Smart Structures
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Week - 03
Lecture No - 15
Induced Strain Actuation - Static Analysis

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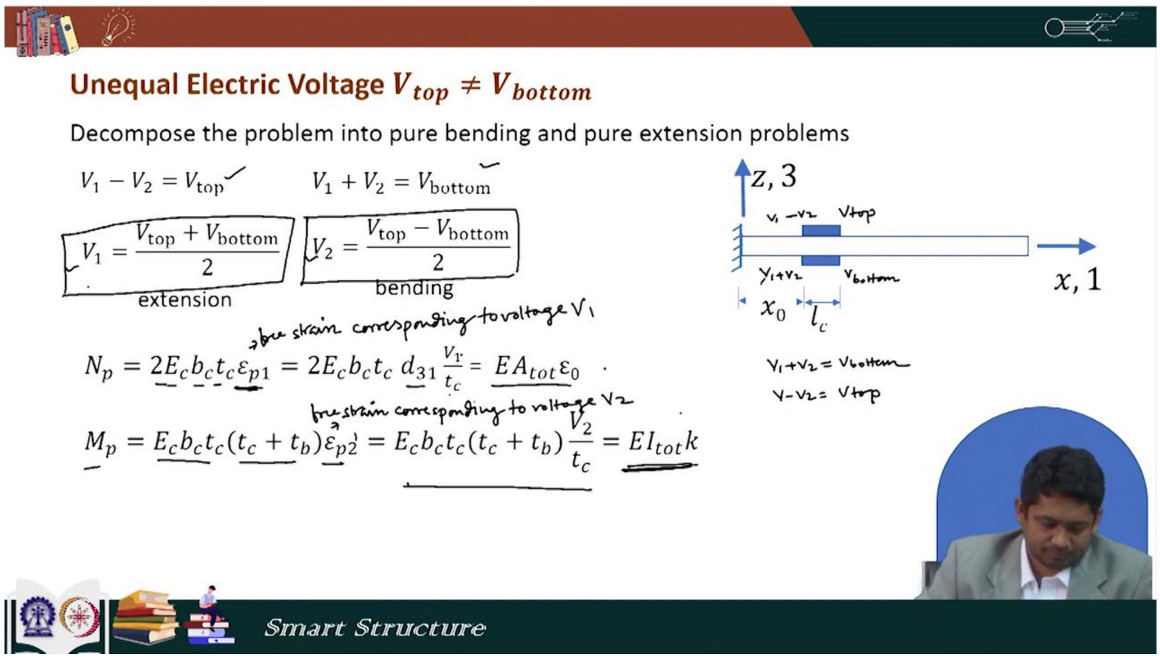

Welcome to the third lecture of week 3.

We are talking about Induced Strain Actuation and its analysis for a one-dimensional beam and the analysis was static analysis. And we started with the Euler Bernoulli Beam method. We looked into few cases and here we will see one more case. Now the beam is again fitted with. It has two actuators at the top and bottom and they are similar actuators, but the voltage applied is different.

So, the voltage at the top and voltage at the bottom is different. So, they can be same direction wise. They can be opposite in direction, but their magnitude are different and that creates a lot of difference. So, it would induce both extension and bending.

So, what we can do is, we denote the voltage at the bottom as V bottom and the voltage at the top as V top. Now, what we do is, we decouple these voltages into a pure bending and pure axial case. So, we assume that there is a voltage V_1 and same voltage V_1 and there is voltage V_2 and a voltage minus V_2. So, V_1 and V_1 they induce pure extension or pure contraction and V_2 and minus V_2 they induce pure bending and their combination gives us the actual voltage V top and V bottom. So, when we superimpose these two, we get the V bottom and V top.

So, V_1 plus V_2 is equal to V bottom as per our consideration and V_1 minus V_2 is equal to V top which is shown here. Now if we solve these two equations, we get V_1 and V_2 in terms of V top and V bottom. So, V top and V bottom are given to us. V_1 V_2 we do not know. So, by solving this we get V_1 and V_2 in terms of V top and V bottom.

So, this is what is going to cause a pure axial effect and this is what is going to cause a pure bending effect. So, as we have decoupled the problem, now we can solve this problem separately as we have solved the pure axial problem and we can solve this problem separately as we have solved for the pure bending problem. So, for the axial problem we can find out Np. Np becomes 2 multiplied by Ec, Bc, tc into epsilon P_1 epsilon P_1 is the free stress corresponding to voltage V_1 free strain corresponding to voltage V_1 and this epsilon P_1 is in terms of V_1 d_{31} multiplied by V_1 by tc. This is the actual trafficness and this can be written as EA total into epsilon 0 which we have been doing so far.

So, by solving this equation EA total epsilon 0 is equal to this Np where Np is this we can find out epsilon 0. Now you find out Mp. Mp is Ec, Bc, tc, tc plus tb epsilon P_2. So, epsilon P_2 is free strain corresponding to voltage V_2. So, we have epsilon P here we have the opposite epsilon P here and that creates a moment and if you find out that moment we get this.

So, the corresponding to this epsilon P_2, we find out the moment here. So, we have to have this elastic modulus and that should be equal to EI total multiplied by kappa and if we solve this equation EI total multiplied EI total kappa is equal to Mp we can find out kappa. So, if we separately find out epsilon 0 and kappa and then we can superimpose the results and that would give us the actual solution.

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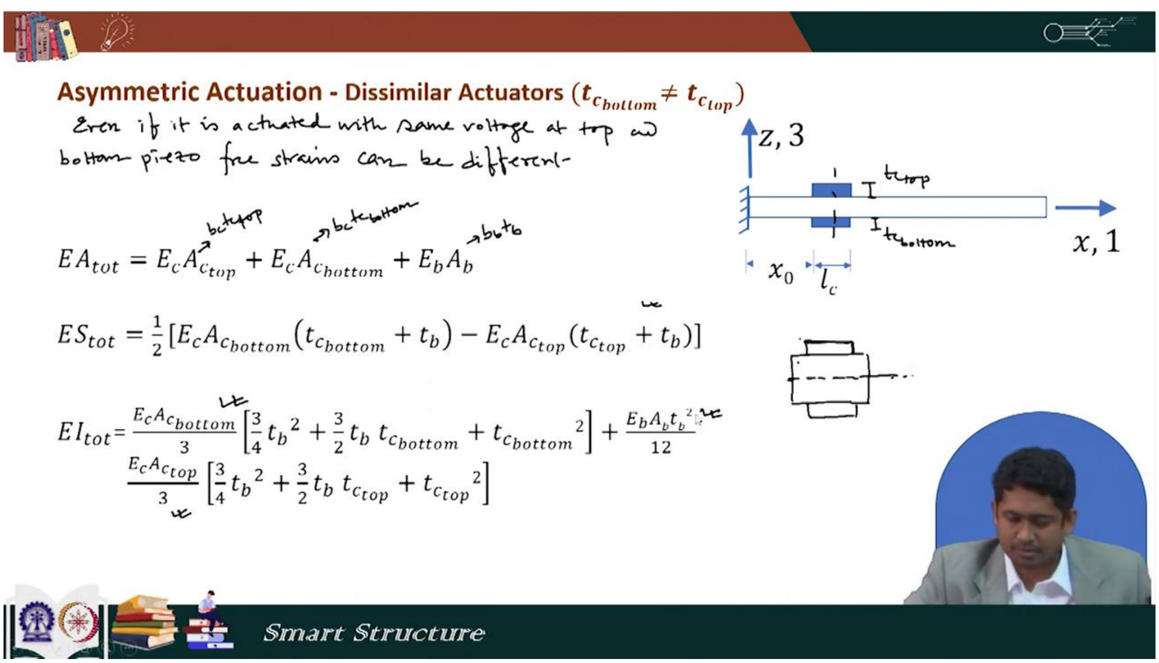

Now, we look into another problem where we have asymmetric actuation. It is asymmetric because although we have two piezoelectric patches, but their thickness are different. So, this thickness we may call as tcb bottom and this thickness can be called as tc top. Now even if it is actuated with same voltage at top and bottom, piezo free strains can be different because if you look at the expression of the free strain, there is tc sitting at the denominator. So, that would make the free strain different. So, to solve this problem again if we look at the cross section, the cross section can look like this. To solve this problem we take z is equal to 0 at the mid of the beam. So, EA total becomes Ec Ac top. Ec is elastic modulus of the top piezo, Ac top is the area of the top piezo. So, Ac top is bc into tc top. So, Ac top is bc into tc top and Ac into Ac bottom where Ac bottom is bc into tc bottom. Their width is same.

So, both are bc and then we have Ab area of this beam section which is bb into tb and now we find out ES total. Now as this section is not symmetric with respect to this line, this z equal to 0-line, ES total is not going to be 0. This is going to be nonzero. So, we have some ES total which is nonzero and that is this and then we find out the EI total again with respect to this line. So, in the EI total this is the contribution from the bottom piezo from the top piezo and from the beam. So, we found out our EA total, ES total and EI total and because the section is not symmetric ES total remains here.

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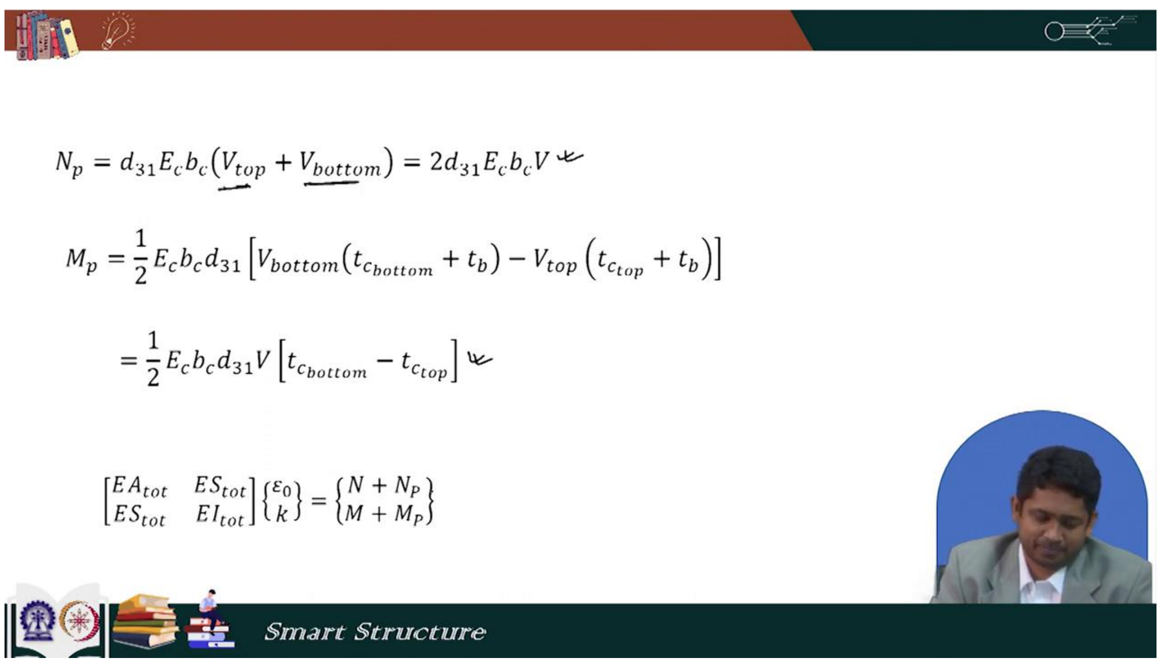

Now we find out our Np and Mp in the similar process that we have adopted so far we can find out our Np. So, this is the Np and although the top and bottom voltage are same. So, just to show, it has been separately V top and V bottom and Mp is this. So, you find out the moment with respect to the z equal to 0 line and that becomes our Mp. Now when V bottom and V top is equal to V, we get this as our Np and this as our Mp and finally, this is the equation that we have to solve. Here because ES total are nonzero, so, these two equations are coupled and by solving this coupled equation, we can find our epsilon 0 and kappa.

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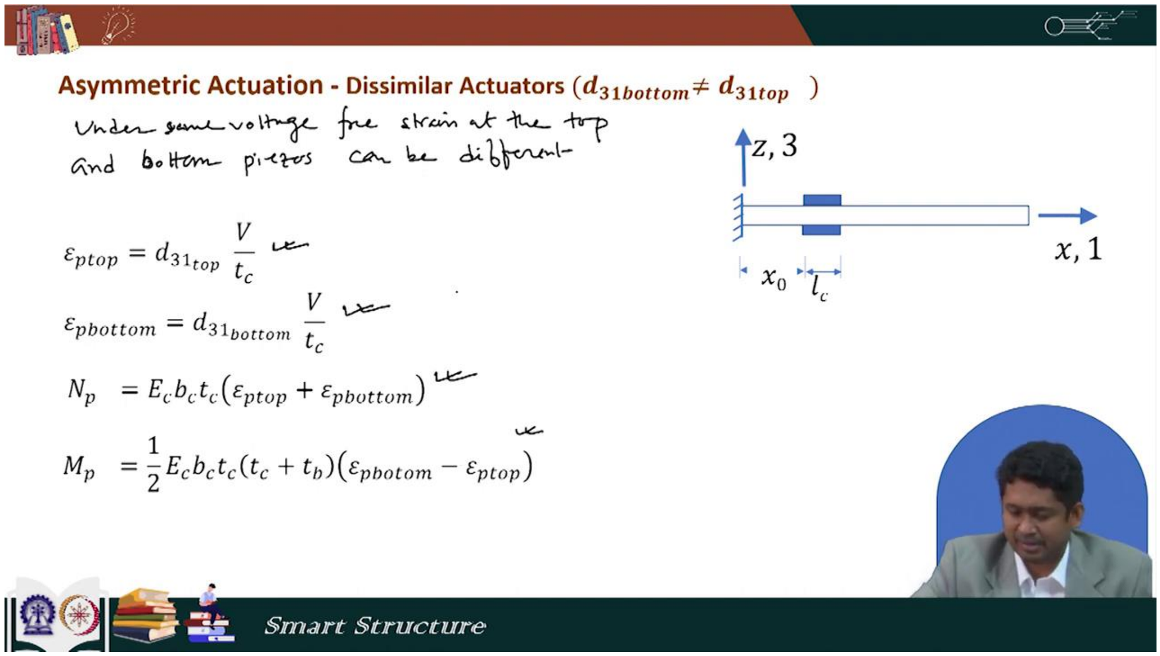

Now in this case again we have dissimilarity, but here the dissimilarity in terms of the d 31. So, the d 31 at the bottom piezo and top piezo are not same. So, again we can say that under same voltage, free strain can be different at the top and bottom piezo can be different.

So, epsilon p top is the free strain at the top piezoelectric patch which is this epsilon p bottom is the free strain at the bottom piezoelectric patch which is this. Np the axial force corresponding to the free strain we can get in the similar fashion and Mp the bending moment corresponding to the free strain. Again we can get in the similar fashion. So, this is what we get as Np and Mp and then after that finally, the equation looks like this.

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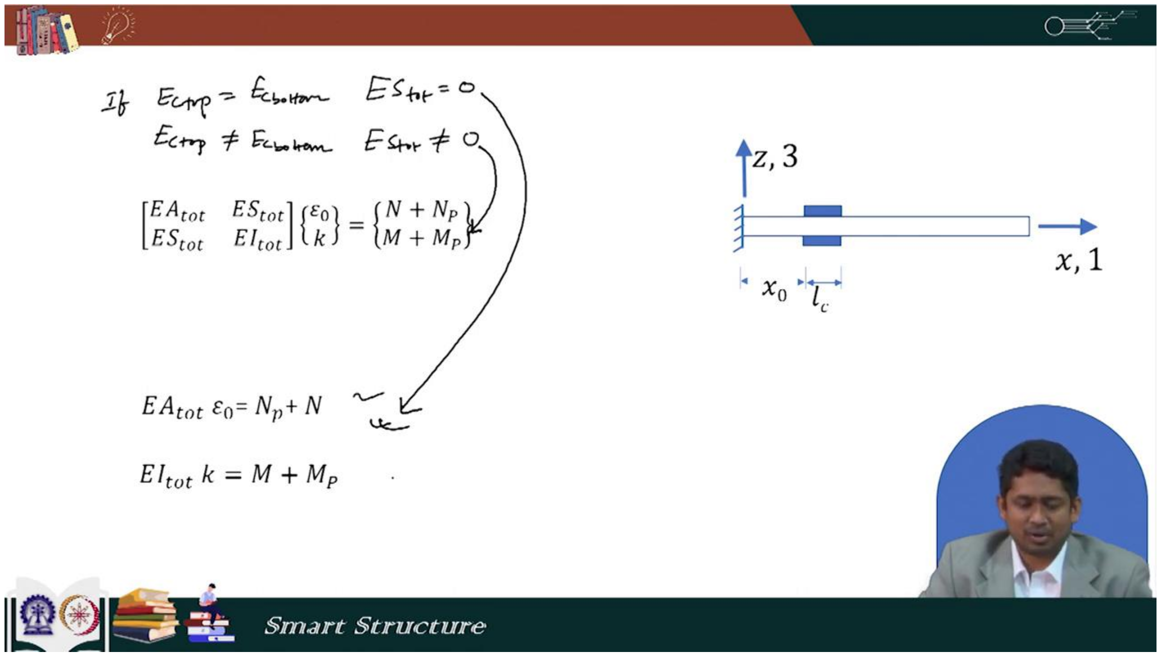

Now here one thing to note is that, we would find out ES total, EA total and EI total in the same way. Now the two pieces are different. So, if Ec top is equal to Ec bottom in that case ES total is going to be 0 because the section becomes symmetric in terms of the elastic properties, but when Ec top is not equal to Ec bottom, ES total is a nonzero value. So, in that case we have to solve the coupled equation if ES total is 0 then we can get rid of these two terms and two equations get decoupled like this and if they are not same, then we have to solve the coupled equations. So, this leads to this situation and this leads to this situation. So, with that our static analysis under induced strain for one dimensional case using the Euler Bernoulli beam technique is finished.

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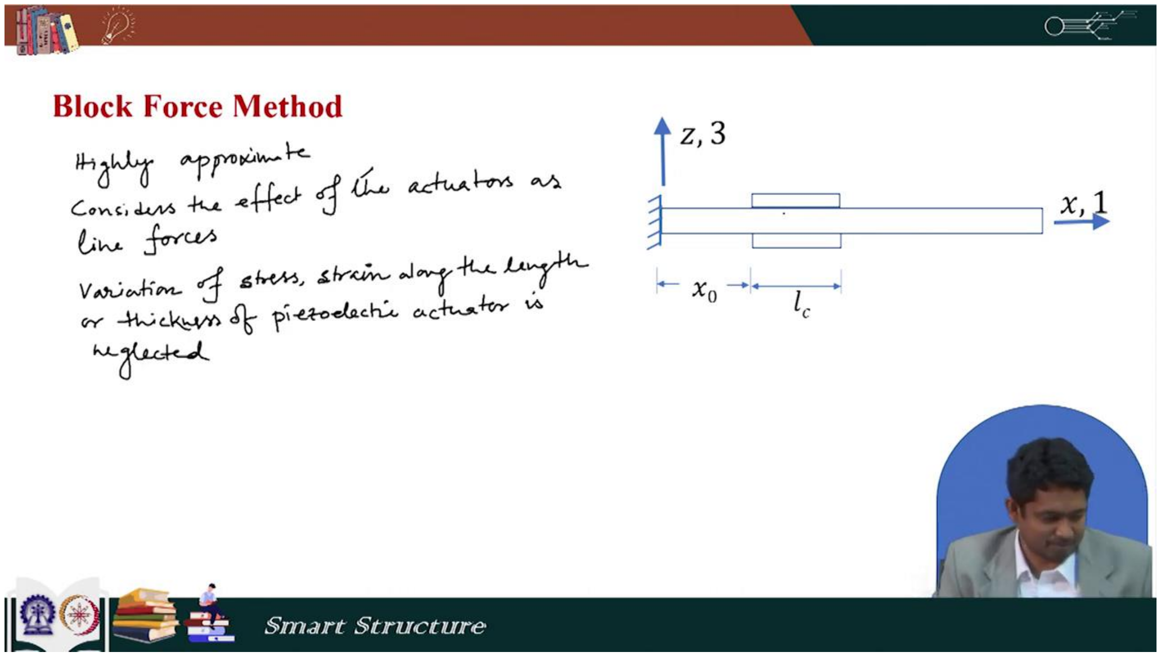

Now we will go to our next technique called the Block force phase method. Block force phase method is a very approximate method. It is highly approximate and it has some more assumptions. So, first of all it is highly approximate. It considers the effect of the actuators as line forces and variation of stress strain along the length or thickness of the piezoelectric actuator is neglected. So, the stress strain along the length and thickness is assumed to be same and effect of this actuator is considered as a line force. We will see that in the next slide.

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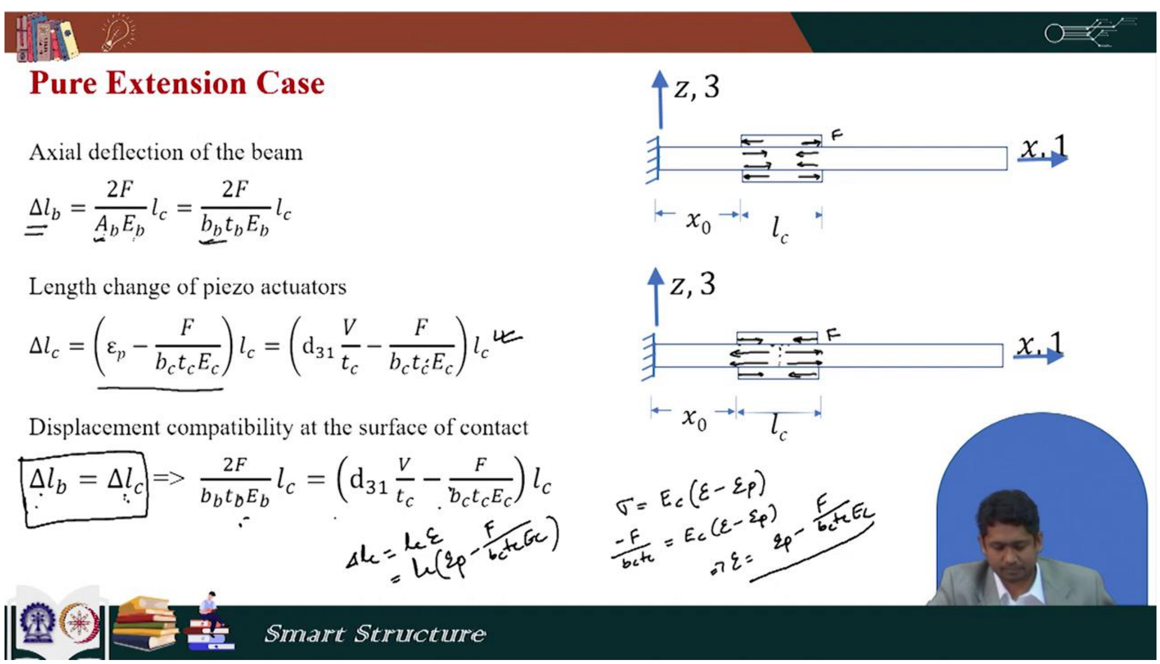

Again we look into those same cases that we did for the Euler Bernoulli beam based technique. So, the first is a extension case as we know. They are actuated with same voltage.

Now if the actuation is such that this piezo contracts both the piezo, that is why the piezoelectric patch is under tensile force and the beam is under compressive force and it happens for both the top and bottom piezo and all these forces have same magnitude F. In these problems, the unknowns are our F's which we if we can find those we can solve the problem. The case can be reverse the actuation can be such that the piezoelectric patch tries to expand. If the piezoelectric patch tries to expand the effect just becomes reverse the piezoelectric patch experiences a compressive force and the beam experiences tensile force similarly here and here and again the magnitudes are we denote as F. So, as I said in this approach our unknown is F and we try to find F by imposing the condition that the displacement at these junctions between the piezoelectric patch and the beam is same.

The compatibility of displacement we impose and by imposing that condition, we try to find out our unknown actuation force F. So, if we try to do it for the pure extension case, we have to find out the axial deflection of the beam, I mean the expansion of the beam, I would say, or the contraction at the junction between the piezoelectric patch. Now it is because it is axial effect, so, the effect is same whether at the top junction or the bottom junction or in between those. It does not matter everywhere it experiences same expansion or contraction and that is denoted as delta lb. Now, delta lb is if we see we have a force F and we have force F.

So, the total force experienced by the beam at here is 2 F. So, if we divide this 2 F by this area of the beam which is Ab that gives us the stress and then if I divide by the elastic modulus of the beam that gives me the strain and because the strain is same in this actuated zone, so, we can just multiply this by lc the length of that actuated zone and that gives me of my expansion or contraction of the beam that is delta lb. Now I can replace Ab by bb tb and this is my full expression for the expansion or contraction of the beam or the axial deflection of the beam. Now we need to find out the length of change of piezo actuators. So, for the piezo electric actuators, the change in length is this epsilon p minus F by bc tc Ec multiplied by lc which we can verify here. We know that the stress in the piezoelectric patch is Ec multiplied by epsilon minus epsilon p. Now epsilon is the stress. To get the stress, we have to get it from the force from this force F. Now we are denoting the force in the beam as F.

So, in the actuator it is minus F. So, it is minus F divided by bc tc is equal to Ec epsilon minus epsilon p and then by solving this we get the expression as epsilon is equal to epsilon p minus F by bc tc Ec. Now if this is my epsilon p from here I can find out the changing length we just have to multiply with lc. So, we get delta lc is equal to lc multiplied by epsilon and that is equal to lc the entire thing multiplied by epsilon minus F by bc tc Ec. So, this is what we can see here and then epsilon p can be written as V by tc and this is our final expression for change in length of the piezoelectric actuators. Now that we have found out the change in length of the beam and change in length of the actuators.

We have to apply the condition of compatibility. So, we apply the condition here. These our compatibility condition or compatibility of displacement and that says that delta lb is equal to delta lc. Delta lb is equal to delta lc. Delta lb is taken from here. Delta lc is taken from here and they are equated.

$$\varepsilon = \varepsilon_p - \frac{F}{b_c t_c E_c}$$

$$\Delta l_c = l_c \varepsilon = l_c(\varepsilon_p - \frac{F}{b_c t_c E_c})$$

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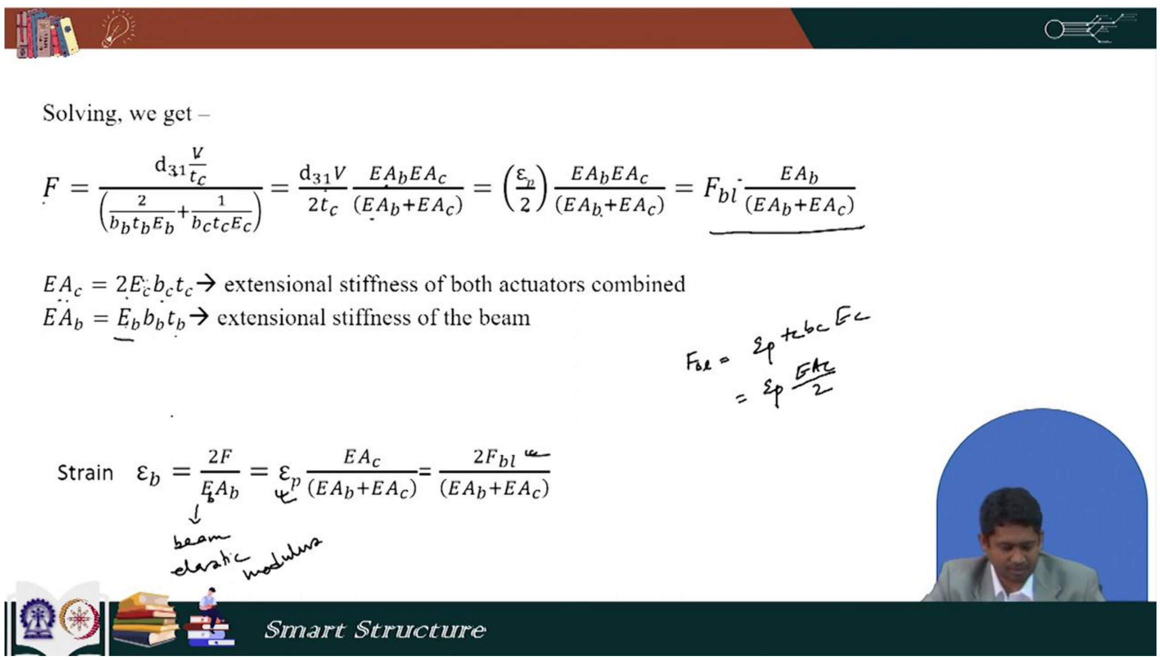

And after equating those, we get F as this. Now this expression can be written in a more compact way. So, we take V by tc by 2 here and here I have E Ab E Ac EAb plus EAc where EAb is the area of cross section of the beam and EAc is the combined area of cross section of the 2 actuators. So, 2 Ec Vc tc, I mean, it is EAc. So, apart from area we have to factor it by elastic modulus also. So, they are here. Now dc Vc by tc is my epsilon p say epsilon p divided by 2 and this remain same and then we can write this entire thing as in terms of the block force also. We know that block force is Fbl is equal to epsilon p multiplied by tc bc into Ec and tc bc is the area of one actuator. So, tc bc Ec can be written as EAc divided by 2.

So, epsilon p is equal to twice Fbl by EAc. If we put it here then the expression in terms of block force becomes this. So, this is our actuation force in terms of the free strain or in terms of the block force. Once we get our actuator force we know everything. Now we can find out how the beam response to the actuation. So, if we just go back to the previous slide after we know the actuation force. So, the actuation force acts here then. So, this beam is under force twice F. So, twice F divided by the area is our stress and that divided by its elastic modulus is the strain. So, it is Eb beam elastic modulus and then again it can be further simplified in terms of either epsilon p or Fbl.

$$F_{bl} = \varepsilon_p t_c b_c E_c = \varepsilon_p \frac{EA_c}{2}$$

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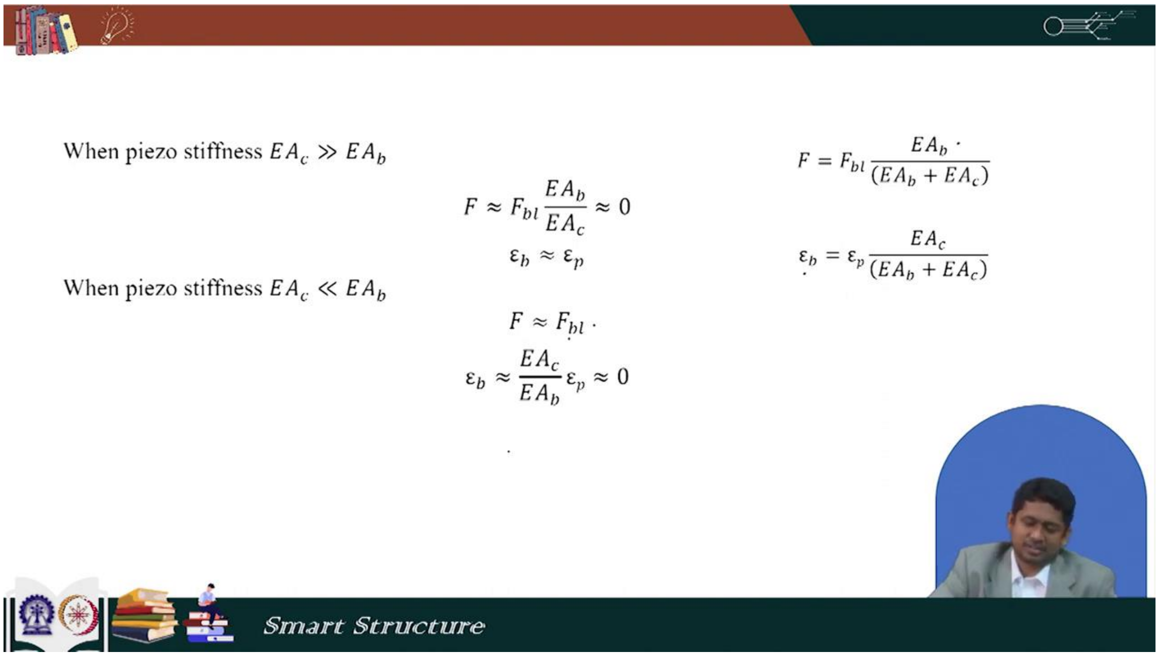

Now that we know the expressions for the force and the strain, we can see some extreme cases now. If our EAc is much more than EAb then this expression for block force if EAc is much more than EAb then this expression becomes 0.

This tends to 0. So, block force becomes 0 and when block force is 0 which means the actuator is expanding freely and that is also evident from the expression for epsilon b. If I put the condition that EAc is much greater than EAb then this tends to be 1. In that case epsilon b becomes epsilon p. So, here because piezo stiffness is much higher than the beam stiffness, so, the piezo experiences less resistance by the beam or the bar. So, it can expense in its own way. So, it does not feel any obstruction. That is why the block force is 0 and it expense freely. So, the strain is equal to the free strain. In the reverse case when the piezo stiffness is much less than the beam stiffness, in that case, it experiences a quite a bit of obstruction by the beam.

So, the force is equal to the block force. So, again if you put the condition that EAb is much higher than EAc this becomes 1. Then F becomes Fbl. So, that is what we see here. On the other hand when the force is just equal to the block force which means the piezo is not able to deform or expand. In that case, epsilon b is equal to 0. So, that is what we see here. So, in that case, it is 0. So, this is what we see here. So, these two are the reverse cases the effects are opposite.

Now, with that I would like to conclude this lecture here. In the lecture, we will see some development of this case itself and then we move on to some few next cases.

Thank you.