Aircraft Structures - 1

Prof. Anup Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No -42
Membrane Analogy for Torsion Problem (Contd.,)

Welcome back to aircraft structures one course this is Professor Aanp Ghosh from Aerospace Engineering Department IIT Kharagpur. In due process we have come to the last lecture of the total course the eighth week is going to end with this lecture the 42nd lecture in sequence and here we will have some understanding will follow or put some insight into the membrane analogy for the torsion problem.

And we will end the course but before we end the course it is we go into the lectures it is my duty to thank many people who has helped me from the audio visual section of IIT Kharagpur who has helped me to record this lecture in a very, very well beautifully record this lecture beautifully and presentable to everyone. My sincere thank to our departmental retired professor P K Dhakta without his help probably it was a very difficult task for me to do it.

I also would like to thank my two students who will be teaching assistant in this course Mr Vikash Kaushik as well as Mr Supen Sah without their help also it is not a possible task. Not only that there are many other people who are involved from CET section of our institute I would like to thank each and everybody and with that note I would like to start today's lecture which is the final lecture.

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So let us proceed so in this recapitulation slide what we see is that history of aircraft and aerospace structural analysis we have discussed. We have also discussed various types of external loads, conceptual structural details how a wing is fabricated what are the members structural members, how does a rib looks like? How does a spar looks like? How thin wall sections are used to fabricate those things.

And we have seen where from loads come to aircraft and it experiences different critical conditions we have come across to the flight envelope. We have also seen that if we consider the fuselage separately or the wing separately how does it bends? How does it experiences shear force its changes. We have done unit load analysis method so that for any practical case it can be achieved that particular value.

Then three dimensional structures are given stress with as example which is very common in aircraft without which an aircraft cannot land. Landing is very, very important so landing gear analysis with help of three dimensional structures. Flying is a tough job fine but unless we are able to land it is of no use. So landing gear analysis we have done and then we have done various methods to find out deflection you are given the concept to find out deflection.

We have covered methods like complementary energy method total potential energy method. We have learned Castiglione’s theorem we have learned unit load method, dummy load method. We have also learned how to solve indeterminate structures statically indeterminate structures if there are more like the cropped cantilever that is a very common example. Not only that even in truss with more members are also solved.

Then we got introduced to a very, very important method known as the Rayleigh ridge method of analysis which lays the foundation stone probably for the numerical analysis world from where probably it starts. And it goes further and the world is now running in computer in digital world unless we have a computer unless we are able to solve we cannot design anything we need to find solution.

But see even the there is computer there are advanced methods of analysis like finite element analysis method or many other different methods to analyze structures. Unless we have the insight into the development of stresses and deflections it is difficult to design So to have that insight to have the feel of development of stresses and strengths and displacements we need to learn the theory of elasticity.

And in that connection we have learnt theory of elasticity. We have learnt that there are compatibility conditions compatibility or continuity of stresses and strains while we are formulating a problem we need to maintain mathematically otherwise it will not going to represent the stress strain with the physical matching with the physical world or the practical world. So those things we have learnt we have solved very, very typical problem of circular hole in a plate.

Then we have solved torsion problem got introduced with the phenomena known as warping it is a very beautiful phenomena to observe you may think of experiments wherever you are in your college you may ask for a torsion experiment it is not very difficult one. But before you go for the torsion element a torsion experiment you should mark parallel lines so that you can observe the warping after the break after by torsion.

And then in the last lecture we got introduced with the membrane analogy and we will continue with the membrane analogy to find out torsional stresses and deflections.

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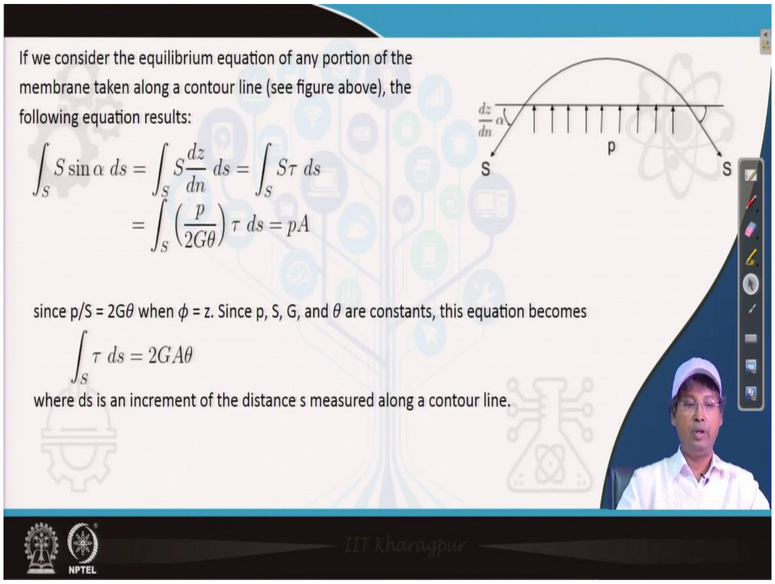

So in that connection we come to the relation between the stress and theta with respect to membrane. So in this we are considering again one stretched membrane which is under pressure P. The stress member is stressed on the surrounding or the edges of the structure which is under torsion. If we consider the equilibrium equation of any portion of the membrane taken along a contour line. The following equation results so what we can get from this equations along a contour line is that S sine alpha sine alpha is equals to dz dn.

Integrate it along a contour line which is the I have shown you in the last lecture we can easily write that this is equals to integration over that contour line for ds length considering a small length ds is this one and that dz dn is equals to tau the slope already we have put and since we have found out a relation between this p by S equals to 2G J sorry 2G theta that if we substitute here we finally see that that whatever the pressure that total pressure is equals to p into A.

The total force acting in this direction and once we complete the integration we get the relation something like tau ds integration over the S tau ds is equals to 2G A theta and where ds is an increment of the distance S measured along the contour line. So with that concept we move further for practical cases.

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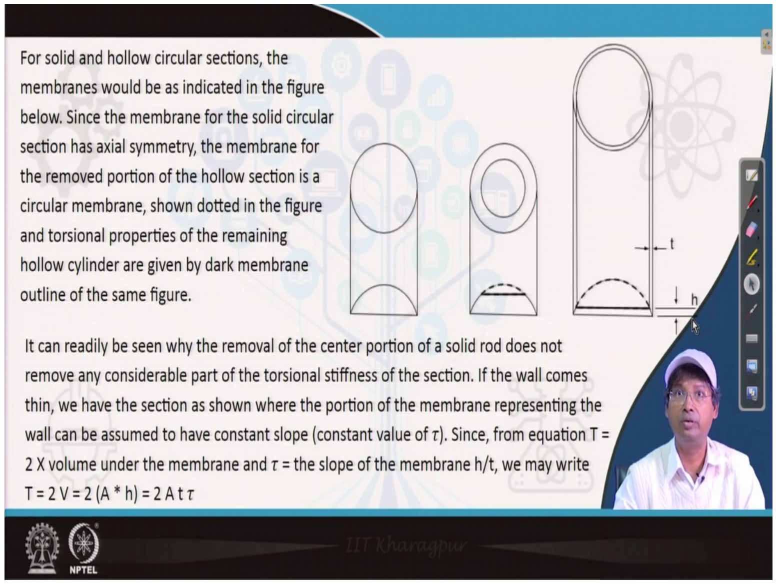

How do we implement this membrane analogy. For solid and hollow circular sections the membranes would be as indicated in the figure below since the membrane for the solid circular section has axial symmetry the membrane for the removed portion of the hollow section is a circular membrane shown dotted in the figure this one. And torsional properties of the remaining hollow cylinder are given by the dark membrane outline of the same figure.

So this will govern the torsional property this membrane so with that note we move further for a solid section this is the way this total membrane for a annular section where there is a hole axial hole cut in the shaft. This portion of the section we are supposed to consider for thin wall let us see what is the relation we get with respect to theta. It can be readily be seen from seen why the removal of the center portion of solid rod does not remove any considerable part of the torsional stiffness of the section.

So if the wall comes thin we have the section as shown where the portion of the membrane presenting the wall can be assumed to have a constant slope this particular portion. So that tau may be equated with that slope value that tau is a so since from the equation T equals to twice the volume under the membrane what we can see that the volume under the membrane is this height and h A multiplied by h t twice A multiplied by h and T is equals to h by T as we can see this is the slope. And then if we substitute the value of h here we get that T is equals to twice At tau.

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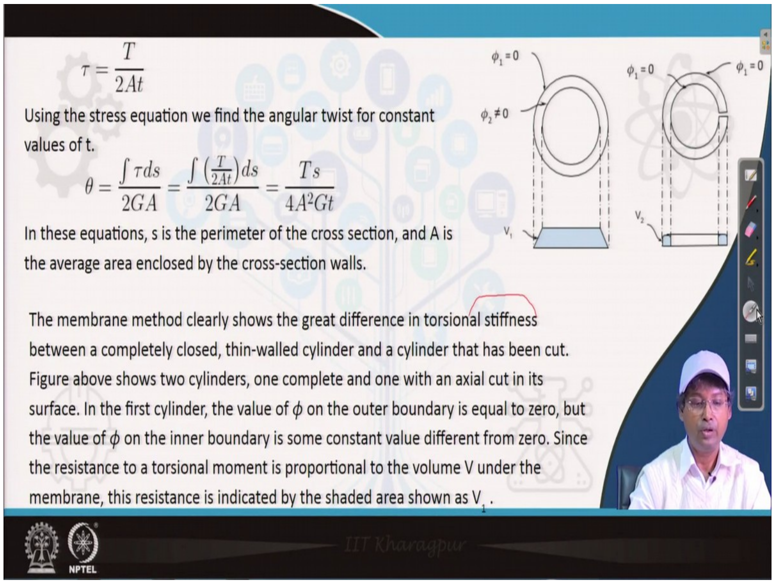

So with that note we come to the relation as tau is equals to T by twice A t using the stress equations we find the angular twist for constant value of t and theta may be written as integration tau ds 2GA or this may be again from the relations whatever we have seen may be written as Ts by A4A square G t. So in this equation S is the perimeter of the cross section and A is the average area enclosed by the cross section wall.

The membrane method so membrane method clearly shows the great difference in torsional stiffness between a completely closed thin wall cylinder completely closed and thin wall cylinder and a cylinder that has been cut. Figure above these are the two examples we are considering now to discuss with membrane analogy and how the torsion torsional resistance varies that will try to discuss.

In the first cylinder the value of phi on the outer boundary is equal to 0 but the value of phi on the inner boundary is some constant value different from zero. This boundary so this is not equals to 0 if we put some internal pressure. So the membrane as we have discussed in the last slide membrane is something like this. So, that is the reason we have some definite value here. Since the resistance to a torsional moment is proportional to the volume V under the membrane here V 1 this resistance is indicated by the shaded area shown as V 1.

So, we are supposed to find out this volume for this closed ring so what happens if it is not a closed ring.

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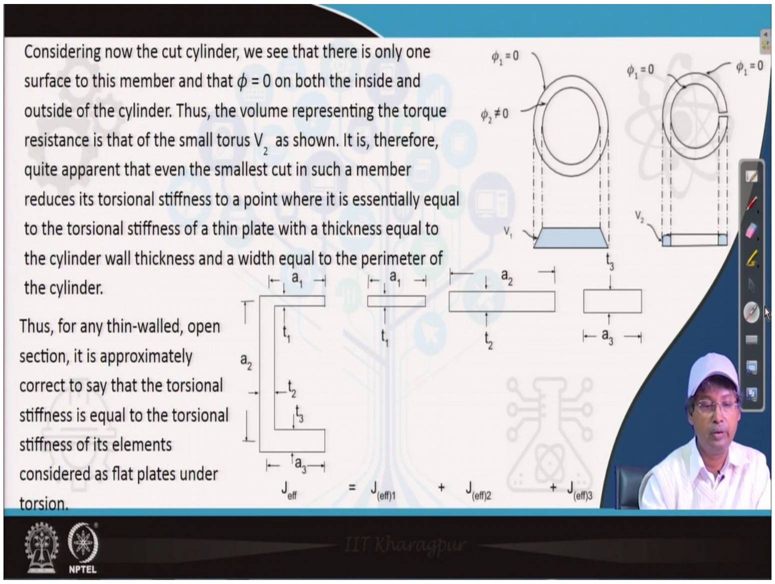

So in case of cuttering considering now the cut cylinder we see that there is only one surface to this member there is one surface to this member and that phi equals to 0. So this total edge if we look at starting from here to here it is a complete single edge and there phi is equals to 0 on both the sides of the and outside of the board both inside and outside of the cylinder. Thus the volume representing the torque resistance is that of the small volume V 2 this volume.

This volume covering this area it is therefore quite apparent that even the smallest cut in such a member reduce its torsional stiffness to a point where it is essentially equal to the torsional stiffness of the thin plate with a thickness equal to the cylinder wall thickness and width equal to the perimeter of the cylinder. So it is saying that if we stretch it, it is becoming almost similar case a thin wall section and with that scenario the capability to resist torsion reduces in great way.

So that is what from the membrane analogy we can easily conclude and keeping in mind that analogy we also conclude that now if we are able to find out J effective separately for this sections which makes the C section a 1 is this one a 2 is this one and a 3 is this one then we can easily have summation of those J effectives to find out the J effective for the total C section thus for any thin walled section open section it is approximately correct to say that the torsional stiffness is equal to the torsional stiffness of its elements considered as flat plate under torsion.

So with that concept it gives us some approximate method of analysis that helps a lot for the initial iteration and with that note we move to the next slide.

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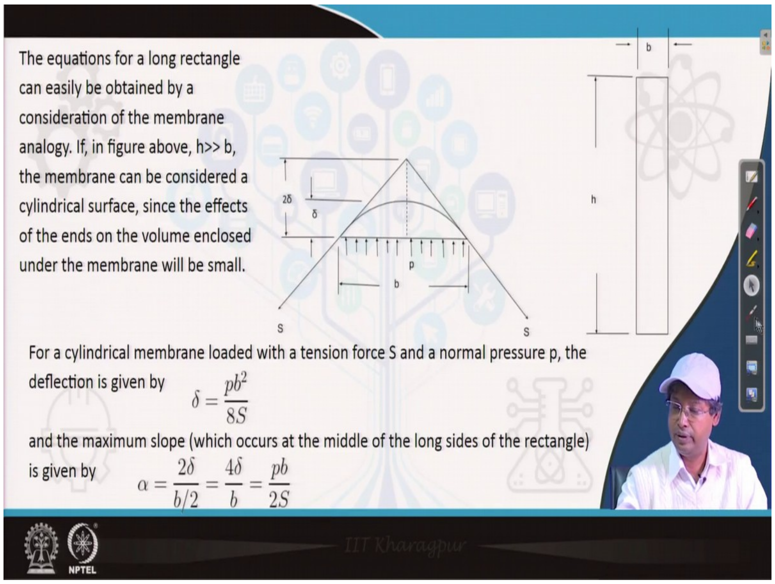

We will try to find out solution for a very easy problem. The equations for a long rectangular rectangle can easily be obtained by a consideration of the membrane analogy if in figure above h is very, very greater than b in this figure is talked about the membrane can be considered a cylindrical surface one to mean if we look from this side this becomes something like this there will be changes definitely at this corner this corner this corner and this corner but we can easily neglect that part since we are going for the approximate analysis.

So that cylindrical surface we will get since the effects of the ends on the volume enclosed under the membrane will be very small as these corners as we said we can easily consider that that as a cylinder. And if we do that for a cylindrical surface what is mentioned here this is the cylinder this cylinder is talked about. For a cylindrical membrane loaded with a tension force S and a normal pressure p the deflection is given by delta equals to pb square by 8S this is a standard value what we are using we are not going to derive this. nd the maximum slope is alpha equals to twice delta by b by 2 and which leads to pb by twice s.

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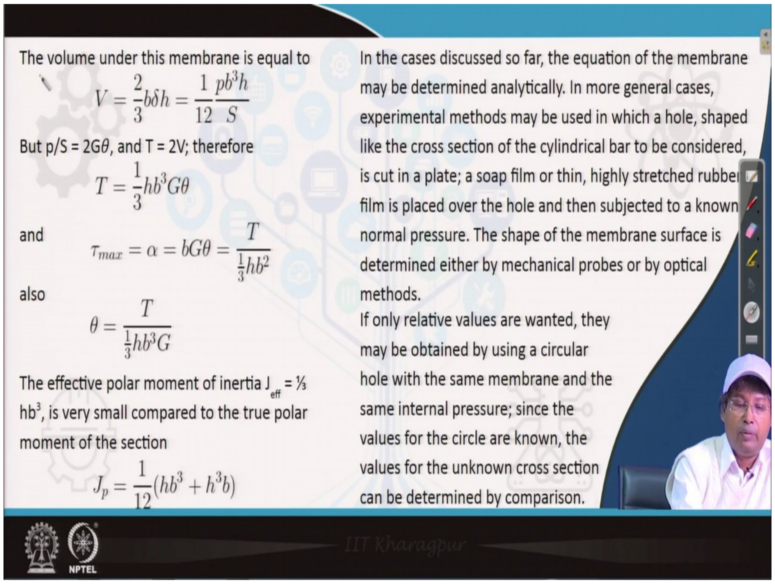

So we the volume under the membrane is equal to V two third of b delta h and we have this value 1 by 12 pb k by h by S but since we have the relation p by S equals to 2G theta and t equals to twice V if we combine those two we get the equation t is equals to one third h b cube G theta and the tau max which is equals to alpha and if we substitute the values whatever we have observe here is equals to T by one third h b square and theta from the membranology theta or the rotation for unit length.

Per unit length is given by T by one third h b cube G the effective polar moment of inertia J effective is equals to one third h b cube as it is obtained in this relation in is very small compared to the true polar moment of inertia of the section which is h b cube +s h cube b divided by 12. So there is a big difference as the section becomes more thin it becomes less in value and as the it is less in value it shows more theta.

So in this connection we go further in the case discussed so far the equation of the membrane may be determined analytically. In more general cases experimental methods may be used in which a whole shepherd like the cross section of the cylindrical bar to be considered is cut in the plate is so film or thin highly stretched rubber film is placed over the hole and then subjected to a known normal pressure the shape of the membrane surface is determined either by mechanical probes or by optical methods.

If only relative values are wanted they may be obtained by using a circular hole with the same membrane and the same internal pressure since the value of the circle are known. The value of the unknown cross section can be determined by comparison. So with this note we come to the membrane analogy method and we come to the end of our course aircraft structures one.

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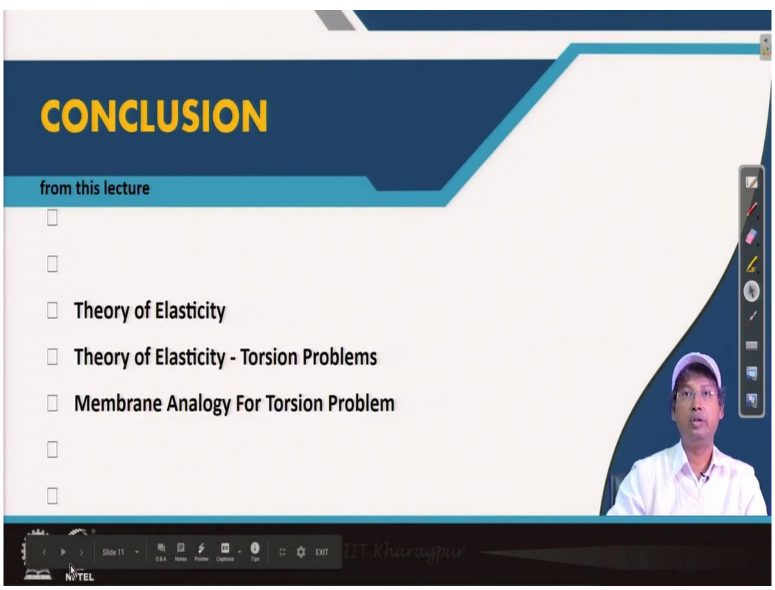

And moving further we see the usual slide what consisting of the reference books and with this note we would like to say I would like to thank you for attending the course and hope it will help you a lot to understand the subject well. And to move further in academic world as well as in the physical design or engineering world whatever we call it. And it will be helpful for your career, thank you for attending the course, thank you.