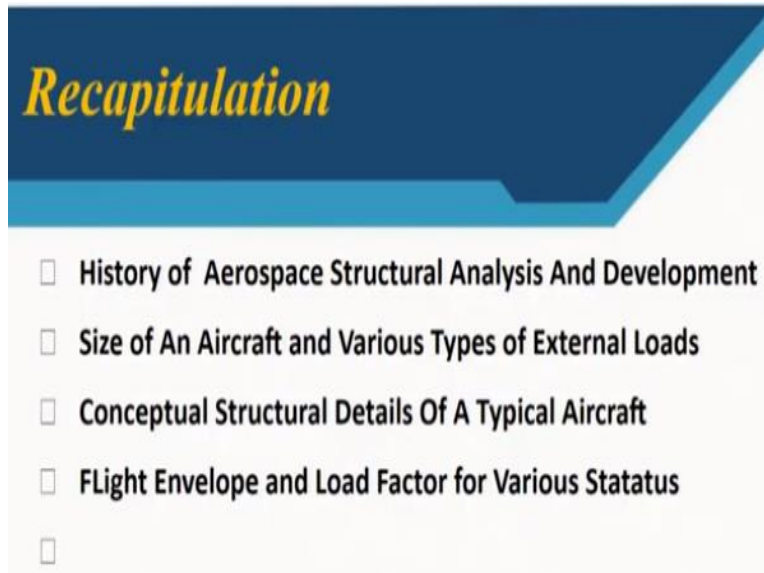


**Aircraft Structures - 1**  
**Prof. Anup Ghosh**  
**Department of Aerospace Engineering**  
**Indian Institute of Technology, Kharagpur**

**Module No - 02**  
**Lecture No - 08**  
**Loads from a symmetric Maneuver of an Aircraft**

Welcome back to aircraft structures one course this is Professor Anup Ghosh from aerospace engineering department IIT Kharagpur. Today is the eighth lecture we will be following that title of the lecture is said as loads from a symmetric maneuver of an aircraft.

**(Refer Slide Time: 00:50)**



What we will learn today is what is maneuver is but before we go into details better to have some recapitulation. Recapitulation includes as we have already discussed history of solid mechanics or the structural analysis applicable to aerospace engineering different sizes and ways the structures are in different configuration in case of aircraft structures then we have seen the development of aircraft structures since the beginning by Wright brothers then flight envelope and load factor for various status we have already learnt.

So today what we will learn is that we will learn what is a; flight maneuver and we will learn symmetric flight maneuver.

**(Refer Slide Time: 01:44)**

# CONCEPTS COVERED

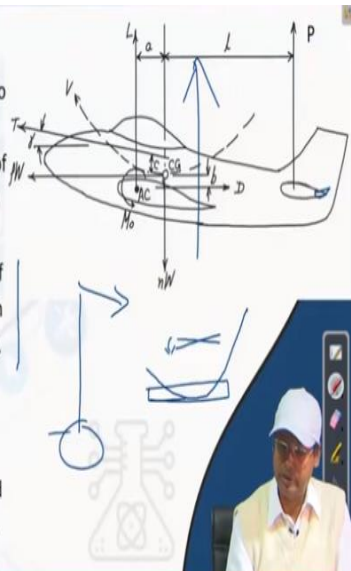
- 
- 
- A Symmetric Manoeuvre**
- Loads From a Symmetric Manoeuvre**
- 

Along with that we will solve one problem to find out loads in that case. So today's lecture we will cover symmetric maneuver loads from a symmetric maneuver equations we will form and then we will solve one example.

**(Refer Slide Time: 02:02)**

**General case of a symmetric manoeuvre**

- In a rapid pull-out from a dive a downward load is applied to the tailplane, causing the aircraft to pitch nose upwards.
- The downward load is achieved by a backward movement of the control column, thereby applying negative incidence to the elevators, or horizontal tail if the latter is all-moving.
- If the manoeuvre is carried out rapidly the forward speed of the aircraft remains practically constant so that increases in lift and drag result from the increase in wing incidence only.
- Since the lift is now greater than that required to balance the aircraft weight the aircraft experiences an upward acceleration normal to its flight path.
- This normal acceleration combined with the aircraft's speed in the dive results in the curved flight path shown in Figure.



Before we will solve one example before we go into detail it is time to have some understanding of symmetric maneuver. So, on the right hand side if you look at air this is an aircraft it is in pullout maneuver before we look into the diagram on the right hand side better we read these bullets and along with that we follow it. So in a rapid pullout from a dive a downward load is applied to the tail plane causing the aircraft to pitch nose upward.

So what happens if an aircraft if we draw these way I get better I remove things here what it says that causing effort for aircraft to pitch up to pitch nose upwards. So elevator is going upward if we look at the section these way this is the way this elevator comes in this is the tail plane and the rapid pull out from a dive a downward load is applied to the tail plane. So that applies the downward load in the total tail plane and causing the aircraft to pitch nose upward that means it is moving this way that is what the V diagram is shown this way that is what is explained in the first topic.

So next if we look at the downward load is achieved by backward movement of the control column thereby applying negative incidence to the elevator. So that is what just now we have drawn but control column moving backward this terminology is used with respect to the pilot and control what happens there is a control rod something like this and then that is moved this way so to give it a pitch up movement that is what is said here achieved by backward movement of the control column is moved back.

Thereby applying negative incidence to the elevator this way the elevator will move and negative incidence to the elevator or horizontal tail if the latter is all moving. So this is one case elevator is if it is not then the total tail plane moves like this in normal case it is like this but for pitching up movement it its rotates this way. So next if you talk about the maneuver is carried out rapidly. The forward speed of the aircraft remains practically constant so that increase in the lift and drag result from the increase in wing incidence only.


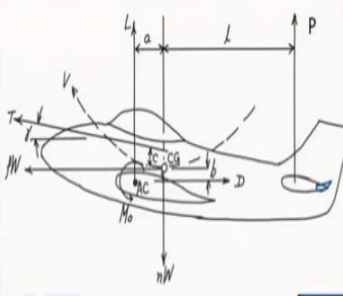
So it is a kind of assumption this is a kind of way we follow it that means we assume the forward speed practically constant and we assume also that the lift and drag result from the increase in the wing incidence only. Since the lift is now greater than the required to balance the aircraft weight. The aircraft experiences an upward acceleration normal to its flight path so the total aircraft moves upward that is what is saved since the lift is now greater than that required to balance the aircraft weight the craft experiences an upward acceleration normal to its flight path.

This normal acceleration combined with the aircraft speed in the dive results in the chord flight path shown in the figure. So what is happening while it is coming down an aircraft is coming down then the pitch up movement is given and then it is following this way. Here only this

portion is shown in this case how it is going on. So if we look at the other parameters let us go to the next slide.

**(Refer Slide Time: 07:59)**

- As the drag load builds up with an increase of incidence the forward speed of the aircraft falls since the thrust is assumed to remain constant during the manoeuvre.
- It is usual, as we observed in the discussion of the flight envelope, to describe the manoeuvres of an aircraft in terms of a manoeuvring load factor  $n$ .
- For steady level flight  $n = 1$ , giving 1 g flight although in fact the acceleration is zero.
- What is implied in this method of description is that the inertia force on the aircraft in the level flight condition is 1.0 times its weight.
- It follows that the vertical inertia force on an aircraft carrying out an  $ng$  manoeuvre is  $nW$ .



As the drag load builds up with an increase of incidence forward speed of the aircraft falls since the thrust is assumed to remain constant during the maneuver. So if the thrust is assumed constant the forward speed is definitely supposed to fall so that is what is stated it is usually it is usual as we are you observed in the discussion of the flight envelope to describe the maneuvers of an aircraft in terms of the maneuvering load factor  $n$ .

So we supposed to find out the maneuvering load factor in terms of ways it is generally talked about the maneuver of how much  $g$  are depending on that the design is carried out for steady level flight  $n = 1$ . This is simply to compare the  $n$  values giving 1 g flight although in fact the acceleration is 0. So this is a very important understanding you may do there is no acceleration that is the reason it is saying that  $n = 1$  and it giving 1 g.

What is implied in this method of discussion is that the inertia force on the aircraft in the level flight condition is 1 times its weight. It follows that the vertical inertia from force on an aircraft carrying out an  $ng$  maneuver is  $nW$ .

**(Refer Slide Time: 09:37)**

- We may therefore replace the dynamic conditions of the accelerated motion by an equivalent set of static conditions in which the applied loads are in equilibrium with the inertia forces.
- Thus, in Figure,  $n$  is the maneuver load factor, while  $f$  is a similar factor giving the horizontal inertia force. Note that the actual normal acceleration in this particular case is  $(n-1)g$ .
- For vertical equilibrium of the aircraft, we have, referring to Figure where the aircraft is shown at the lowest point of the pull-out
- $L + P + T \sin \lambda - nW = 0$  *Vertical  $\gamma = \lambda$*
- For horizontal equilibrium,
- $T \cos \lambda + fW - D = 0$  *Horizontal*
- and for pitching moment equilibrium about the aircraft's CG,
- $La - Db - Tc - M_0 - Pl = 0$  Last equation contains no terms representing the effect of pitching acceleration of the aircraft; this is assumed to be negligible at this stage

We may therefore replace the dynamic conditions of the accelerated motion by an equivalent set of static condition in which the applied loads are in equilibrium with the inertia forces. So we have expressed in terms of  $n$  how does it affect or affect the flight. So that is what we say that the dynamic condition is replaced equivalent set of static condition thus in figure  $n$  is the maneuver load factor while  $f$  is the similar factor giving the horizontal inertia force.

Here we see what the  $f$  comes this portion this is because of the maneuver in the vertical direction this is in the horizontal direction. Note that the actual normal acceleration in this particular case is  $n - 1$  please note that this is because while there is no state and level flight it is under  $1g$ . But while it is in  $ng$  maneuver actually the normal acceleration is  $n - 1g$  for vertical equilibrium of the aircraft we have referring to figures, where the aircraft is shown at the lowest point of the pullout, this is considered as the lowest point of the pull out if it is moving this way the figure is shown for this position.

So we are supposed to consider the static condition and we are supposed to find out the equation that is what is shown here. Before we go into the equation better we have some discussion on the drive as we have already mentioned. If we look at this is the aerodynamic center about which the moment is acting aerodynamics moment  $m_0$  is acting. Thrust is acting along this line which is a at a distance  $C$  perpendicular distance  $C$  from the  $cg$ .

Drag is acting along the AC in this for direction backward thrust is definitely acting in this line and the distance between the drag and the cg is  $D$  and  $fW$  is as I said but what we have mentioned about the  $fW$  the horizontal inertia force that is acting in this direction. So  $P$  is the lift acting on that the tail plane so with respect to that if we consider the vertical equilibrium equation is  $L + P + T \sin \gamma = W$  I think there is a mismatch with the symbol here it is  $\gamma$  here it is  $\lambda$ .

Please consider that  $\gamma$  is equals to  $\lambda$  otherwise again there will be some problem, please consider this. So let  $L + P + T \sin \lambda = W$  this the  $\sin$  thrust  $\sin$  component acting vertical and then  $nW$  is acting downward. Now for the horizontal equilibrium as we have seen the thrust component will come the thrust component has come this is the horizontal inertia force  $fW - d$  drag is acting on the others equation.

So this is for vertical this is for horizontal equilibrium now about taking the moment and the pitching moment equilibrium about the aircraft cg is  $L \times A$  this is the distance this I missed from figure it is quite clear lift multiplied by  $A$  drag multiplied by  $b$  it is acting this way it is acting the other way  $tC$  is also acting in this direction rotation direction. So this and this both are acting in this direction and  $M_0$  is definitely in the same direction  $P \times L$  lift is acting and we get the moment equation.

Here one thing point we should note that effect of pitching acceleration is assumed to be negligible. So please note that we are not considering the pitching acceleration these pitching accelerations are generally considered for other aspects other study for control consideration but here our consideration is not critical. So we have neglected that point so with this understanding with a symmetric maneuver now we understand what is a general symmetric maneuver is and in that generic symmetric maneuver.

What are the force components act this is the flight path followed it is at the bottom position of the flight and what are the forces acting on it lift acting on the wing  $P$  force acting on the tail plane forward inertia force  $fW$  particle inertia force  $nW$  is acting on it thrust is acting with an angle  $\lambda$  or  $\gamma$ . So with that consideration we have got 3 equations this is the moment

equation vertical and horizontal equation. Let us try to solve a problem and try to see how we can find out the force components lift track okay.

**(Refer Slide Time: 15:35)**

**How to Solve**

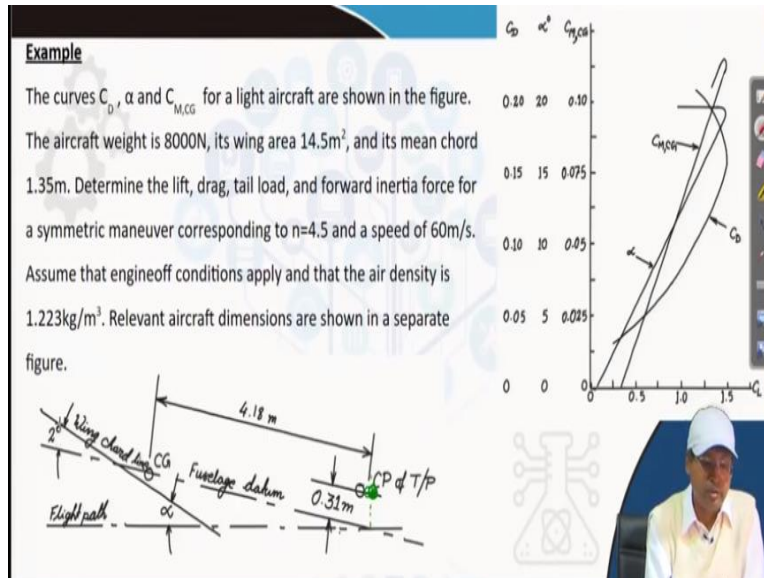
- Method of successive approximation is most convenient for the solution of these equations.
- It may be noted that there are a few differences with respect to the steady level flight case.
- The engine thrust  $T$  is no longer directly related to the drag  $D$ , as the latter changes during the maneuver.
- Generally, the thrust is regarded as remaining constant and equal to the value appropriate to conditions before the maneuver began.

The diagram shows an aircraft in a maneuver. The center of gravity is labeled  $CG$ . The lift force  $L$  acts perpendicular to the flight path. The drag force  $D$  acts opposite to the flight path. The engine thrust  $T$  acts along the fuselage. The weight  $W$  acts vertically downwards. The pitch angle is  $\theta$ . The distance from the CG to the lift application point is  $a$ . The distance from the CG to the drag application point is  $b$ . The distance from the CG to the thrust application point is  $c$ . The flight path angle is  $\alpha$ . The angle between the fuselage and the flight path is  $\beta$ . The angle between the fuselage and the vertical is  $\theta$ . The angle between the fuselage and the horizontal is  $\theta + \beta$ . The angle between the lift force and the vertical is  $\alpha + \theta$ . The angle between the drag force and the horizontal is  $\alpha + \theta$ . The angle between the thrust force and the horizontal is  $\theta + \beta$ . The angle between the weight force and the vertical is  $\theta$ .

How to solve method of successive approximate approximation is most convenient for solution of these equations. Successive approximation we will follow we will see how do we get one after another the values and we will solve those it may be noted that there are a few difference with respect to the steady level flight cases. So that successive approximation method has some difference it depends on what type of case? What how many data are available depending on that we need to change the scheme of approximation the engine thrust is no longer directly related to the drag as the letter changes during the maneuver.

So drag changes during the maneuver because of it is orientation at different position the drag changes and it is not directly related to the  $T$  the steady level flight it is related to  $T$  but whereas in this particular case it is not related it changes. Generally the thrust is regarded as remaining constant and equal to the value appropriate to condition before the maneuver began. So with this understanding we consider the thrust and in some specific case we also assume some other way let us say for a particular problem how do you solve do we solve problem.

**(Refer Slide Time: 17:09)**



This is one example in this example this is CL let us see what the example says the curves CD alpha and CMCG CD alpha CMCG for a light aircraft are shown in the figure these 3 values are given with respect to CL. The aircraft weight is 8000 Newton its wing area is 14.5 meter square and its mean chord is 1.35 meter. Determine the lift drag tail load and forward inertia force for a symmetric maneuver corresponding to  $n = 4.5$  is a maneuver required for that this type of general symmetric maneuver.

And a speed of 60 meter per second, assuming that engine of condition apply and that the air density is 1.223 kg per meter cube. So for this particular problem it is also specified that the engine is off density is given. So we will use that relevant aircraft dimensions are shown in the separate figure. So this is the figure for the relevant aircraft dimensions. In this figure there is a small correction please note that this is the point sorry on which it acts not this point so keeping a note with that this in the diagram let us try to follow or solve the problem.

The cg distance from the center of pressure of tail plane is 4.18 meter wing chord line is this one the angle between the fuselage datum is 2 degree this is the flight path, this is the angle alpha between chord and flight path. And we need to find the alpha from the diagram depending on some iterative process this angle is given and let us see how can we find out the solution?

**(Refer Slide Time: 19:30)**



there are three equations

$$\text{vertical : } L + P + T \sin \lambda - nW = 0$$

$$\text{Horizontal : } T \cos \lambda + fW - D = 0$$

$$\text{Moment: } La - Db - Tc - M_0 - Pl = 0$$

- We need to follow an iterative method
- First we will assume  $P=0$  and find out  $L=nW$  from first equn. and  $C_L$
- This gives us  $\alpha$ ,  $C_{M,CG}$  from chart or plot.
- The moment equation may be approximated to find out  $P$ .
- Now from the first equn. using new value of  $P$  we get modified  $C_L$

So before we go into the detailed solution let us try to note the procedure followed there are 3 equations vertical equilibrium, horizontal equilibrium and moment. We need to follow an iterative method because as we seen in the last slide that the alpha CMCG CD alpha and CMCG these are dependent on CL coefficient of lift. So with confirming value with respect to the CL will have to consider these values and will have to solve the problem.

So let us see first we will assume that in this particular case that  $P = 0$  and if you consider the first equation if you use  $P = 0$   $T$  is stated to be 0. So the equation boils down to  $L = nW$  and then from the  $L=nW$  from the detailed expression of  $L$  that is  $Q \text{ half } \rho V \text{ square } CL \text{ into } S$  we will find out the value of  $CL$  and that will be our first value of  $CL$ . Now using this first value of  $CL$  from the plot described in the previous page we can find out the value of  $\alpha$  as well as  $CMCG$  and if we have the  $\alpha$  and  $CMCG$  these  $\alpha$  will result in some  $L$  parameter the length calculation and as well as the  $CMCG$  will be used to find out  $CL$  again.

But before that we will need the value of  $P$  so let us try to see what we can do the moment equation may be approximated to find out  $P$ . Now from the first equation using new value of  $P$  we get the modified  $CL$ .

**(Refer Slide Time: 21:44)**

As a first approximation, we neglect the tail load P. Therefore, from vertical equilibrium

$$L + P + T \sin \lambda - nW = 0$$

since  $T=0$ , we have  $L \approx nW$  ----- (a)

Hence,  $\Rightarrow L = \frac{1}{2} \rho V^2 C_L S = nW$

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 S} \approx \frac{4.5 \times 8000}{\frac{1}{2} \times 1.223 \times 60^2 \times 14.5} = 1.113$$

From figure for a value of  $C_L = 1.113$ ,  $\alpha = 13.75$  deg and  $C_{M,CG} = 0.075$ .

The tail arm, from first figure, is

$$l = 4.18 \cos(\alpha - 2) + 0.31 \sin(\alpha - 2) \text{ --- (b)}$$

So let us see for this particular case how do you solve it fast approximation as it is said we neglect the tail load P this is the first approximation as it is mentioned, we use the equation we get a  $L = nW$  as I mentioned it that L with its detail formula that is half rho V square CL S is the wing area, CL is the lift coefficient n omega equals to n omega. So following this equation we get the value of CL n omega W value n is 4.5 and W is already given.

And putting the other value as given 1.223 density of air the speed and 14.5 is the wing area here was not there. So please note S is there that value of s is equals to 14.5 yes it is 14.5 meter square. So with that value of S we calculate the CL the first as a calculated value of CL and now using this CL value from this plot say for 1.113 somewhere here we drop and we find out the value of alpha corresponding to that alpha value we get around 13.75 we get the CMCg 0.75 CMCg is this is alpha this CMCg.

So these values we get and the tail arm L from first figure we need to find out is that  $L = 4.18$  this alpha is the new alpha we are supposed to put here again this is the point and this component is and this small component whatever is this component. So this is this component and this is this component so Cos and Sin part alpha / 2 all these 2.31 that is the reason we have the length corresponding to the tail arm and using this equation let us see how do we.

**(Refer Slide Time: 24:29)**

Substituting the above value of  $\alpha$  gives  $l=4.123\text{m}$ . ✓

In the moment equation

$$L a - D b - T c - M_0 - P l = 0$$

the terms  $(L a - D b - M_0)$  are equivalent to the aircraft pitching moment,  $M_{CG}$ , about its CG.

The moment equation may therefore be written  $M_{CG} - P l = 0$

$$P l = \frac{1}{2} \rho V^2 S c C_{M,CG} \text{ --- (c)}$$

where  $c$  = wing mean chord. Substituting  $P$  from eqn. (c) into eqn of vertical equilibrium

$$L + P + T \sin \lambda - nW = 0$$

we have  $\Rightarrow L + \frac{\frac{1}{2} \rho V^2 S c C_{M,CG}}{l} = nW$

or dividing through by  $\frac{1}{2} \rho V^2 S$

$$C_L + \frac{c}{l} C_{M,CG} = \frac{nW}{\frac{1}{2} \rho V^2 S} \text{ --- (d)}$$

(Note that R.H.S. is the old value of  $C_L$ )

We now obtain a **more accurate value for  $C_L$**  from eqn (d)

$$C_L = 1.113 - (1.35/4.123) \times 0.075 = \mathbf{1.088}$$

$\alpha, C_{M,CG}$

So we have the value of  $L$  calculated that is  $L = 4.123$  meter and from the moment equation if we look at we can consider that this is total  $L a - D b - M_0$  is equivalent to the aircraft pitching moment that is  $M_{CG}$  and if you rewrite the equation in that form that is  $M_{CG} - P l = 0$  and the detail expression or expression of  $M_{CG}$  which consists of half  $\rho V^2 S c C_{M,CG}$  where  $c$  is the wing mean chord.

Now substituting so from here we will get the value of  $P$  and that  $P$  value can be substituted from to the vertical equilibrium equation where initially we assume  $P = 0$  and that we have put here  $P$  is put divided by  $L$  this expression divided by  $L$  is put which is equals to  $nW$  is 0 in our problem.

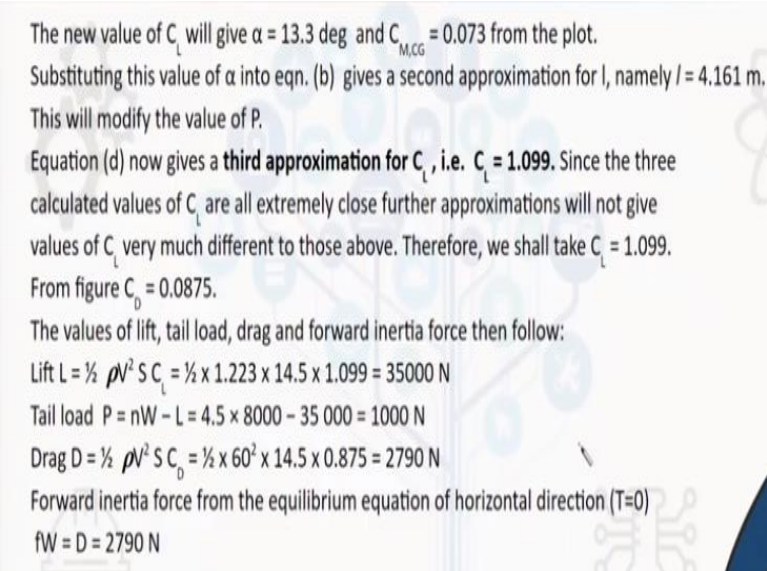
Now again to make it in the coefficient form we divide it by half  $\rho V^2 S$  and we get the expression for  $C_L$  which is equal to  $C_L + \frac{c}{l} C_{M,CG}$  this  $C_{M,CG}$  with respect to the  $C_L$  the previous  $C_L$  value that is previous  $C_L$  value was 1.113 with that value this value is the previous  $C_L$  value we can we have got the  $C_{M,CG}$  value and substituting those value we can have the new  $C_L$  value.

Note that the right hand side is nothing but the old  $C_L$  value this is what is written here the old  $C_L$  value 1.113 and this portion is nothing but this one. So we get a new value of  $C_L$  so one iteration is done so this iteration we can repeat to get a refined value of  $C_L$  so with this  $C_L$  value

with this new CL value from the plot, we can find out again alpha we can again find the value of CMCG.

And with help of alpha and CMCG, we can find out the new modified L we can find out the new value this D we can use this new CMCG value and we can have the new value of CL. So that is what the iteration goes on so let us see in our next slide how the iteration takes place.

**(Refer Slide Time: 27:15)**



The new value of  $C_L$  will give  $\alpha = 13.3$  deg and  $C_{M,CG} = 0.073$  from the plot. Substituting this value of  $\alpha$  into eqn. (b) gives a second approximation for  $l$ , namely  $l = 4.161$  m. This will modify the value of  $P$ . Equation (d) now gives a **third approximation for  $C_L$** , i.e.  $C_L = 1.099$ . Since the three calculated values of  $C_L$  are all extremely close further approximations will not give values of  $C_L$  very much different to those above. Therefore, we shall take  $C_L = 1.099$ . From figure  $C_D = 0.0875$ . The values of lift, tail load, drag and forward inertia force then follow:  
Lift  $L = \frac{1}{2} \rho V^2 S C_L = \frac{1}{2} \times 1.223 \times 14.5 \times 1.099 = 35000$  N  
Tail load  $P = nW - L = 4.5 \times 8000 - 35\,000 = 1000$  N  
Drag  $D = \frac{1}{2} \rho V^2 S C_D = \frac{1}{2} \times 60^2 \times 14.5 \times 0.875 = 2790$  N  
Forward inertia force from the equilibrium equation of horizontal direction ( $T=0$ )  
 $fW = D = 2790$  N

So in this we see the new value of CL will give alpha = 13.3 degree CMCG = 0.073 from the plot. Substituting these values of alpha into equation b gives a second approximation of l namely  $l = 4.161$ . This will modify the value of P and as we have described the new CL value we have equals to 1.099. Since the 3 calculated value of CL are all extremely close further approximation will note give values of CL very much different to those above.

Therefore we shall take CL equals to 1.099 and from the figure we calculate the value of CD that will give us the drag here it gives us the drag the value of lift tail load drag and forward inertia force then as follows. L is calculated from the standard formula all these values are put tail load  $P = nW - L$  those values are put and we have found that it is equals to 1000 newton drag is also put from the value half Rho V square a CD and 2790.

And the forward inertia force which is equals to the drag is 2790 so considering  $T = 0$  so with this what we conclude that for a symmetric general maneuver how the maneuver is what are the

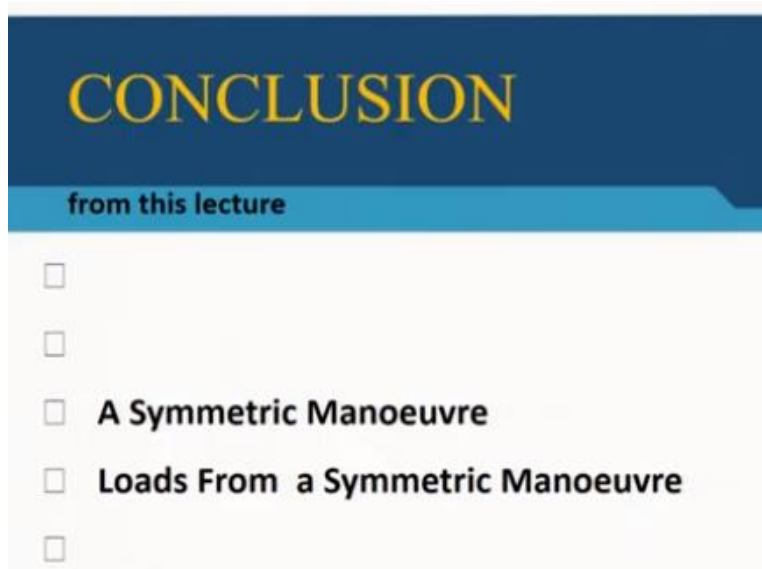
forces act on that aircraft we have learnt and we have considering a standard plots how to solve a problem how to find out lift drag tail plane load forward inertia force all those things we have talked out.

(Refer Slide Time: 29:19)



So references are standard reference what we had followed in the previous slides.

(Refer Slide Time: 29:24)



And at the conclusion slide if we see a symmetric maneuver is we have learnt how a symmetric maneuver takes place what are the condition loads from symmetric maneuver we have to solve and with that we have got a fair understanding at different flight scenario flight maneuver how

do we find out loads experienced by an aircraft? So with this slide we move to our end of today's lecture thank you for attending this lecture.