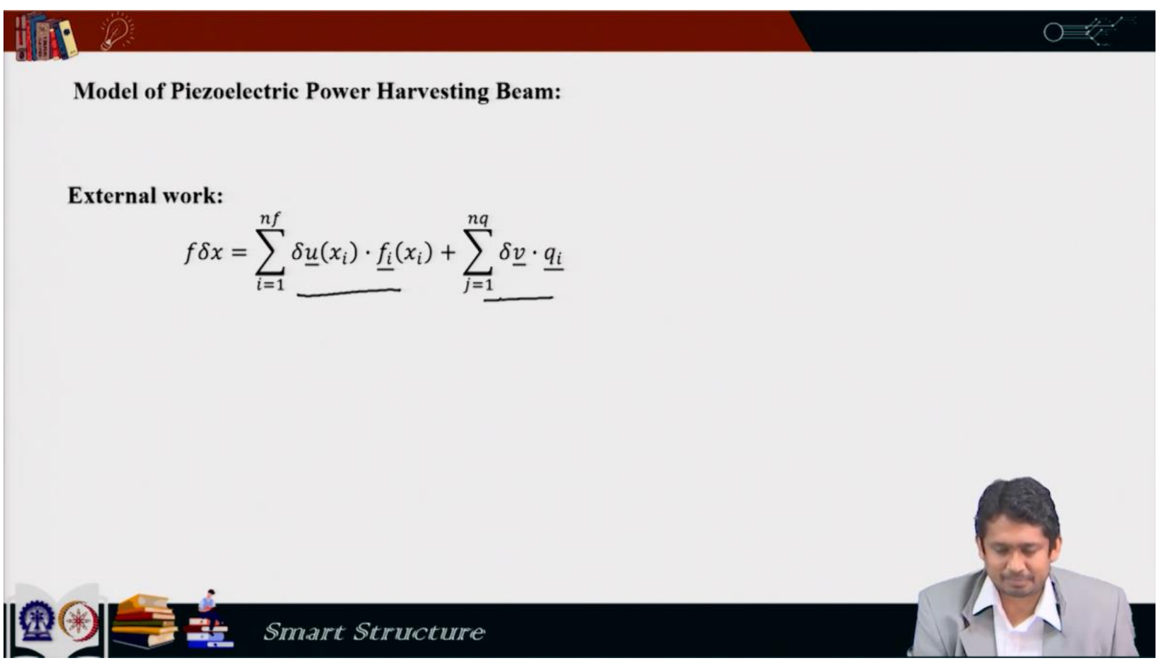
Kinetic energy can be written as this. So, rho s is the density of the host structure and host structure material and rho p is the density of the piezoelectric material. u is vector of displacement components. So, we can write maybe u_1, u_2, u_3, for our case generally we denote u_3 as w and we do not have this out of plane displacement for the beam. But here to be more generic. Everything has been written in terms of the vector and matrix form. Later on, the Euler Bernoulli beam assumption has been applied and things have been reduced.

(Refer Slide Time: 9:29)



External work is this. So, this is the external work due to the applied load and this part comes from this boundary integral. So, we have charge multiplied by variation of the potential and that is what comes here. So, we have all the terms that should be there in the Hamilton's principal equation.

(Refer Slide Time: 10:02)



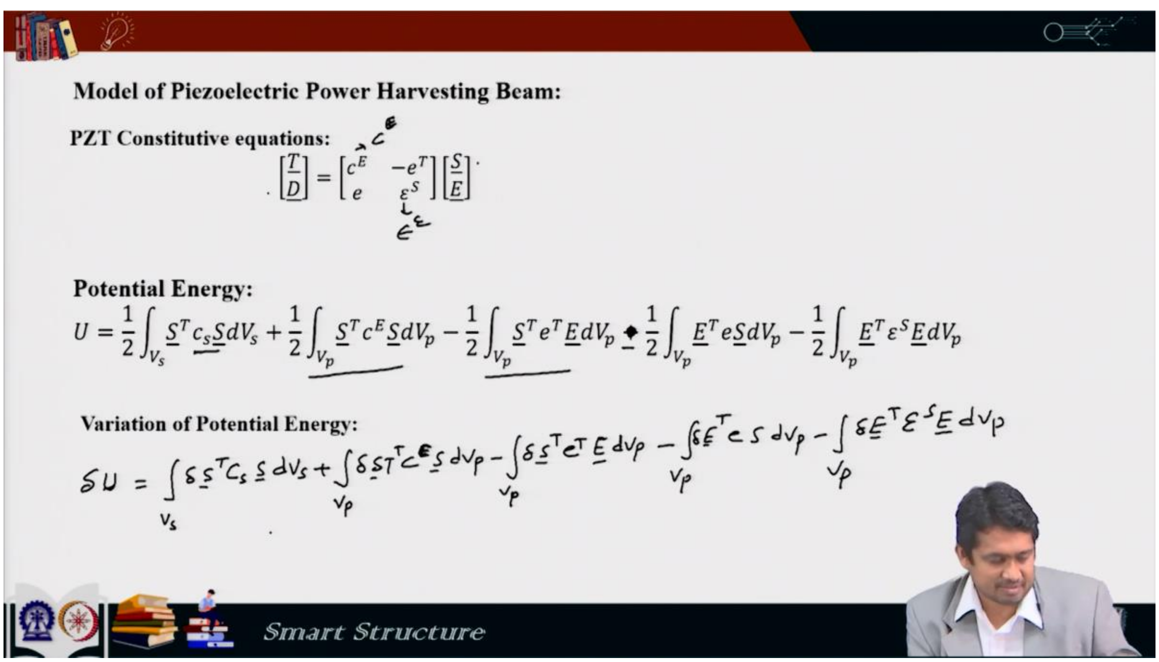
Now comes the constitutive relations. So, we have we have stress and electrical displacement at this side and the states are strain and electric field. c^E we know, E we know. Here this epsilon is this for us, epsilon and this is c with a superscript of electric field. So, this is how we defined the constitutive relations when stress and, sorry, strain and electric field were the state variables. And we also know what are these piezoelectric coefficients E in terms of the D coefficients.

Now, if all these material properties are substituted, then the potential energy expression looks like this. So, here we have c_S because T is equal to c^S multiplied by – because T is equal to c_S multiplied by S that is the stress and here c_S means the stiffness coefficients for the host structure. c^E is this. So, if you put the stress expression as c^E multiplied by S minus e^T multiplied by E, then we get this term minus this term. And then, we have the term for the electrical part which is this. So, this should be minus here and this. Now we need to take the variation of the total potential energy.

So, if the variation is taken, then it looks like this. Delta s^T c^E S dV_P minus V_P delta s^T e^T E dV_P minus variation of E^T e S dV_P minus again integral over the V_P. Variation of E^T epsilon^S E dV_P. So, this is the variation of the total strain energy.

$$\delta U = \int_{V_P} \delta s^T c_s S dV_s + \int_{V_P} \delta s^T c^E S dV_P - \int_{V_P} \delta s^T e^T E dV_P - \int_{V_P} \delta E^T e S dV_P - \int_{V_P} \delta E^T \varepsilon^T E dV_P$$

(Refer Slide Time: 14.04)

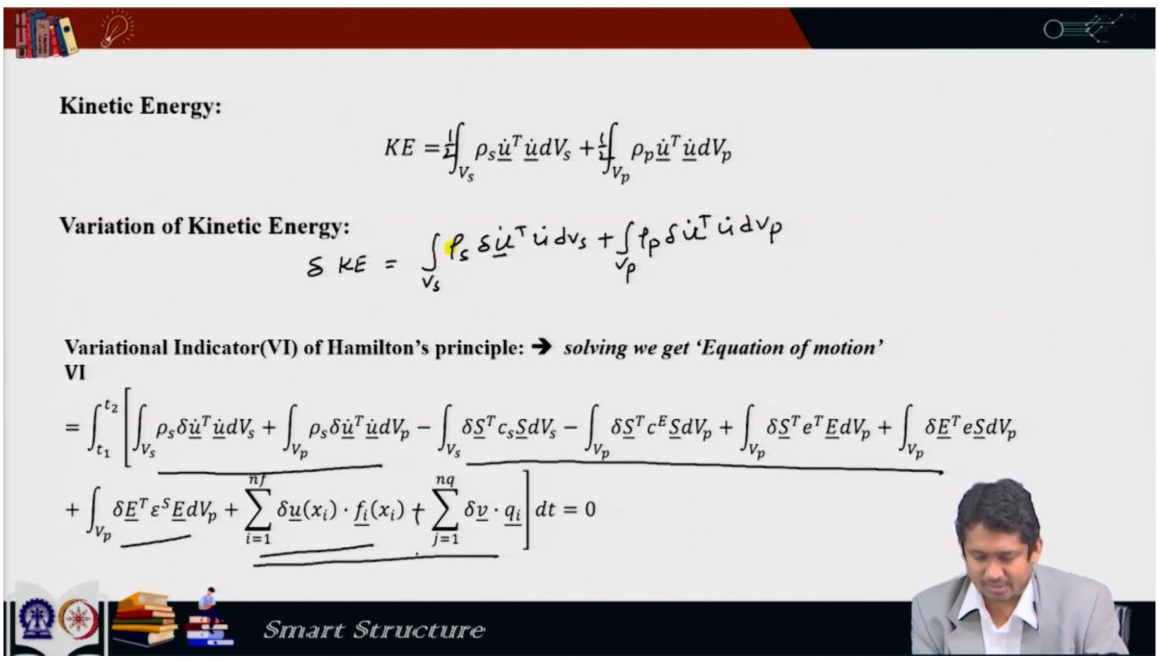


Kinetic energy is this. So, again if the variation of the kinetic energy is taken. So, delta KE that so there should be half everywhere and this becomes – this becomes rho S then delta u dot T u dot dV_S plus again the same thing, but for the piezoelectric domain dV_P.

$$\delta KE = \int_{V_S} \rho_S \delta \dot{u}^T \dot{u} dV_S + \int_{V_P} \rho_P \delta \dot{u}^T \dot{u} dV_P$$

So, finally, after substituting everything the variational indicator looks like this. So, here we have the variation of the kinetic energy and then we have the variation of the total potential energy which has the strain energy and the potential energy of the electrical part. And then we have the terms which are equivalent to the force multiplied by the displacement terms. So, here is the force multiplied by the displacement and here we have the charge multiplied by the variation of the electrical potential.

(Refer Slide Time: 15:43)

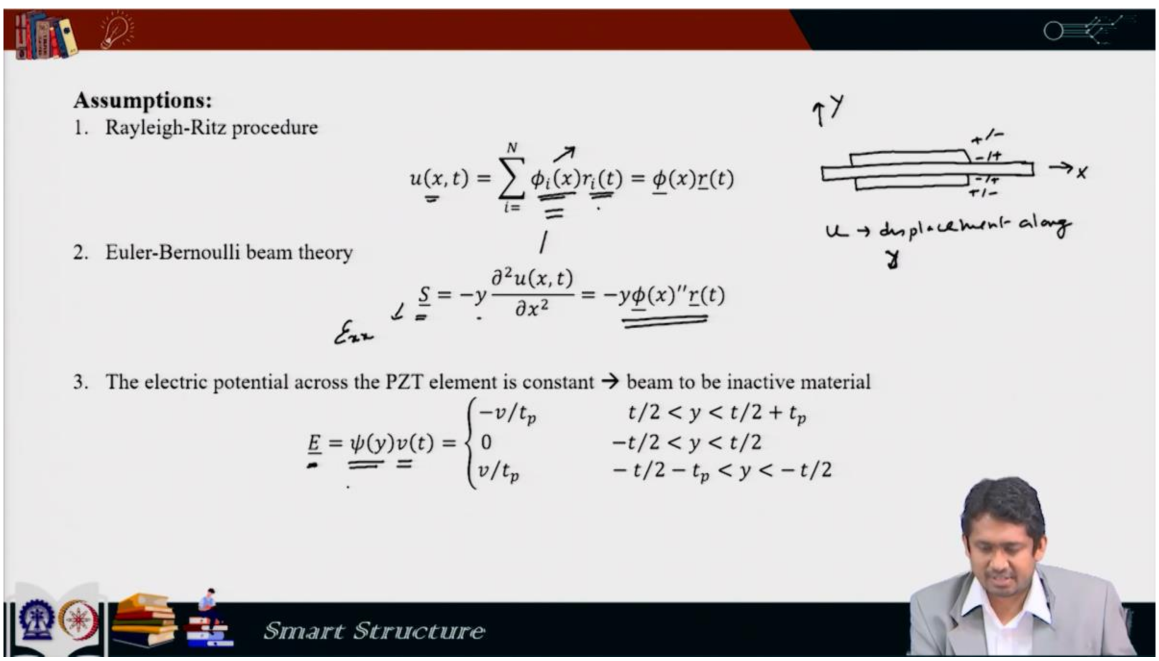


Now, we will put the approximations, kinematic approximations in the expression. So, it is a Euler Bernoulli beam. So, it follows this relation – strain is equal to minus y multiplied by second order derivative of the displacement along z direction. Now this is how the structure looks like. It is spanning along x direction and as per this convention, the vertical dimension is y. And the only non zero displacement component is u which is displacement along y.

So, in our previous formulations, we wrote w as the displacement along the vertical direction, but here just to be consistent with the terminologies used in the paper, we are writing as per them. So, it is u. So, minus y del 2 u by del x 2 is the strain and this is the strain in the x direction, normal strain in the x direction and that is the only non zero strain. And then this vertical displacement is written as a summation of the product of this known function and the unknown coefficient. So, these known functions have been the vibrational modes of the beam has been considered as the known functions here. And as we know that they should satisfy some boundary conditions and those beam mode shapes satisfy those boundary conditions. So, that is why they have been taken as this. Now with this approximation the same expression looks like this. So, phi has been differentiated twice with respect to s and then r has come.

Now the piezoelectric potential across the PZT element is constant. So, the electric potential, I mean the electric field has been written in terms of electrical potential as this. So, electric field is equal to some known function of y which is psi y multiplied by v t. Now, we know that the electrical potential here is constant, here is constant, and here it is 0. And we also know that because it is a beam, so, if I have plus minus here, I would have plus minus here. Or if I have minus here, plus here, I am going to have plus here, minus here. So, this psi is minus v by t_p at the top. Here it is 0 because it is inert beam and here, it is just the opposite v by t_p. And this variation is written as psi y as a function.

(Refer Slide Time: 18:59)



So, these are the approximations.

Now our job is to put the approximations in the variation indicator and they have been put here. Now after they are put, here also it has to be plus, and after they are put, it looks like this. Then we can see here that there are variations of r dot, r, r, and v. Now, after doing integration pi parts, we know that this r dot terms goes away and r double dot, I mean this dot goes away, and r double dot comes here. So, it also becomes delta r and then we separate out all the terms multiplied by delta r and the terms multiplied by delta v and that gives us two sets of equations. But before doing that let us write these coefficients, their values.

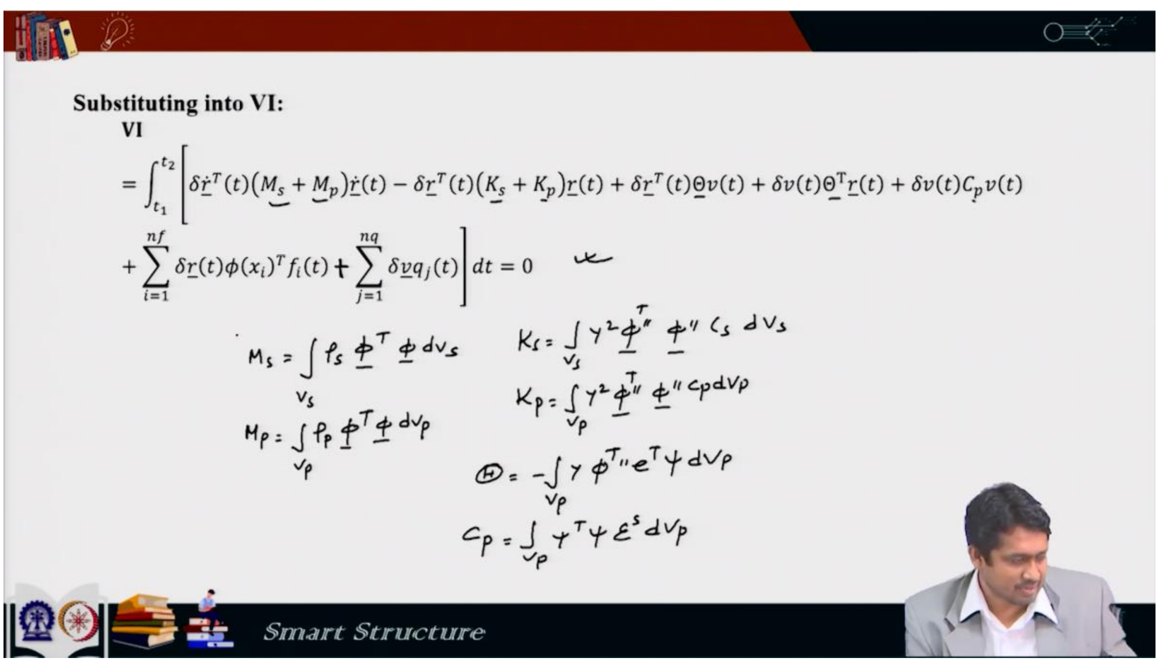
So, M_s is equal to V_s rho s, phi t and phi dV_s. M_p is same thing rho p, phi t, phi dV_p. And then, we have K_s, which is again integral over V_s y square phi prime transpose multiplied by phi prime and we have c_s dV_s. So, we have only one stress component, one strain component. So, c_s is nothing but elastic modulus along the x direction and we know that the beam is isotropic. So, it is just the elastic modulus. K_p is same thing for the piezo domain c_p dV_p. And then, we have this coupling term phi, and phi is minus integral over V_p of y, phi prime transpose e^T, psi dV_p. And then, we have another term c_p, c_p is integral over V_p of psi transpose, psi multiplied by epsilon s dV_p.

$$M_S = \int_{V_S} \rho_S \phi^T \phi dV_S \qquad M_P = \int_{V_P} \rho_P \phi^T \phi dV_P$$

$$K_S = \int_{V_S} y^2 \phi''^T \phi'' c_S dV_S \qquad M_P = \int_{V_P} y^2 \phi''^T \phi'' c_P dV_P$$

$$\Theta = -\int_{V_P} y \phi''^T e^T \psi dV_P \qquad C_P = \int_{V_P} \psi^T \psi \varepsilon^S dV_P$$

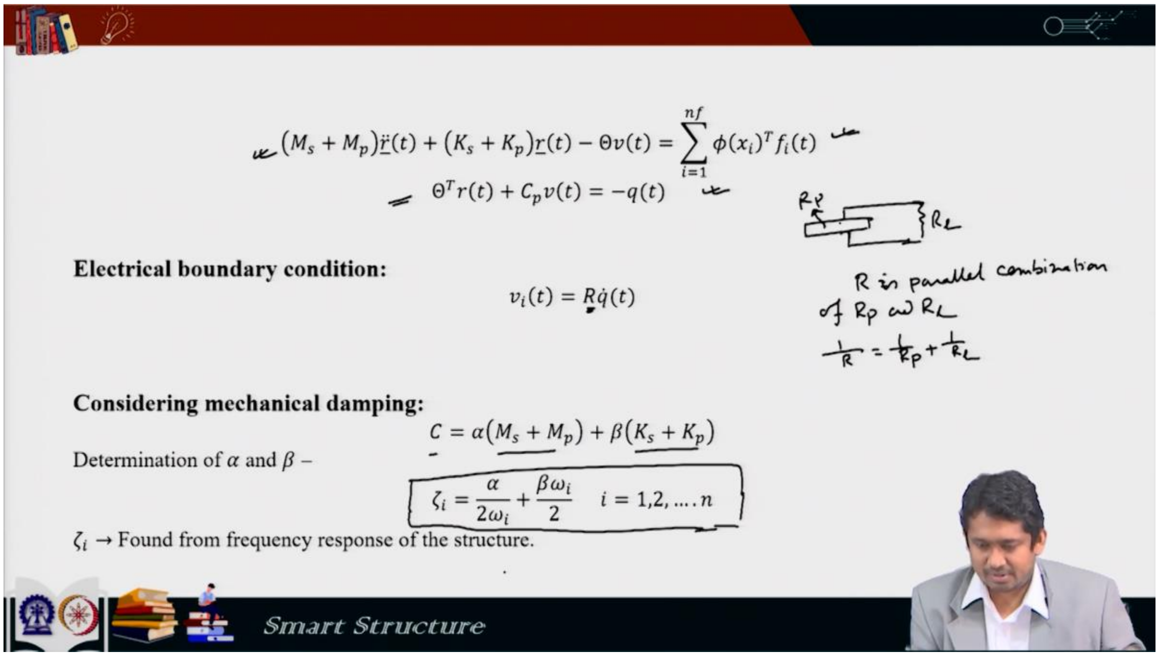
(Refer Slide Time: 22:35)



So, these are the matrices that we found out and then we separate out the terms which has variation of r and variation of v and that gives us two equations of two equations.

This is the first equation this is the second equation. Now here we can do two things, we can write it in this equation in the form of this charge q or we can write this equation in the form of voltage v. If you want to write it in the form of charge q, all we can do is we can write v as R q dot, because what is happening is this piezoelectric is connected to a resistance and that resistance is dissipating some energy. So, we know that the voltage across this is equal to R q dot. In fact, this R is the parallel combination of the load resistance R_L. So, the external resistance that we put is load resistance and the piezo has a resistance which we may like to call R_p. So, R is parallel combination of R_p and R_L. So, we know that 1 by R is equal to 1 by 1 R_p plus 1 by R_L, and that gives us R. So, this is the R using which we can express v in terms of q dot. And then, in the first equation which signifies the mechanical motion we do not have any damping, but structures are supposed to have damping as we discussed before also. And we also discussed that the damping is generally written as a linear combination of the mass matrix and stiffness matrix, alpha into mass matrix plus beta into the stiffness matrix and these are called Rayleigh damping coefficients. And if we know the damping factors they can be found out. So, this is how they are related the damping factor the natural frequencies and the Rayleigh damping coefficients.

(Refer Slide Time: 25:23)



So, incorporating the damping, the first equation looks like this, and incorporating v in terms of q dot, the second equation looks like this. So, these are the two equations which can be solved and that will give us the variation of the displacement and the electrical charge. And if we know the electrical charge, we can find out electrical current also. Electrical current is nothing, but I equal to q dot. So, we can find out the variation of the displacement charge or current with time. Now here this q dot term which is multiplied with R, so this R and this C what they are doing as they are acting as a damper and they are taking out energy from the system.

Now this system can be tuned for maximum energy harvesting by tuning R. So, by tuning the load resistance, we can tune R and accordingly we can tune the system for the maximum amount of energy harvesting. Now we said that we could have written this expression in terms of v also for that all that we do is – we differentiate this equation. If I differentiate this equation I have r dot here, I have v dot here and I have q dot here. And then, this q dot, we replace by v. So, this I have r dot v and v. That way the equation is written with respect to v.

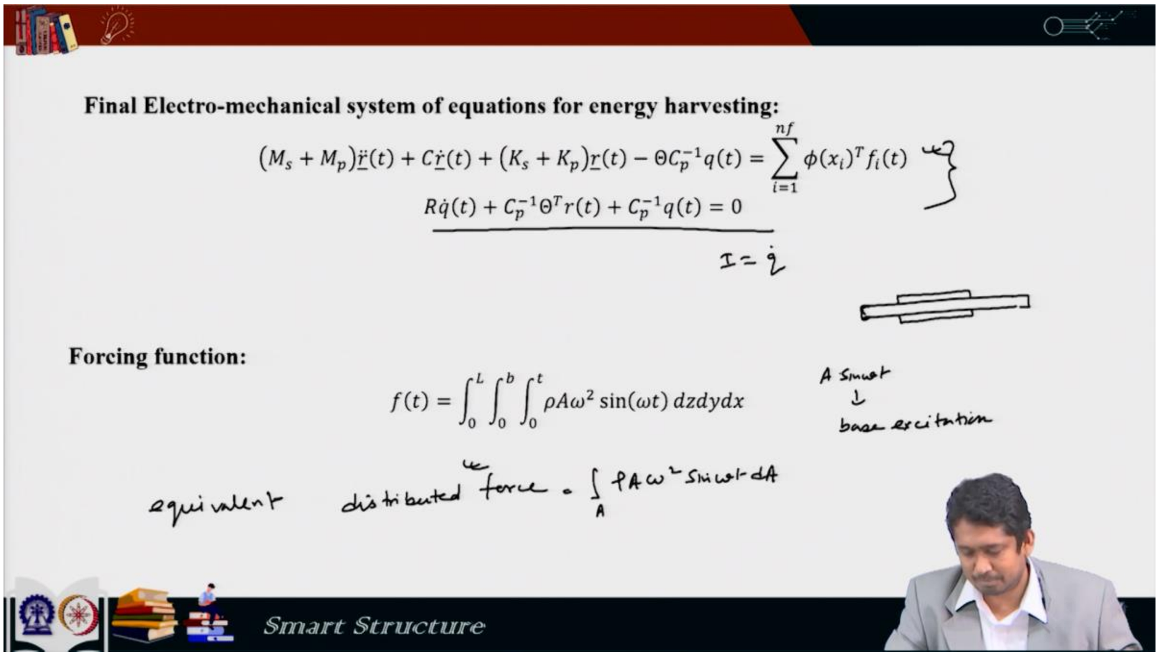
Now in this problem, the external load that has been taken as in this study as a base excitation. So, this entire thing can be put in a shaker and it can be excited that is how the they did the experiment. So, the base excitation if we think that the base excitation is A sine omega t which is base excitation. So, it is not a fixed end anymore, it is just excited freely. So, this end is free and which is just free, but clamped in the sense that it does not allow any rotation and this entire thing is excited.

Now one way of doing it is writing the motion here as a boundary condition and solving for it. The other way of doing it is using the acceleration due to this base excitation and finding out the corresponding inertia force distributed over the beam and taking that as a force in the opposite direction and that is what has been done here. So, the inertia force due to the base excitation motion which is distributed throughout the body is written as here. Here t is the thickness, b is the width, L is the length and this is the force. So, the distributed force is – equivalent distributed force is A rho A omega square sine omega t dA, where A is the cross-section area of the beam at any x.

$$Distributed\ force = \int_A \rho A \omega^2 \sin \omega t \, dA$$

And this is this has to be taken in the negative direction that is already taken care of in the sign because if I find out acceleration from here that gets a negative sign. So, we have got rid of the negative sign here. So, the force is taken in the opposite direction. So, that is how this problem is solved and solving the coupled equation, the displacement and the charge or the charge current or the voltage can be found out.

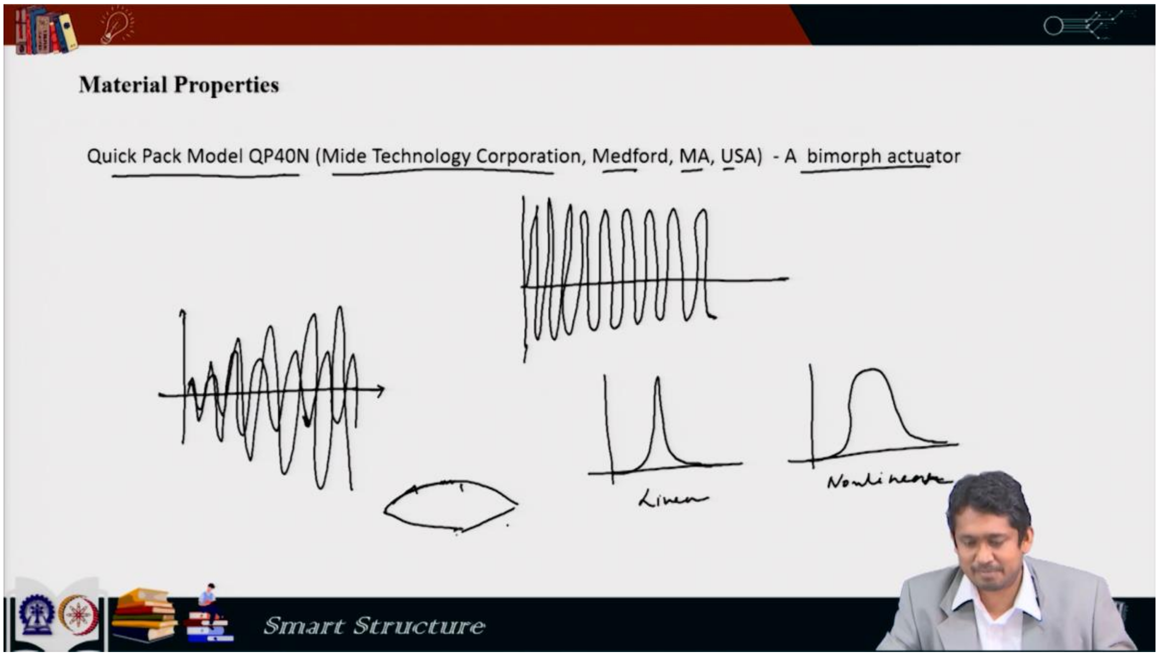
(Refer Slide Time: 29:52)



Now, for this study in the paper that we are referring a quick pack model QP40N that was made by Mide Technology Corporation, Medford, Maryland, USA was used it is a bi-morph actuator. So, what it means that it already has the host and the two pieces at the two sides. So, it was just put in a shaker and it was tested and the mathematical model developed what tested against the experiment and a good match was found. So, generally when we look into the displacement or the current or any response here, it looks like this. Initially there is a transient part and after some time depending on the value of the structural damping as well as the load resistance, it gets stabilized and it does a stable oscillation.

So, the amount of energy that we can harvest depends a lot on the load resistance that we put. So, for that a parametric study is presented in the paper at different frequencies, different amount of load resistance was given and it was seen how much it is efficient in extracting energy and it was found out that if the load resistance matches with the impedance of the piezo, then the best efficiency is obtained. So, this was about an that I mean the experimental scenario. In real life, if this has to be applied, then to do the analysis the force has to be known. Now one good application is the control of aero elastic instability. So, in aero elastic system sometimes it goes to dynamic instability because of the negative, I mean because of the effect of the aerodynamic load. So, the aerodynamic load depends on the structural displacement as well as velocity. So, it affects in such a way that the structural system starts taking energy from the aerodynamic system and the effective damping becomes negative. So, in that case if the energy can be dissipated taken out by the load resistance, then the structural vibration can be stabilized and the extracted energy can be used for some meaningful work. So, that is where a lot of research has been done and is also going on and there are many others finding by other researchers also. So, the system that we discussed where a linear harvester. So, this kind of harvesters work very well when the excitation frequencies match with the structural structures natural frequency which means the system is in near resonance. But when we go away from the resonance, its response dies down. But in a non-linear system near the resonance, we see a wide band here. So, a wide band energy harvesting can be done in case of non-linear system. So, that is where also a lot of research has been done and also there are findings like a structure which is bi-stable. A bi-stable structure means like a buckled column. So, it has two stabilities at the top and at the bottom. So, this bi-stable structures can have small amplitude vibration around one stable equilibrium, it can have large amplitude vibrations around one stable equilibrium and it can have very large oscillation when it snaps between the two equilibriums. So, in the when this happens a lot of more energy can be extracted and a good amount of research has been done in this field also. So, these are the possibilities. So, that is it about the mathematical model of the piezoelectric beam-based energy harvester.

(Refer Slide Time: 34:25)



We will discuss more about this and also vibration control applications in the next lecture.

Thank you.