Smart Structures
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Week 03
Lecture No: 18
Induced Strain Actuation – Static Analysis – Numerical Examples
Part 06

Welcome to the sixth lecture.

So, in this lecture, we will solve some numerical problems based on whatever was covered this week.

In the first problem is based on the Euler Bernoulli theory. It says that there is a beam, and it has two piezoelectric actuators, and one is PZT 5H and PZT 5A. They vary by their piezo properties. Their lengths are fifty point eight millimeters, width is twenty five points four millimeters, and thickness is zero point three one seven five millimeters, and they are they are bonded to the top and bottom of a thin aluminum cantilever beam of length l is equal l_b is equal to one. So, this is the beam to which it is bonded; its length is one, its width is fifty point eight millimeters, and thickness is zero point seven nine three seven five millimeters. The thickness of the bond layer is zero point one two seven millimeters, and the bond layer is assumed to be uniform.

The configuration is shown below here, and the material data is given. These are the material data. The seventy two point four GPa has been considered to be the elastic modulus of both Piezo and the beam. And d_{31} of the PZT 5A is minus one seven one multiplied by ten to the power minus twelve meters per volt, and d_{31} for the PZT 5H is this.

So, this is our 5A at the bottom, and 5H is at the top, and bond elastic modulus is given. So, there is a thin bond layer between the piezo, and the beam properties are given.

So, we using the Euler Bernoulli beam theory, we have to derive the mid plane strain and curvature, which is epsilon zero and kappa, and plot the span wise variation of slope and vertical displacement. How the vertical displacement and slope varies along the length and plot the span wise variation of axial displacement. The variation of axial displacement along the span wise length. And if these two patches are replaced with a PVDF polyvinyl difluoride of the same size, then calculate the new surface actuation strain and actuation bending moment for a voltage of one fifty volts to the both top and bottom piezos. And the property of the PVDF is given to us.

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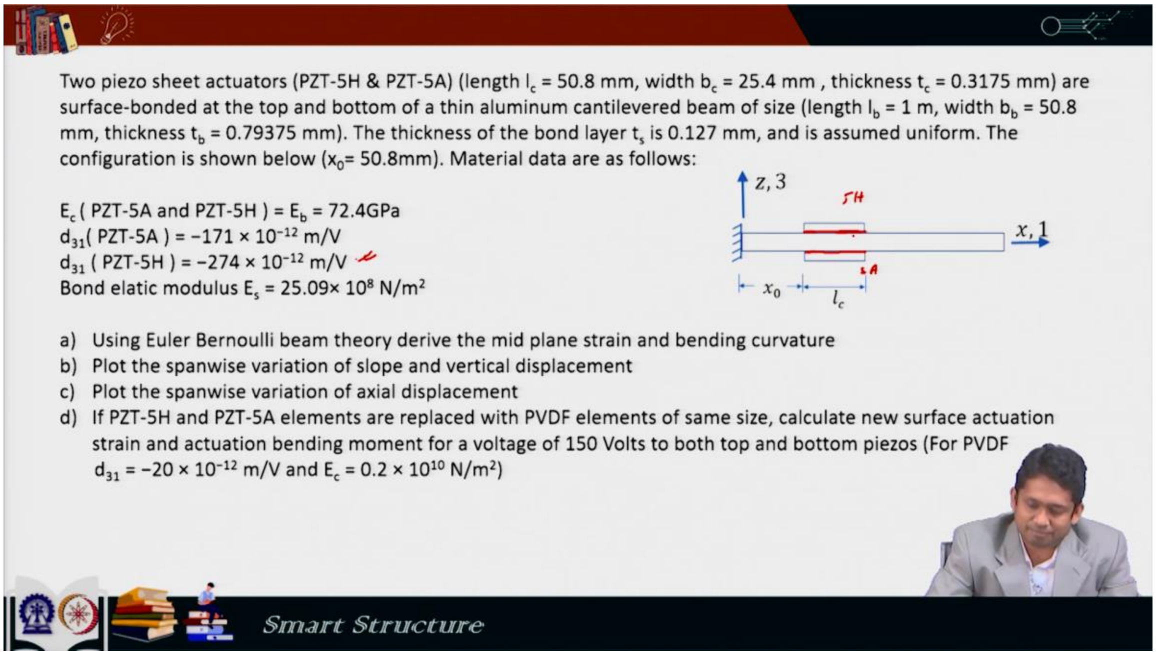

Now, we have to solve the problem.

So, for the first part, now the beam is symmetric; as we can see, they vary only in d_{31}. So, that does not affect our EA, ES, or EI. So, for this configuration, ES is zero. EA_{total}, if we want to find out. So, there are five layers: two piezo layers, one bond layer in between, and a beam. So, the formula that we discussed was this, and finally, that leads to this expression. And then, we put the numbers. If you put the numbers, then we get this. So, we are writing all the given properties of the piezo bond and the beam. And then finally, the expression comes to be four point one zero three multiplied by ten to the power nine Newton. So, this is our EA_{total}.

Similarly, EI_{total}, if we want to find out, then the expression is two E_c b_c t_c. So, there are two piezo layers, and we are finding out everything with respect to the midline of the beam. So, when we find out for the bond and the piezo, we have to find out their moment of inertia with respect to their centroid, and then we shift it to our desired location, that is the midline of the beam. Now, please understand that the bond is connecting the beam and the piezo patch. So, the width of the bond and the width of the beam is the same. So, that is why it is b_c plus; then we do it for the beam. And after we apply all the properties, then it comes to this value.

Now, we have to find out the free strains in the piezo patches. So, epsilon p bottom, the free strain in the bottom piezo patch in terms of the applied voltage is $d_{31_{bottom}}$ V by t_c, and that becomes minus one seven one multiplied by ten to the power minus twelve V divided by zero zero zero three one seven five and epsilon p top, the same thing we have to just change the d_{31} and then it becomes minus two seventy four multiplied by ten to the power minus twelve divided by point zero zero zero three one seven five.

Now, let us assume that the applied voltage is one fifty volts.

So, when the applied voltage is one fifty volts, it tells us that the force Fp is, as we know, E_c b_c t_c multiplied by epsilon P one, epsilon P two. So, in the expression of epsilon P one and epsilon P two, we have to apply V is equal to one fifty volts, and we know these quantities. So, after we do that, the value comes to be this.

And similarly, the expression for the bending moment becomes. So, while writing the bending moment, we have to incorporate the bond also bond thickness. So, that is incorporated here, and the value comes to be point zero one nine four Newton millimeter.

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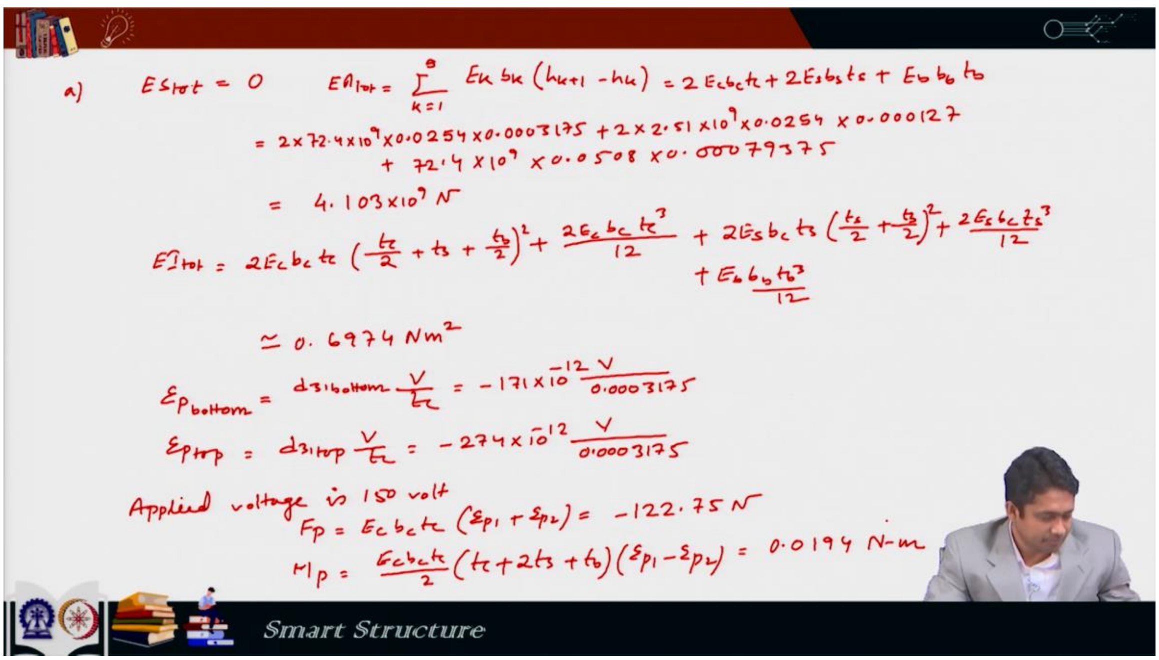

So, we have found out Fp and Mp, and we have found out the properties of the sections. So, now we can write the equations, and we can find out the mid-plane strain and bending curvature. Now, here, ES_{total} is zero.

So, we can separately find out the mid-plane strain and the bending curvature. So, to find out the mid-plane strain, we write EA_{total} multiplied by epsilon zero is equal to Fp. EA_{total}, we have found out, Fp we know. So, epsilon zero comes to be Fp, Fp divided by EA_{total}, and that is equal to minus twenty-nine point nine one into ten to the power minus six.

Similarly, we can separately find out bending curvature, and for that, we can solve the equation EI_{total} multiplied by Kappa, which is equal to Mp. And from there, we get EI_{total}, and that gives the expression as this. That gives the value as this.

Now, in the next part of the question, we are supposed to find out the displacement and slopes.

So, we have del two W by del x two is equal to zero when it is between the fixed end and the piezoelectric patch. And it is equal to Kappa when it is between the, I mean, in the zone of the piezoelectric patch. And again, it is zero when it is beyond that. So, after integrating, we get the expression as this. It is zero; we have discussed it that we need to apply some boundary conditions and the continuity and compatibility conditions, when where the piezo ends and starts. So, after doing that, we get this as the expression. And similarly, to find out W, we have zero, and we have Kappa multiplied by x two. And we have Kappa multiplied by l_c square by two plus Kappa l_c multiplied by x minus x-zero minus l_c. So, if you plot it, first, if you plot the slope variation, it looks like this.

So, this is our x-zero, this part is l_c. So, before the piezo zone starts, the slope is zero. In the zone of the piezo, the slope varies linearly, and after that, the slope remains constant. And that slope is one point four zero six multiplied by ten to the power minus three. If we want to plot the deformation, it would look like this. Again, it is zero at the beginning, and then, there is some curvature. And after that, the slope remains constant. So, there is no curvature, just a constant slope.

So, this part has a bending curvature, and whatever the slope is at the end, this continues. So, this is x-zero, and from here, it is l_c. Now, at the end of the piezo zone, where the curvature ends, the displacement is three point five nine into ten to the power minus five meters. And at the end of the beam, i.e., at the end of 1 meter, the displacement is one point three into ten to the power minus three meters.

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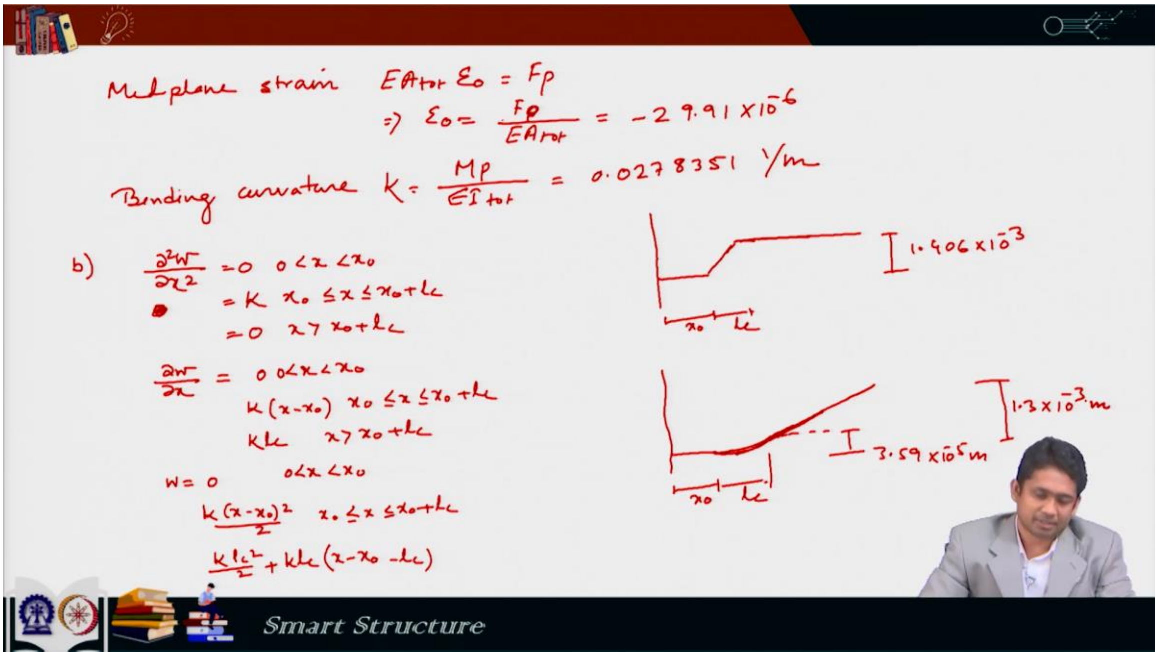

So, we have found out the deflected shape.

Now, we have to solve the C part of the problem, where it asks for the extension.

Again, similar thing that we discussed before. So, del u-zero by del x is zero before the start of the piezo, and then this quantity is epsilon zero in the zone of the piezoelectric material, and then again, this quantity is zero. So, we apply the boundary condition and the continuity condition and solve the problem.

So, u-zero at the beginning is zero. And then, u-zero in the zone of the piezoelectric material is this. And then, u-zero after that is constant, which is this.

Now, if you plot it, the plot looks like this. The epsilon zero is a negative quantity here. So, the displacement is negative. So, this is a zero line. So, this is u-zero is equal to the zero line, and the maximum axial displacement is fifteen point one nine multiplied by ten to the power minus seven meters.

Now, in the last part of the problem, it says that the two piezoelectric patches are replaced by a PVDF. PVDF at the both top and bottom, and that makes the configuration symmetric. And if it is symmetric in terms of the piezo properties as well as geometry and the other material properties, then the Mp is zero, and Fp becomes one fifty multiplied by... And finally, Fp comes to be minus point three zero four eight. So, there is no Mp. There is only Fp.

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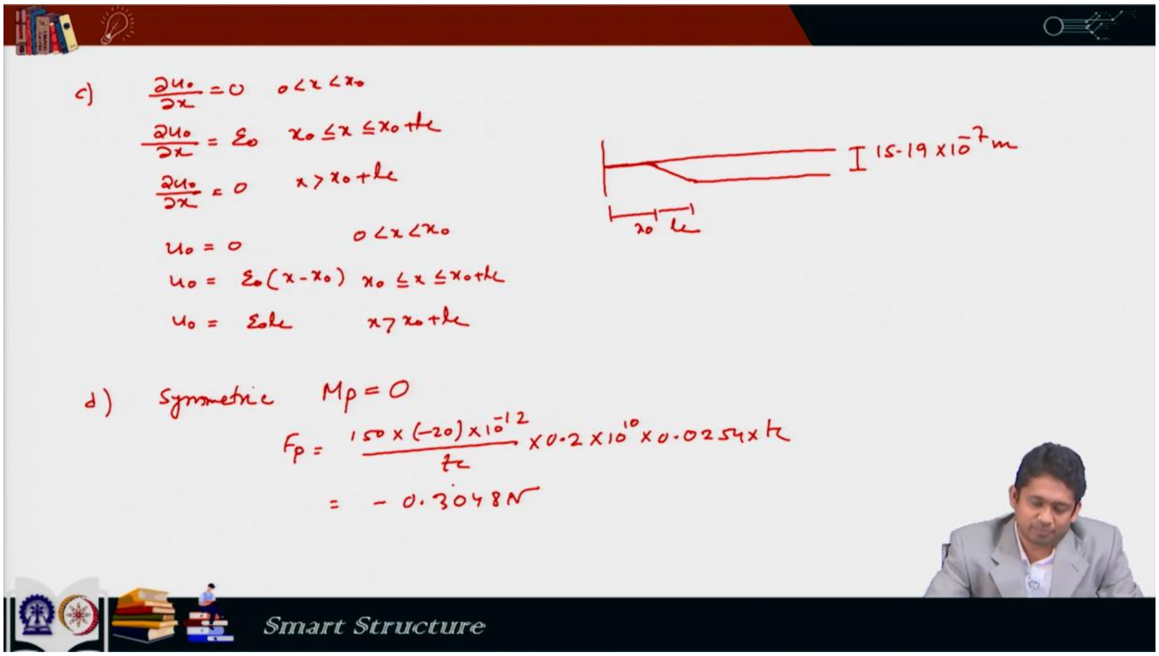

So, if it can be solved as a pure extension problem in the same procedure.

Now, we will look into another problem.

Here, again, we have two piezoelectric patches at the top and bottom surfaces. And they are two different patches, PZT 5H and 5A, and their properties are given. And we have to find out the free strain variation with voltage for each piezo and the variation of piezo strain with axial force for each piezo. And we have to find out the actuation surface force of bending moment for a voltage of one fifty volts applied to both the top and bottom piezo, and we have to find it out using the block force method as it is mentioned here. And span-wise distribution of bending slope and beam displacement for this excitation. And again, there is the same statement: if these two are replaced by PVDF, then we have to find out the actuation strain and actuation bending moment for a voltage of one fifty volts.

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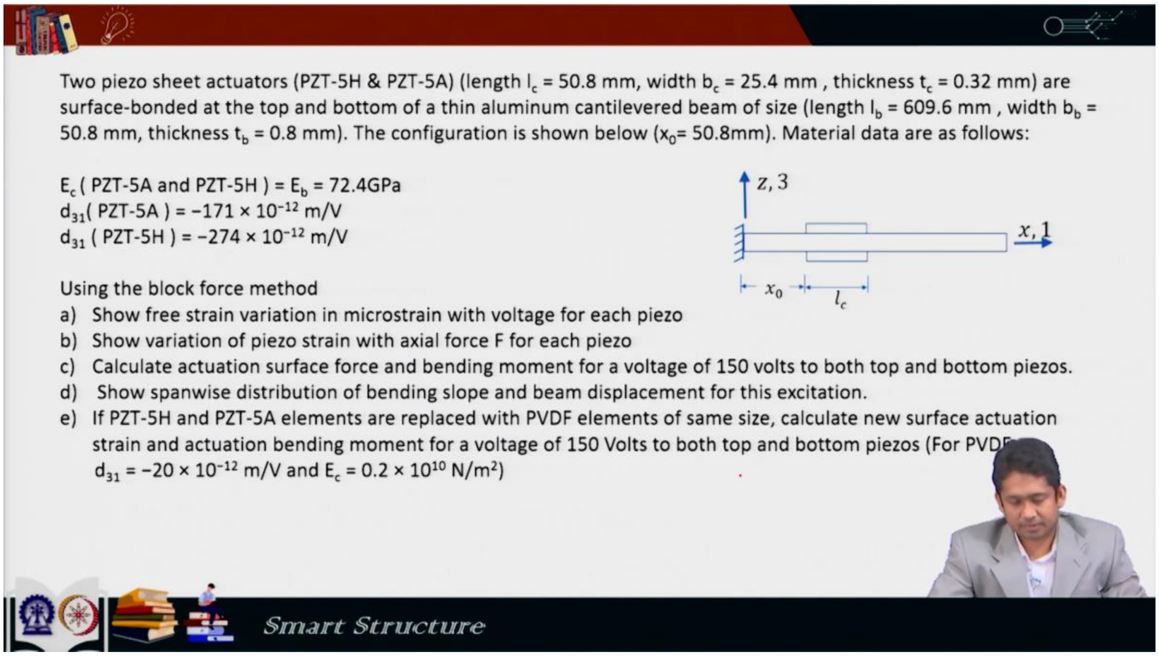

To solve the first problem for PZT 5H, which is at the top, the free strain epsilon p top is equal to d_{31}, as we know E by, so it is $d_{31_{top}}$, E by, d_{31} E. So, if you write it in terms of voltage, it is – we have to replace E by V by t_c. And if you apply the properties, the value comes to be minus point eight six three V. So, for any voltage involved, this is this. And the strain is in microstrain. And then, PZT 5A same thing epsilon p bottom $d_{31_{bottom}}$ V by t_c, and the value comes to be minus five three eight V microstrain.

Now, we have to find out the relation between the strain and the force. So, for that, we can use the constitutive relation in this form. Now, here, epsilon one and sigma one denote the stress in direction one, so they are not for piezo one and piezo two. So, from this equation, we can just get the relation minus eighty-six point three plus F_{top} divided by point five eight four, and this gives us in microstrain, and the force F top comes to be point five eight four strain. Let us see, call it, this is not this is just an epsilon top; we cannot use p because it is not a free strain. So, F^e plus fifty point four. Now, we just got by putting the values here. We know t_c, we know d_{31} and s_{11} E, it is just one by E_c. And from there, we can get the relation.

So, from here, if we put the epsilon top is equal to zero, that gives us the block force $F_{bl_{top}}$ as fifty-point four Newton. And if we put F_{top} is equal to zero, that gives us with the free strain. Epsilon p top is equal to minus eighty six points three microstrains. Now, in all this analysis, the value of V that has been assumed as a hundred volts. In all this analysis, and even whatever is going to proceed now, for the bottom piezo also. Now, similar quantities can be found out for the bottom piezo.

So, F_{bottom} can be written as point five eight four epsilon bottom plus thirty one point four two, and from here, we can find out the F block force at the bottom as thirty one point four two Newton. And free strain at the bottom is minus fifty three point eight micro strain. And again, a hundred volts of voltage is assumed here.

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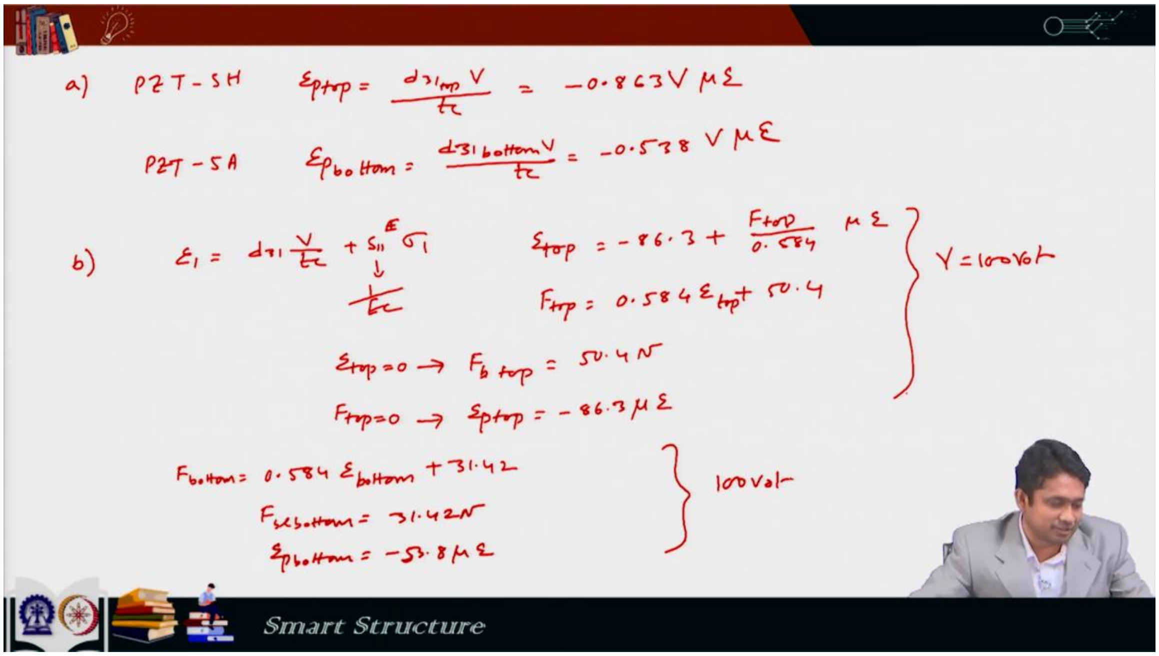

Now, we have to find out the actuation forces to proceed further.

So, we already discussed that this is a case of asymmetric actuation, where d_{31} are different for the top and bottom piezos. And we saw that the relation comes in terms of alphas and the free strains. So, we have to calculate the alphas. And alpha1 is EA_{c1}, where it is EA_{c1} is EA_c of one of the piezo patches. And EA_{c1} plus four by EA_b. Alpha2 is minus two by EA_b. And using these alphas, we can find the extensional actuation force as alpha1 plus alpha2, epsilon p top plus epsilon p bottom, and that comes to be minus forty three point eighty two Newton. And the moment comes as t_b by alpha1 minus alpha2 epsilon p bottom minus epsilon p top, and the value comes as five point one three into ten to the power minus three Newton meter.

Next, we have to find out the deflected shape. So, the governing differential equation is this. And, is equal to M by EI_b, where M is zero when x is less than x-zero. And when M is five point one three multiplied by ten to the power minus three when x is between x-zero and l_c. And M is zero when x is beyond lc plus x-zero. And again, if we apply the boundary conditions and the continuity at the beginning and the end of the piezo, we get these relations: that del W by del x is equal to zero and W is equal to zero in the first zone. And then, we have del W by del x, which is slope is equal to point zero three three five x minus x-zero, and W is equal to point zero one six seven x minus x-zero square when we have l_c. And then, in the last region we have, del W by del x is equal to some constant, and that is in radian, and W is equal to four point three one into ten to the power minus five plus one point seven into ten to the power minus three x minus zero point one zero one six. And these values will come in meter. So, these are all in radian and this in meter. So again, if we plot the slope distribution, it would look like this. And if we plot the displacement, it would look like this. So, this is our x-zero, and this is $l_{c;}$ similarly, this is x-zero, and this is l_c.

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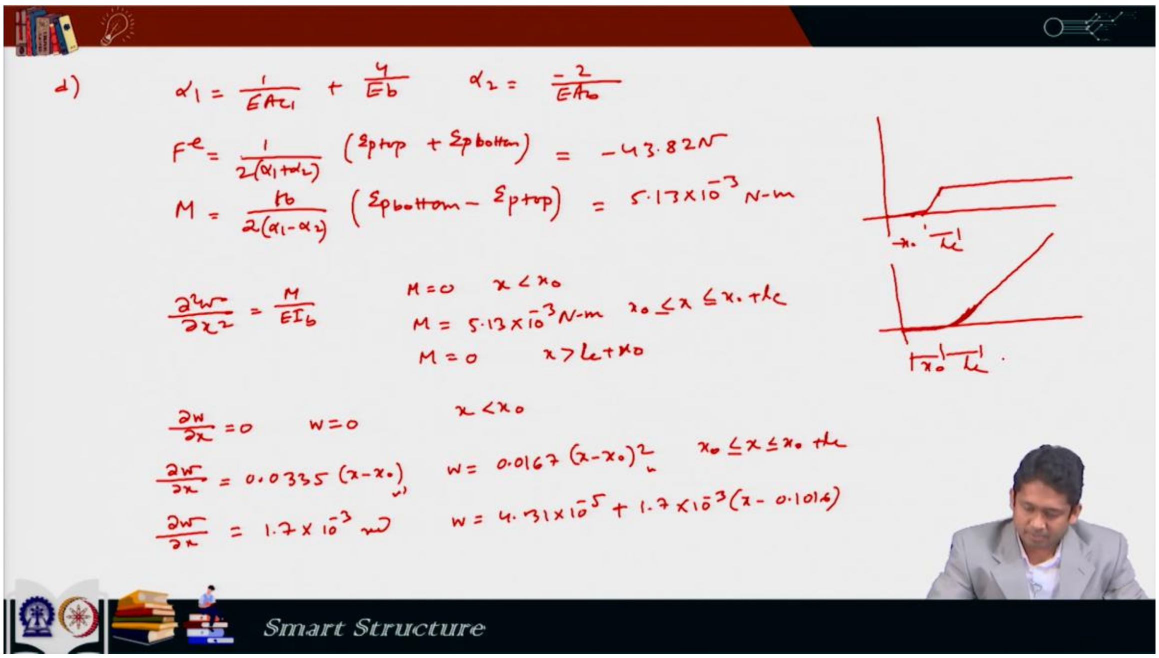

Now, the last part of the problem says that we have replaced the piezo patches by two PVDFs.

Now, for PVDF, the free strain is this. And that gives us minus point zero six three volts, and this is microstrain. So, V is equal to one fifty volts, that gives us epsilon p is equal to minus nine point five micro strains. Now, again, it is a symmetric case because the properties are symmetric, so there will be no bending actuation, so M would be zero. So, bending actuation would be zero. There will be only an extensional case. And that also says that Kappa is zero. It is only a pure extensional actuation, and for that, we get epsilon p based on the expressions that we derived before; for a pure extensional case, the mid-plane strain comes to be minus point one zero four microstrain.

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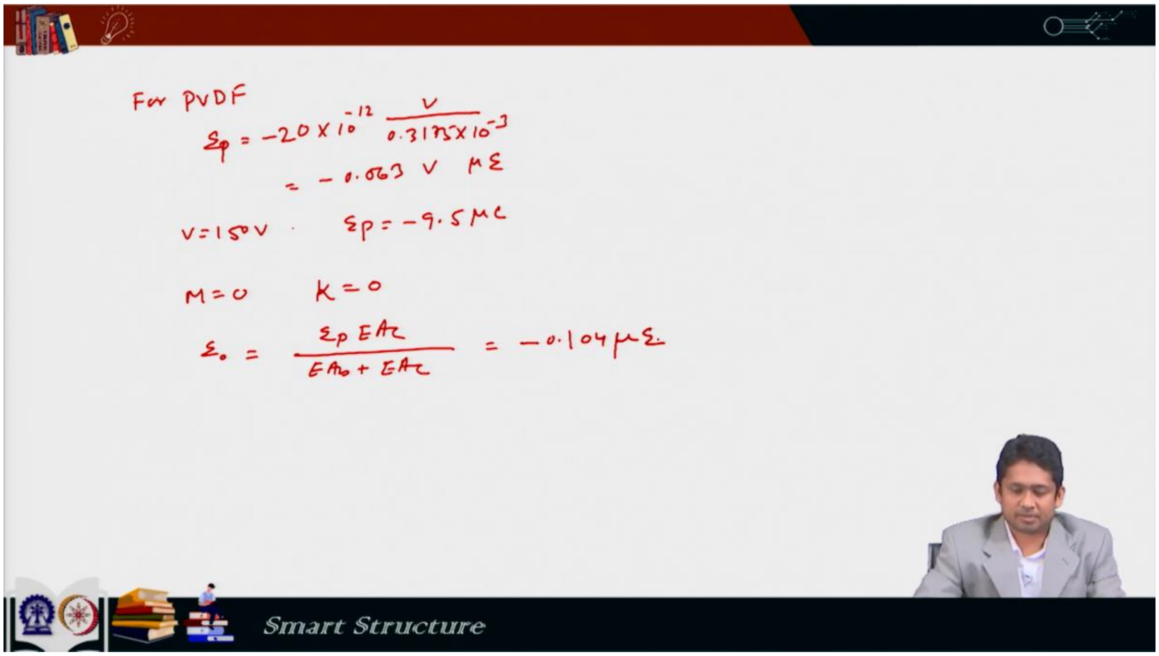

Now, just one point to remember in this analysis: the voltage, as stated in the question, was taken to be one fifty volts.

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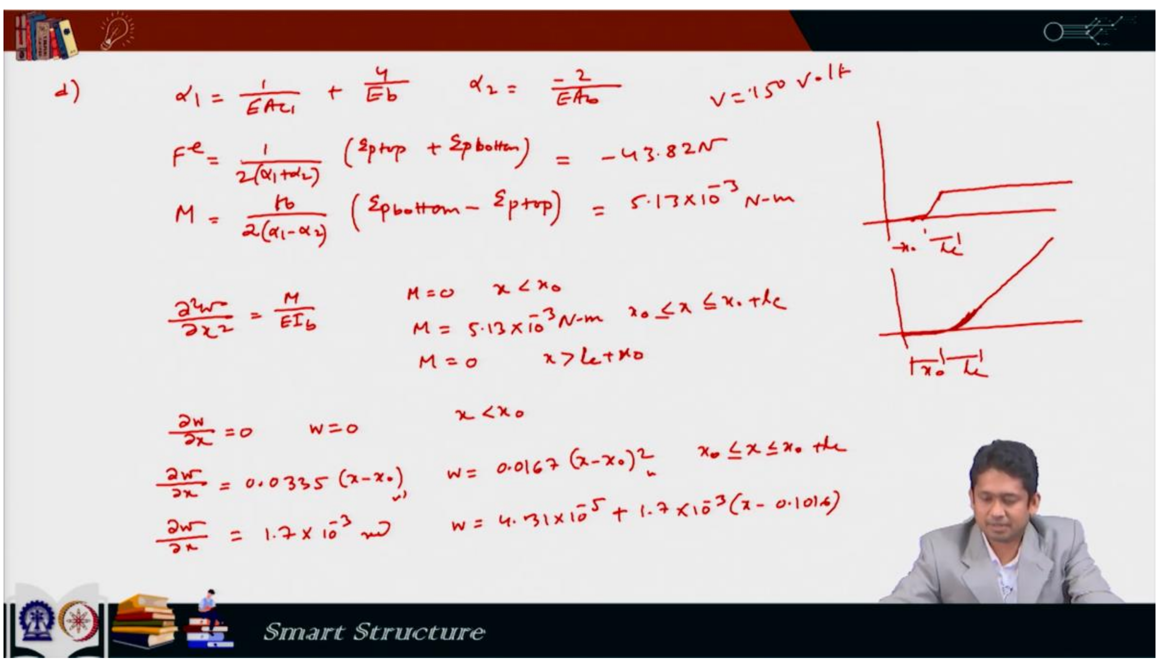

So, with that, I would like to conclude this lecture.

Thank you very much.