Smart Structures
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Week - 02
Lecture No - 11
Piezoelectric Sensors and Actuators

Welcome to the fifth lecture on Piezoelectric Materials.

So far, we have derived the constitutive relations of piezoelectric materials and we have seen their interrelations.

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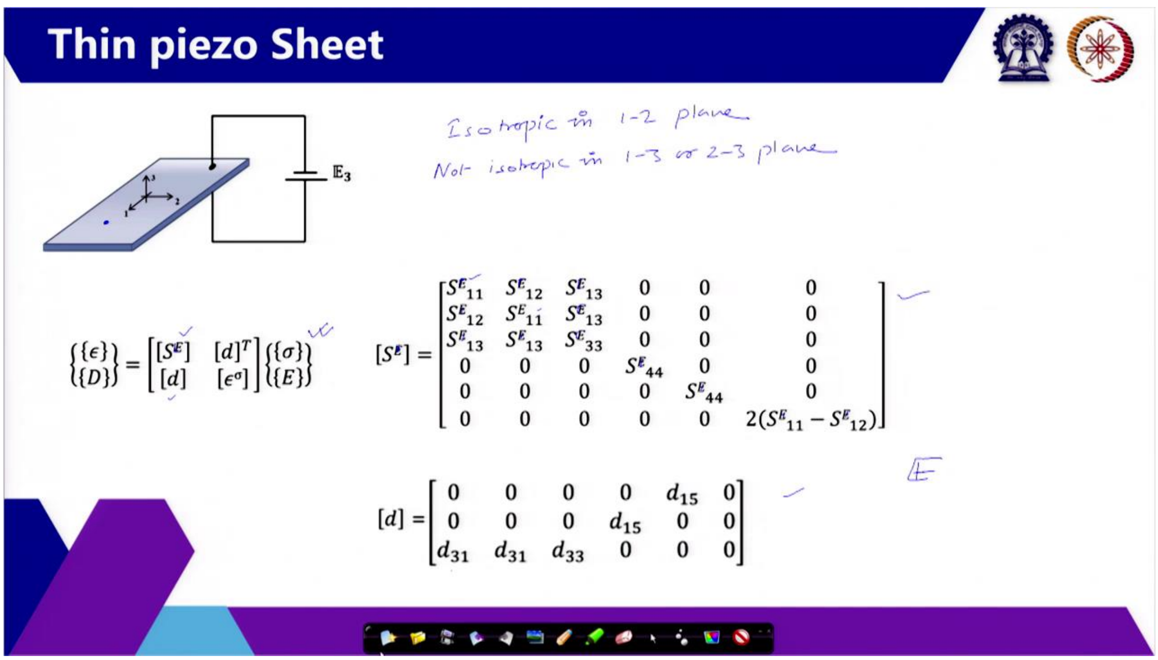

Now, we will talk about how these piezoelectric materials are used as sensors and actuators and we will look at some simplified cases. So, a piezoelectric materials is mostly available in most of the applications as this kind of thin sheets. Now, these thin sheets are polarized along a certain direction. So, these sheets are mostly polarized along the vertical direction, i.e. direction 3. They can be polarized in other directions also, but what we get is mostly in this form and that is what mostly useful to us. So, the polarization is shown by different marks for example, it can be shown as a dot like that which shows that it is polarized along this third direction i.e. z direction. Now, these sheets are generally isotropic in 1-2 plane which means the properties are same in 1-2 plane. So, whatever the elastic modulus or the other properties are there in this direction i.e. in direction 1, it can be same in direction 2 also, but they are not isotropic in other planes i.e. not isotropic in 1-3 or 2-3 plane. So, with that if you want to write the constitutive relation in this form, then these constants SE and d comes to be like that. So, it is isotropic in 1-2 plane.

So, S_{11} is equal to S_{22}. So, we see S_{11} here also and similarly S_{13} is equal to S_{23}. We see S_{13} and S $_{23}$ here also and again please understand that these SE are considering that the other state variables is electric field. So, better we put a double strokes to be consistent with all of our notation that we have. So, it is all are double stroke is everywhere. Now, similarly these are d matrix and in the d matrix also because of its symmetry we have d_{31} and d_{32} same and d_{24} and and d_{15} are same.

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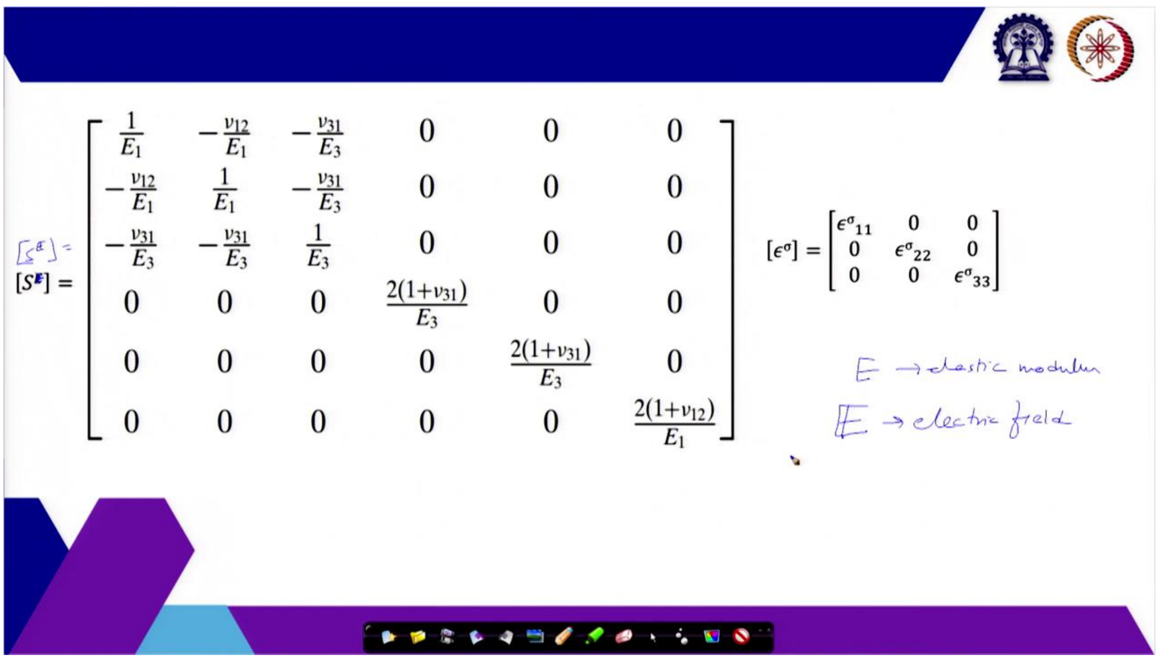

$$[S^{E}] = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_1} & -\frac{\nu_{31}}{E_3} & 0 & 0 & 0 \\ -\frac{\nu_{31}}{E_3} & -\frac{\nu_{31}}{E_3} & \frac{1}{E_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu_{31})}{E_3} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+\nu_{31})}{E_3} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+\nu_{12})}{E_1} \end{bmatrix}$$

$$[\epsilon^{\sigma}] = \begin{bmatrix} \epsilon^{\sigma}{}_{11} & 0 & 0 \\ 0 & \epsilon^{\sigma}{}_{22} & 0 \\ 0 & 0 & \epsilon^{\sigma}{}_{33} \end{bmatrix}$$

If we look at this SE matrix in an expanded form it looks like this. So, in terms of the elastic modulus of the material this SE matrix can be written. So, these E's are the elastic modulus. So, we have now two E's, one E is for elastic modulus and a double stroke E for electric field.

So, these E's inside this matrix are all elastic modulus and whereas, this E this superscript is considering the fact that in this constitute relation the other state vector is electric field. So, that is why it is a double stroke E. Now, in terms of this elastic modulus in different directions the entire SE matrix is written in this way and this is our epsilon sigma matrix. So, this is what we can use for these thin piezoelectric sheet materials.

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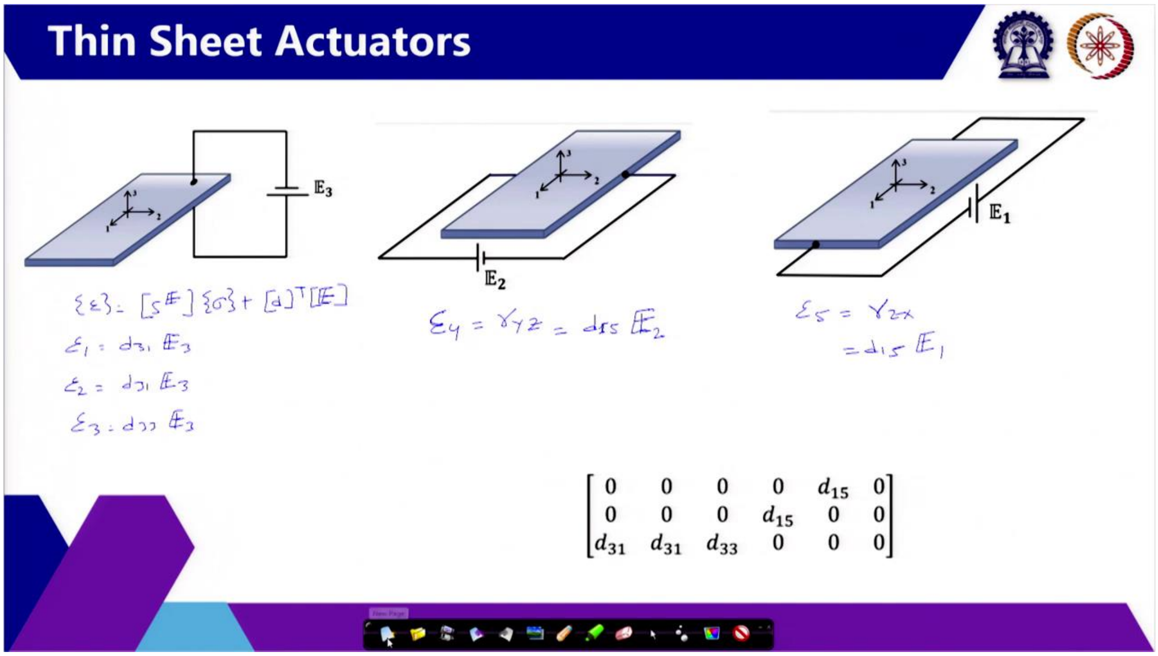

So, let us assume that we are using these thin piezoelectric sheets as our actuators. So, this is the first case where we have this piezoelectric sheet and it is polarized along z direction and this piezoelectric element has electrodes deposited at the top surface and the bottom surface. So, along the three axis we have the top surface and at the bottom surface and at those two surfaces electrodes are deposited. So, that we can apply voltage to it. Now assume that it is under electric field E and if that is the case then from the constitutive relation that we are supposed to use here is epsilon is equal to SE multiplied by sigma plus d^T multiplied by E.

$$\{\varepsilon\} = [S^{\mathbb{E}}]\{\sigma\} + [d]^T[\mathbb{E}]$$

So, with this relation we can write epsilon 1 is equal to d_{31} multiplied by E_3.

$$\varepsilon_1 = d_{31}\mathbb{E}_3$$

So, E_3 is the voltage applied. So, because of this voltage applied E_3 it will induce a strain along direction 1. Similarly, along direction 2 it will induce another strain and along direction 3 it will induce another strain that is d_{33} into E_{33}.

$$\varepsilon_2 = d_{31}\mathbb{E}_3$$

$$\varepsilon_3 = d_{31}\mathbb{E}_3$$

Now please consider the fact that sigma is 0 here because this element is free to expand or contract. So, because we are not restricting its expansion or contraction under the electric field it is not going to experience any stress. So, because this is 0. So, we can just solve this equation epsilon is equal to d^T multiplied by electric field and that gives us this.

Now let us move to the next case. In this case we are applying a voltage along the second direction i.e. direction 2. Now this is possible to apply it, but again for this to happen, we have to have electrodes deposited at the 2 faces along direction 2. So, at this face as well as at the opposite face. So, at the positive 2 face and at the negative 2 face we have to have electrodes deposited. Now for this case again applying the same equation we get epsilon 2 we have we get epsilon 4 and we apply electric field 3 here. We get epsilon 4 which is equal to gamma yz and that comes to be d_{15} multiplied by E_2.

$$\varepsilon_4 = \gamma_{yz} = d_{15}\mathbb{E}_2$$

And similarly, here we are applying a voltage along E_1 direction again for this to happen we have to have electrodes deposited at that face positive and negative face of one direction. And under this electric field epsilon 5 which is epsilon zx that is what we get and that is equal to d_{15} multiplied by E_1.

$$\varepsilon_4 = \gamma_{zx} = d_{15}\mathbb{E}_1$$

So, these are several modes of actuation and under those different modes of actuation these are the different strains that we get.

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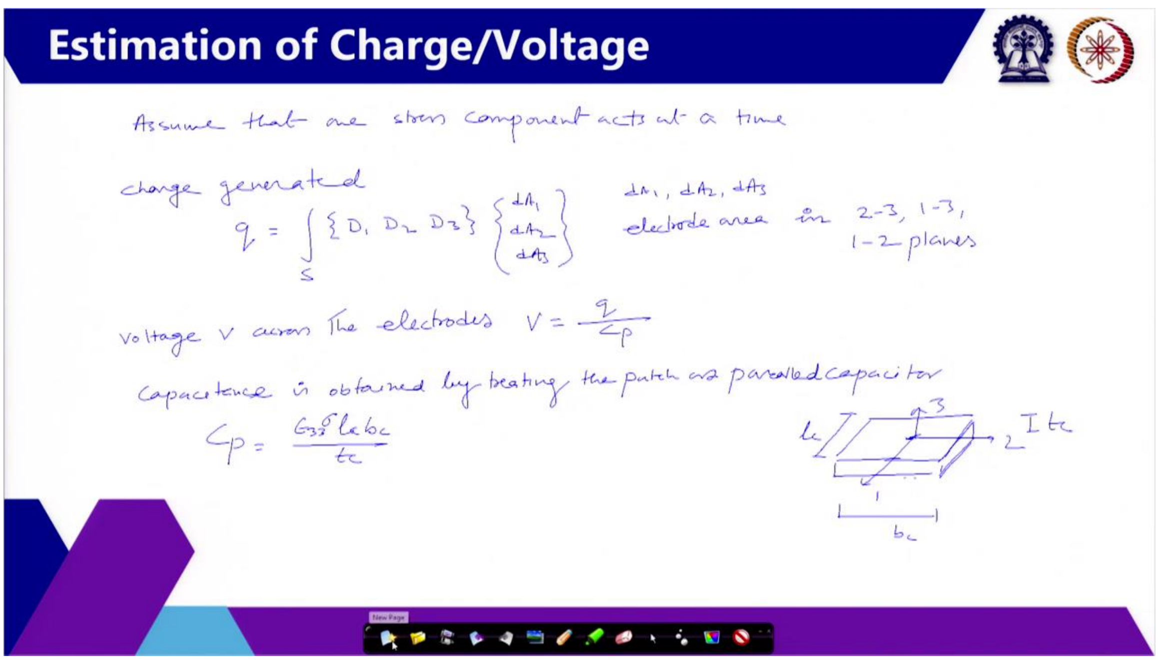

Now we will move to the next discussion that is on estimation of electric charge and voltage. Now we will talk about estimation of electric charge and voltage. So, here we are talking about direct piezoelectric effect. So, first we will assume that I mean our first assumption is that one stress component acts at a time just to keep things simple. And from that we will solve the equation and finally, by solving the equation we will get the electrical displacement and from the electrical displacement we can get the electrical charge and from the charge we can find out what is the amount of voltage it is generating. So, charge generated q can be related to the electrical displacement component as D_1 D_2 D_3 and that multiplied with this vector and integrated over the surface. So, dA_1 dA_2 dA_3 are electrode area in 2-3, 1-3 and 1-2 planes.

$$q = \int_S \{D_1 \quad D_2 \quad D_3\} \begin{Bmatrix} dA_1 \\ dA_2 \\ dA_3 \end{Bmatrix}$$

Now voltage generated can be related to the charge as q divided by the capacitance Cp.

$$V = \frac{q}{C_p}$$

Now capacitance is found out by treating the patch as a parallel capacitor. So, if these are thin piezo sheet which we call piezo patch also and suppose it is electrodes are deposited here and here at the top surface along direction 3 at the positive and negative surface of the 3 axis. Then we can treat this sheet to be a parallel capacitor and accordingly we can find out the capacitance. So, that gives us the expression of capacitance as epsilon 3 d sigma Lc bc divided by tc.

$$C_p = \frac{\varepsilon_{33}^{\sigma} l_a b_c}{t_c}$$

So, lc is our dimension over this, bc is our dimension over direction 2 and tc is the thickness dimension over direction 3. So, lc bc is the area and tc is the thickness. So, that gives us the expression for the capacitance. So, from by solving the constitutive equation I can find out our electrical displacement and from the electrical displacement we can find out the charge by using this equation and if I put the capacitance expression here that gives us an estimate of the voltage generated.

So, now we look into few cases.

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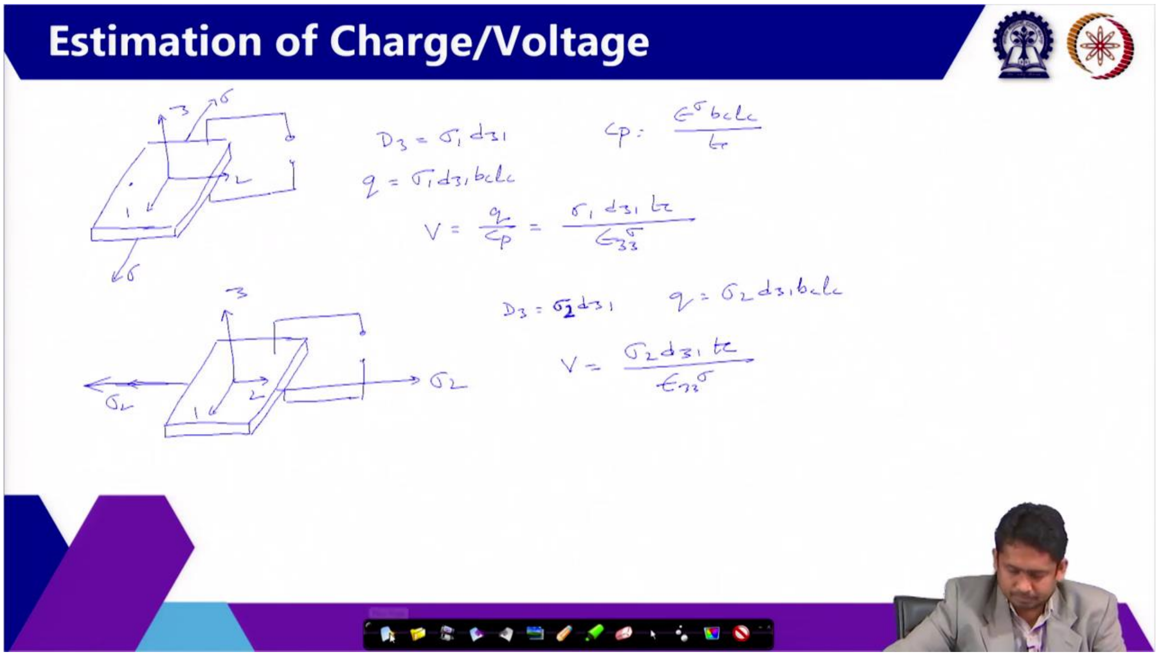

So, the first case is this. We have a piezo patch and these are our directions 1, 2, 3 and suppose we are finding out the voltage here across the 3 directions and this patch is under a normal stress sigma and it is polarized in direction 3. So, in this case by solving the same equation, we can get D_3 to be is equal to sigma 1 into d_{31}.

$$D_3 = \sigma_1 d_{31}$$

And if I find out the capacitance treating this as a parallel plate capacitor where this surface and the opposite surface are my parallel plates. In that case we can find out the expression for the capacitance as we saw before and then so, it would be as we just derived epsilon sigma and then bc lc by tc.

$$C_p = \frac{\varepsilon^{\sigma} l_c b_c}{t_c}$$

And from this D3 we can estimate the charge and that is sigma 1 multiplied by d_{31} which is basically capital D3 multiplied by bc lc.

$$q = \sigma_1 d_{31} b_c l_c$$

Now, we can put everything here q by Cp and that gives us voltage equal to sigma 1 d_{31} tc divided by epsilon 33 sigma.

$$V = \frac{q}{C_p} = \frac{\sigma_1 d_{31} t_c}{\varepsilon_{33}^{\sigma}}$$

Now we look into another case where we have again a similar piezoelectric patch, property wise it is same. It is the same patch, only difference is that the stress is applied in direction 2 and this is direction 1, this is direction 2 and this is direction 3 and we are measuring the voltage across the direction 3. So, everything remains same. Our D_3 is also sigma 1 d_{31} because the plane is isotropic because it is isotropic in the 1-2 plane. Stress is now sigma 2, it is not sigma 1 and q is sigma 2 d_{31} bc lc and expression for the capacitance remain same.

$$D_3 = \sigma_2 d_{31}$$

$$q = \sigma_2 d_{31} b_c l_c$$

So, finally, voltage becomes sigma 2 multiplied by d_{31} tc divided by epsilon 33 sigma.

$$V = \frac{\sigma_2 d_{31} t_c}{\varepsilon_{33}^{\sigma}}$$

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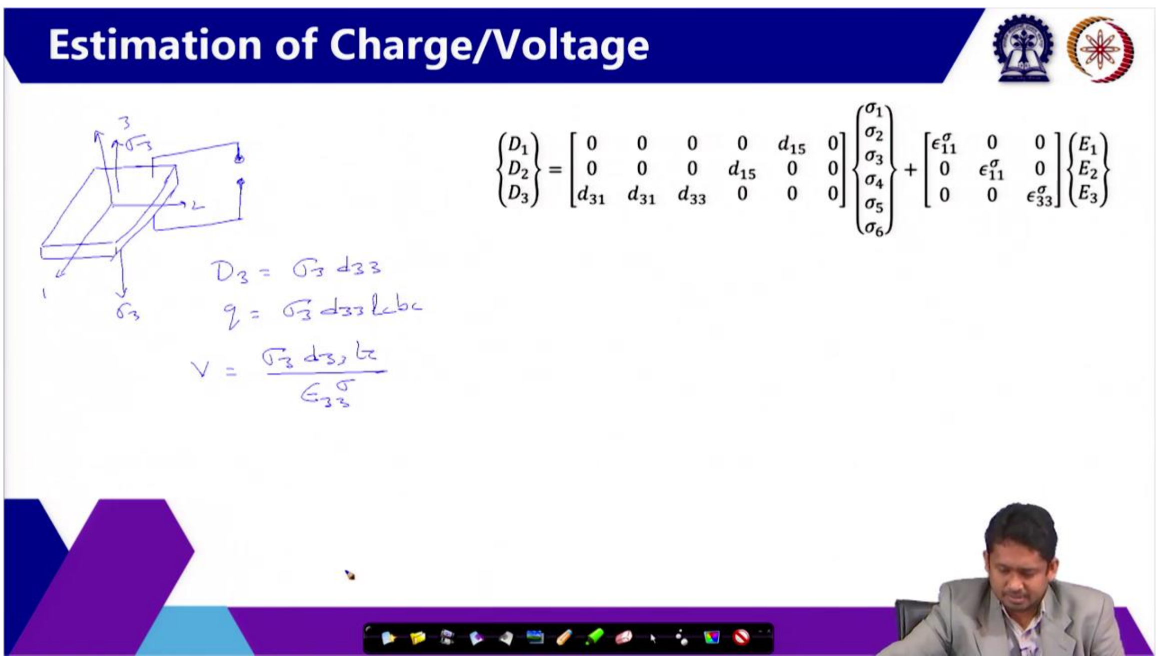

Now, we will look into few more cases. So, suppose we have again the same piezoelectric patch. So, this is direction 1, 2, 3 and then it is under a stress sigma 3 that means, normal stress along direction 3. And again the voltage is measured across the third direction i.e. third axis. In this case, my D3 expression is sigma 3 multiplied by d_{33}. So, q becomes sigma 3 d_{33} multiplied by lc bc and expression for the capacitance remain same.

$$D_3 = \sigma_3 d_{33}$$

$$q = \sigma_3 d_{33} b_c l_c$$

So, that gives me that voltage is equal to sigma 3 d_{33} tc by epsilon 33 sigma.

$$V = \frac{\sigma_3 d_{33} t_c}{\varepsilon_{33}^{\sigma}}$$

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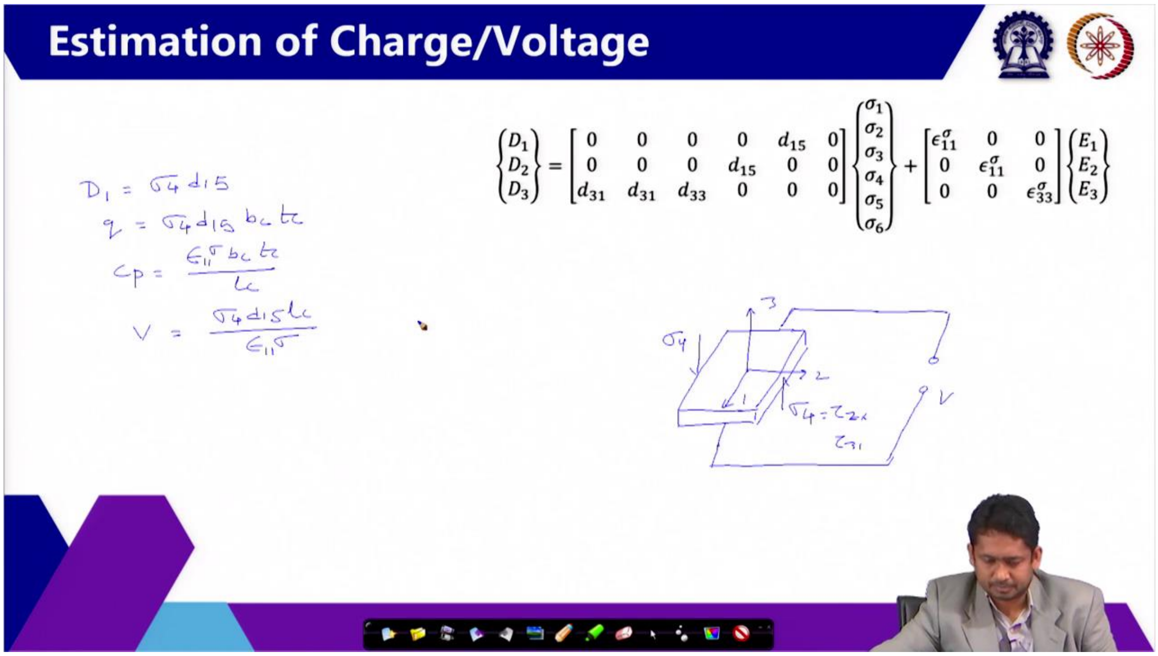

Next case is somewhat different. So, here we can write here. So, we have again the same piezoelectric patch, but now we are applying a shear. So, the shear is sigma 4 is equal to tau zx or we can call it τ_{31} as per our nomenclature 1, 2, 3 of the axis.

$$\sigma_4 = \tau_{zx} = \tau_{31}$$

And, then we have we have sigma 4 and we are measuring the electric field along the axis 1. So, here these faces have to have electrodes deposited to measure it and we are trying to measure the voltage here. If that is the case, then here we have D_1 is equal to sigma 4 multiplied by d_{15}.

$$D_1 = \sigma_4 d_{15}$$

So, that gives me that q equal to sigma 4 multiplied by d_{15} bc tc and then we get Cp is equal to epsilon 11 sigma multiplied by bc t c divided by lc that is the capacitance and with that we get an expression for the voltage as sigma 4 d_{15} lc by epsilon 11 sigma.

$$q = \sigma_4 d_{15} b_c t_c$$

$$C_p = \frac{\varepsilon_{11}^{\sigma} t_c b_c}{l_c}$$

$$V = \frac{\sigma_4 d_{15} l_c}{\varepsilon_{11}^{\sigma}}$$

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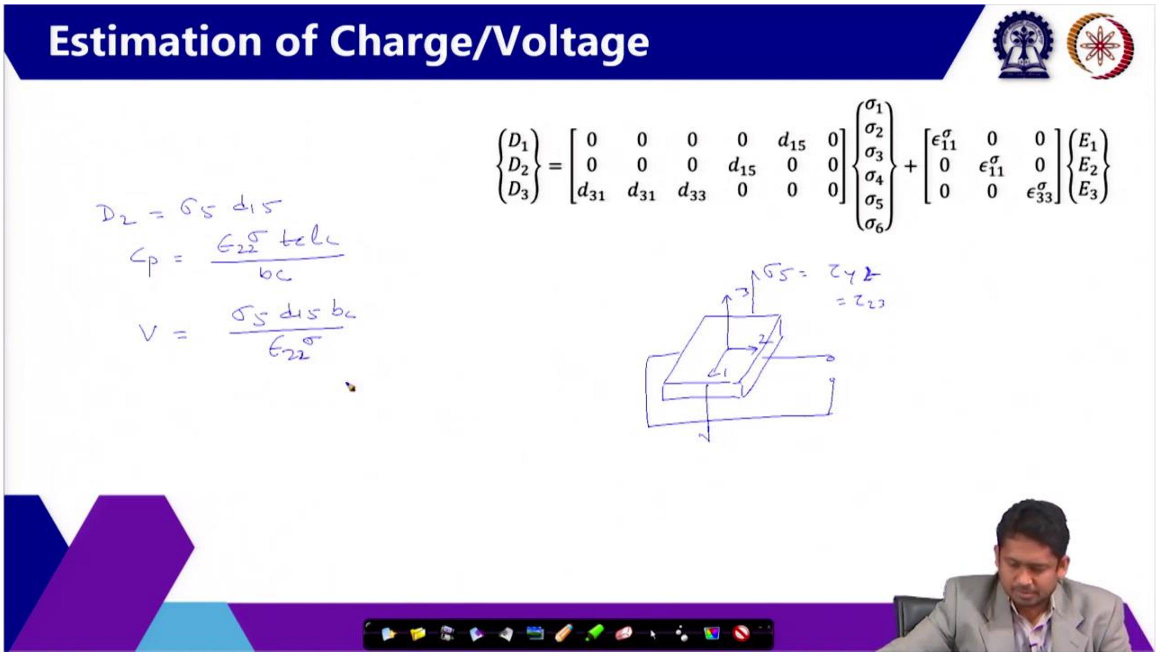

Now the last case, for that we have again a piezoelectric patch like this same piezoelectric patch only thing is that we are measuring voltage across axis 2 and the stress is this. We have sigma 5 applied. So, sigma 5 is equal to tau 32 yz we which we can call tau 23 as well again it is the same thing.

$$\sigma_5 = \tau_{yz} = \tau_{23}$$

So, D2 becomes now sigma 5 d_{15} and our capacitance changes because our voltage is measured along direction 1 now.

$$D_2 = \sigma_5 d_{15}$$

So, this is our capacitance and voltage become sigma 5 d_{15} bc by epsilon 22 sigma. So, that is about the measuring the voltage or charge.

$$C_p = \frac{\varepsilon_{22}^{\sigma} t_c l_c}{b_c}$$

$$V = \frac{\sigma_5 d_{15} b_c}{\varepsilon_{22}^{\sigma}}$$

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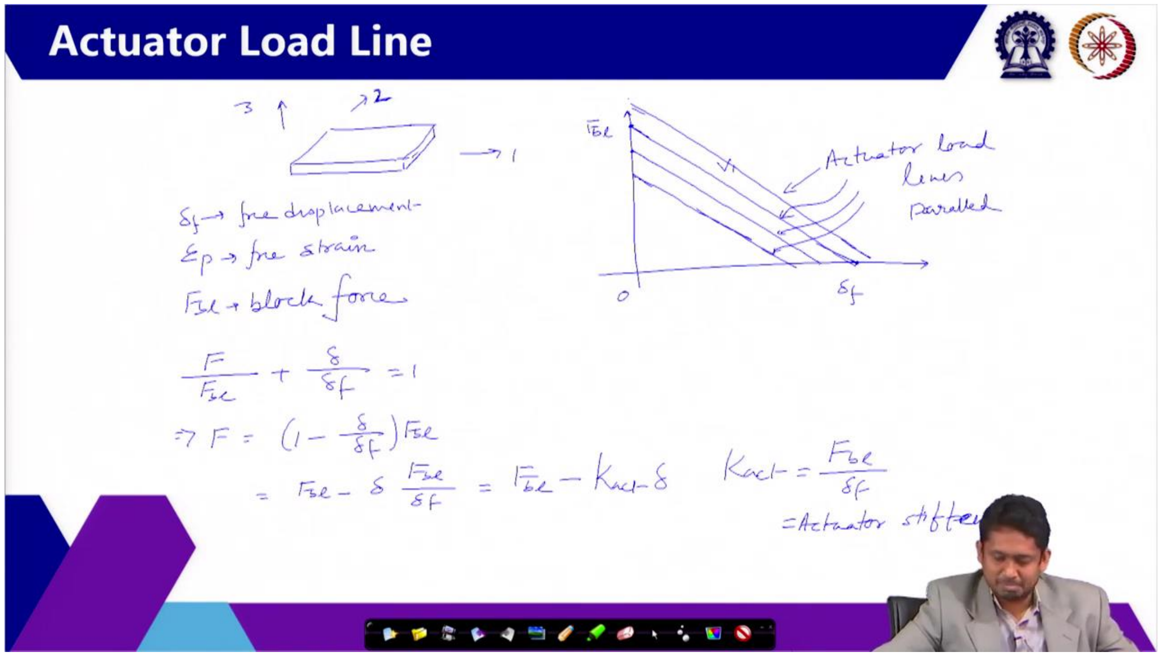

Now we will talk about another concept that is related to piezoelectric actuation. Now we will talk about actuator load line. So, imagine we have a piezoelectric sheet and our entire phenomena are assumed to be one dimensional.

See if I apply electric field, suppose it is direction 1, this is direction 2, this is direction 3 and to keep things simplified we are concerned with direction 1. So, we are giving the actuation in direction 3, but we are getting the result in direction 1. Now if we apply electric field along this third direction then we see that it will try to expand or contract depending on the direction of the electric field and polarization. Now if it is free to expand or contract then it will have a full expansion or contraction and let us call it delta f maybe. So, that delta f is called free displacement and the corresponding strain is called maybe free strain.

Now if I restrict it from expanding or contracting if we hold it tightly in its place then it would try to expand or contract, but it would not be able to. So, it will exert a force to the surroundings and the surroundings also would exert a reaction force to it and that force is called the block force. So, that is the maximum actuation force that we get from the actuator. So, when the force is 0 we get the maximum amount of displacement when the force is maximum we get 0 displacement and in between them it has some variation. Now if our actuator is linear then the variation in between them is also linear.

If the actuator is non-linear and then the variation would be non-linear. We are not going there. We are assuming our actuator is linear. So, it will have a linear variation and this is called actuator load line. So, this is for a certain voltage. So, maybe for V1. So, if I reduce the voltage and make it V2. So, at the reduced voltage, my del f will have some reduced value and the block force would have some reduced value and if I reduce it further it will again have some reduced value or if I increase it will have some enhanced value. So, these are called actuator load lines and they are parallel.

Fbl is called block force. Now if I want to fit a curve to it, so, it is a straight line. The equation can be written as F by Fbl plus delta by delta f equal to 1 and finally, it gives me expression F is equal to 1 minus delta by delta f multiplied by Fbl or I can write it as Fbl minus delta into Fbl by delta f which we can write as Fbl minus K act into delta.

$$\frac{F}{F_{bc}} + \frac{\delta}{\delta_f} = 1$$

$$F = \left(1 - \frac{\delta}{\delta_f}\right) F_{bl} = F_{bl} - \delta \frac{F_{bl}}{\delta f} = F_{bl} - K_{act}\delta$$

$$K_{act} = \frac{F_{bl}}{\delta f}$$

So, K act is called actuator stiffness which is the slope of the actuator load straight line.

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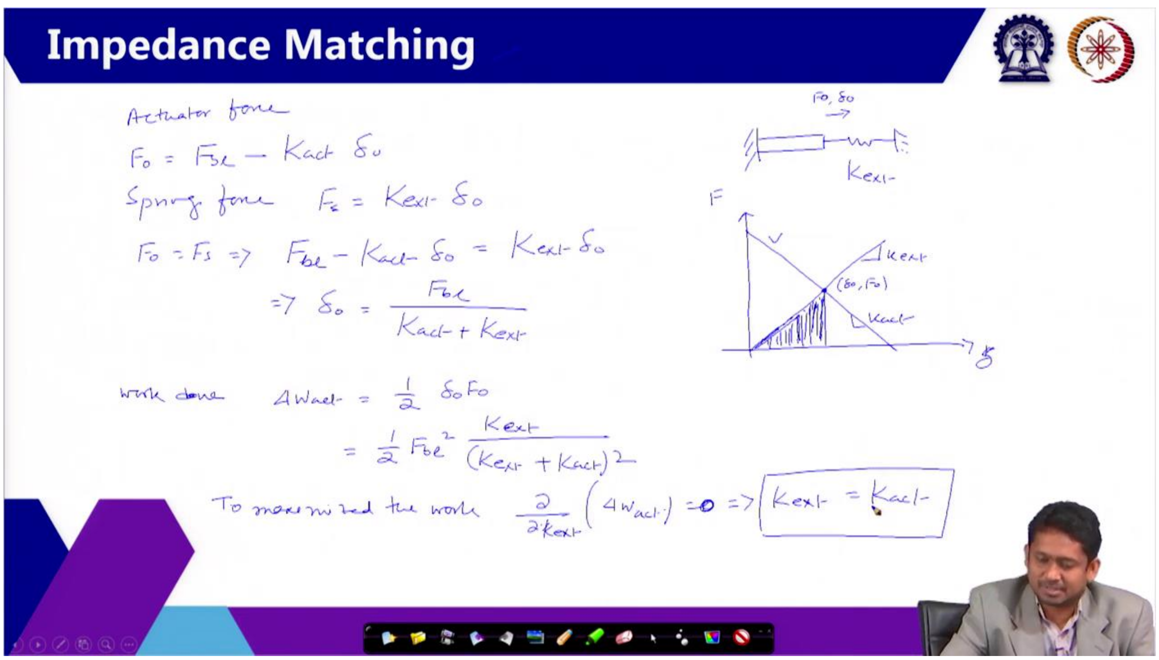

Now we will talk about impedance matching, a very closely related concept. So, imagine that we have the actuator and again we are concerned with a 1 D case to keep it simple and that is attached to a spring. So, we get a force here F0 and del 0. So, this spring does not keep the actuator free. So, the expansion cannot be delta f and apart from that the spring also does not restrain it fully. So, the force also cannot be a block force and the displacement cannot be 0. So, force is between 0 and the block force and the displacement is between 0 and delta f and that depends on the stiffness of the spring. So, from our previous equation we can write F0 is equal to F bl minus k actuator multiplied by delta 0.

$$F_0 = F_{bl} - K_{act}\delta_0$$

So, it is the actuator force and the spring force in terms of the spring constant can be written as K external, which is the stiffness of the spring. So, K external multiplied by delta 0, but F0 and Fs should be same for equilibrium.

So, we can write Fbl minus k act minus delta 0 is equal to K external delta 0 and that gives us delta 0 as block force divided by sum of the two stiffness, the actuator stiffness and the spring stiffness i.e. the external stiffness.

$$F_s = K_{ext}.\delta_0$$

$$F_0 = F_s \Rightarrow F_{bl} - K_{act}\delta_0 = K_{ext}\ \delta_0$$

$$\Rightarrow \delta_0 = \frac{F_{bl}}{K_{act} + K_{ext}}$$

So, it looks something like this graphically we can represent it in this way. So, if you are concerned with voltage V here. So, these are delta side and this is F. So, these are load line for this actuator and then on this if we draw a straight line which has a slope equal to the external stiffness and this is our actuator stiffness. So, the intersection of these two lines tells us about the displacement and the force.

The expression is written here. So, this is a way to graphically represent the same thing. So, now, when as the voltage increases we keep on getting this area. So, the area under this triangle increases. It becomes maximum when we reach our desired voltage and again from here we can come back and we can keep the cycle going on. So, in one half cycle when voltage goes from 0 to V the amount of work done is the area of this triangle. So, we can say that work done by the actuator is half multiplied by delta 0 into F0 and with all these previous expressions we can write this as this.

$$\Delta W_{act} = \frac{1}{2}\ \delta_0 F_0 = \frac{1}{2}\ F_{bl}^2 \frac{K_{ext}}{(K_{act} + K_{ext})^2}$$

Now, if I want to maximize the work. We can just say that the derivative of the external work is 0 and if we said the derivative 0 this gives us this condition that K external is equal to K actuator.

$$\frac{\partial}{\partial K_{ext}}(\Delta W_{act}) = 0 \Rightarrow K_{ext} = K_{act}$$

So, to extract maximum amount of work we have to make sure that the external stiffness and the actuator stiffness are same and this concept is called impedance matching. So, when impedance match we get most of it from the actuator. So, if we know the external stiffness and we need to choose a proper actuator for that we have to choose actuator in such a way that the stiffnesses are same and I mean the impedance matching condition is satisfied.

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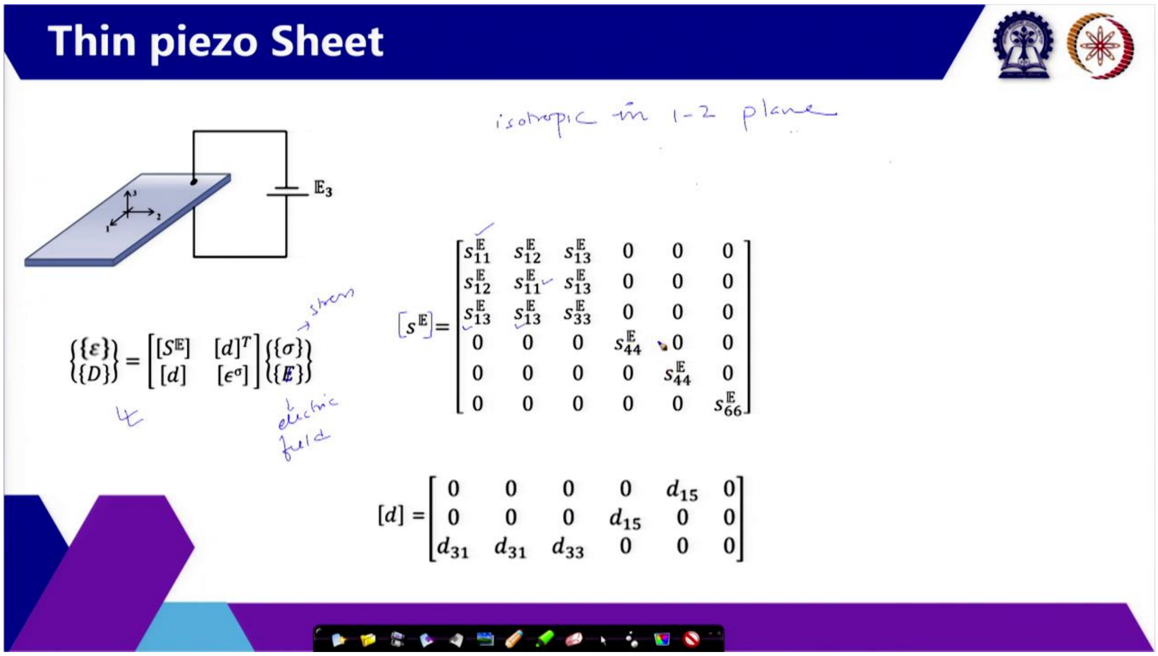

Now, we look into piezoelectric sensors and actuators. The piezoelectric sensors and actuators are generally available in the form of these thin sheets. So, here we can see there is axis 1, 2 and 3. Axis 3 denotes the thickness axis and they are generally polarized along the thickness axis. Polarization is shown by different marks for example, here it is shown by this dot and along the three axis at the top and bottom surface the electrodes are deposited. They can be polarized along other directions also 1 or 2, but for most of the applications we gave these sheets polarized along the direction 3.

Now in these thin sheets the properties are same in 1-2 plane. So, they are generally isotropic in 1-2 plane. So, the properties along direction 1 and direction 2 are same, but they are different in 1-3 and 2-3 planes. So, if we look into the constitutive relation where stress and electric field are the states and so, it is stress electric field. So, when we have these as the as the stress states we have this we have this constitutive relation.

Now, the individual components of the constitutive relation looks like this here. So, because of the isotropic in 1-2 plane we can see that S_{11} and S_{22} are same and similarly S_{13} and S_{23} are same. And also we see that there is no coupling between normal and shear components and there is no coupling within the shear components as well. And also we can see that these two components S_{44} and S_{55} are also same because the properties are same in 1-2 plane. Similarly, d_{24} and d_{15} are also same.

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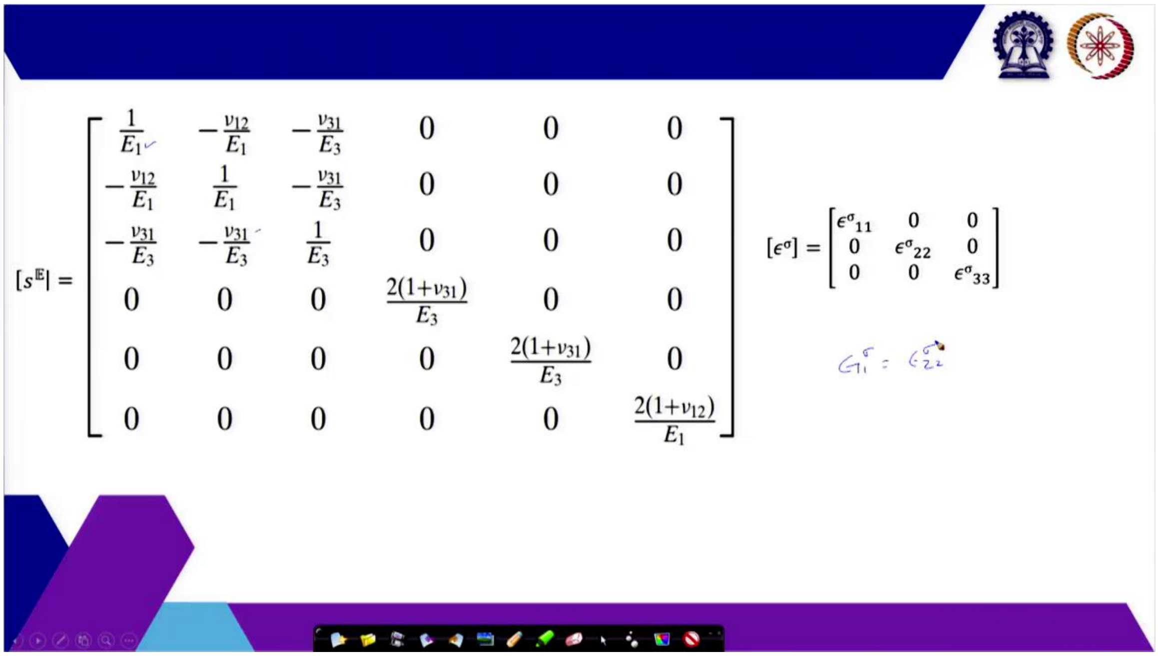

If we look at the compliance matrix the S matrix once again this is how it looks in terms of the elastic moduli and Poisson's ratios. So, again because of the isotropic in 1-2 plane we can see that E_1 and E_2 are same and similarly nu $_{31}$ and nu $_{32}$ are same. So, that is how it looks like the compliance matrix S^E in terms of the elastic constants and the epsilon vector is this, epsilon 11, epsilon 22 and epsilon 33. And, again because it is transversely isotropic epsilon 11 sigma and epsilon 22 sigma can also be same.

$$\varepsilon_{11}^{\sigma} = \varepsilon_{22}^{\sigma}$$