Aircraft Structures - 1
Prof. Anup Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No -41
Membrane Analogy for Torsion Problem

Welcome back to aircraft structures one course this is Professor Aanu Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are in the last week lecture the eighth week lecture module 8 it is named and the topic we will cover in these two lectures last two lectures is the membrane analogy for torsion problems. So keeping in mind that let us proceed.

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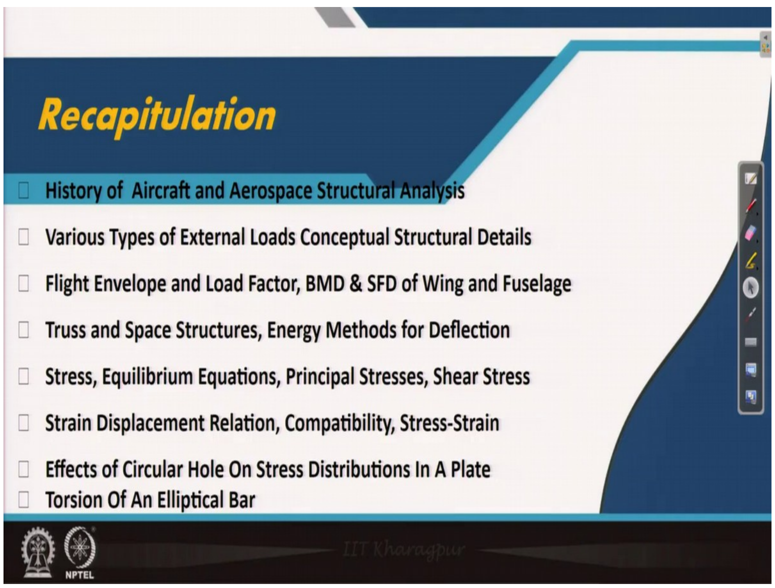

Since we are we have come almost at the end of the course only one topic is remaining that is the membrane analogy with respect to torsion problem to find out torsionals rotation as well as a torsional shear stress developed in a section irregular section. Those things will come later but before that let us have a recapitulation of what we have done. We have done history of aircraft and aerospace structural analysis or we can say that it is related to the development of aircraft from the Kitty Hawk site the 12 second flight.

From there in 1903 from there we have come to the huge aircrafts we have come to the rotary aircrafts, we have come there are developments in rockets also but those things we are not going to discuss our predominant area of discussion was related to fixed wing aircraft. And we have seen how the configuration have configurations have changed from time to time different types of wing configuration tail configurations different types of purpose it serves with those configurations.

And we have seen that from a very small aircraft may be fitting in a classroom to a an aircraft fitting comfortably fitting in a football field. People have developed human men have developed society engineers have developed that and those are in service we are being used. Many aircrafts are there some variation of 737ah and 747 are there which are used for fire fighting also. Huge quantity of water they carry and fire retardant chemicals they carry and spray it on jungle.

So we have seen various developments of aircraft we have also have gone from to through the we have gone through the development of solid mechanics structural analysis from the static analysis how people have come to the concept of modulus of elasticity it is constant. How people have come to the concept of Poissons ratio it was difficult for initially to imagine that for any isotropic material these are the two constants which defines the total material property when we are considering linear elastic region.

This again this linear elasticity also they defined and slowly from different experiments related to the beam it has been done with and then it has gone further in with to the plate and to the cell we have also seen developments in the direction of vibration but those are not of important in this particular course but people have developed side by side physicist scientists have developed side by side. Different equations they have proposed they have proposed relations for strain and displacement the famous relations.

Proposed compatibility equations and from with all those things we are able to design huge aircrafts nowadays. Even huge other structures that knowledge is very, very important unless we develop that insight to a to a analysis it is difficult to have a insight in that problem and to solve. Then various types of external loads and conceptual loads coming to the aircraft structure we have covered a different flight condition which portion of the structure is more prone to experience certain type of loads some part of fuselage.

Some part of wing some part of tail plane at which condition they experience what type of load and what should be the critical design condition we have also come across to the agency known as the air worthiness agency. And they define and help engineers to design the aircraft according to safety norms and then what we have seen we have seen how the bending moment and shear force are experienced by wing and fuselage we have not come done any combined three dimensional way the shear force bending moment.

But we have considered in two dimensional manner; separately as being as fuselage we have drawn shear force bending moment diagram with unit load concept. So that for any kind of unit load we can solve it before that we have come across to the concept of inertia load and how inertia load is very, very important in case of aircraft structures. So with those notes we have come across to the flight envelope for different types of aircrafts.

How flight envelopes are different and we need to prepare it. Then three dimensional truss structures we have seen we have solved a few problems related to related to trusses and then we have seen displacement to find out deflection of different structures we started with determinate structure with energy different energy methods related to complementary energy method Castiglione's theorem unit load method dummy load method.

And also we got introduced to a very, very important energy based solution known as Rayleigh method so after finding out displacement we have started learning theory of elasticity approach theory of elasticity approach is the mathematical way of formulating problems and to find out solution for that different stress conditions strain condition and displacements. So and then that connection we got that for isotropic material there are 15 unknowns to be found out to solve a problem amongst that 6 are stresses 6 are strains.

And three displacement components accordingly 15 equations we have found out in that consequence we have also got introduced to the compatibility condition compatibility condition is really very important condition we need to maintain while we are formulating a problem we are trying to define mathematically a problem unless the compatibility is maintained compiling compatibility means the continuity from point to point one point to the other in terms of strain or displacement and as well as in terms of stress.

So unless you maintain that in mathematically it will be a mistake because in all physical structure physical world it is always continuous there is no discontinuity like that. So that concept is introduced. Then we have solved a few problems with inverse semi inverse method we have come got in we got introduced to the concept of stress function Aris stress function was introduced and then we solved a very typical problem known as the stress concentration on the circumference of a hole if irrespective of the diameter of the hole while the plate is uniaxially loaded with uniform stress.

Then from there we have seen that if even if it is other conditions are there what is the maximum stress may be found out on the circumference of a hole and then we have learned how a crack propagation may be correlated with elliptical hole and how it can be arrested by drilling a hole. After that we have started in last three lectures we have covered the concept of torsion in theory of elasticity approach irregular prismatic sections we have considered not circular sections.

And then at the end we have come to the circular section also we got introduced to the free torsion where warping takes place. Warping is nothing but the out of plane displacement the plane here means the plane which rotates which rotates due to the torsion. So it is better way to imagine mathematically it is difficult to say without drawing a figure so the warping got introduced it is out of plane displacement.

And then a easy way of finding out um torsion properties with same with respect to the membrane stretching to be discussed this two lectures in these two lectures.

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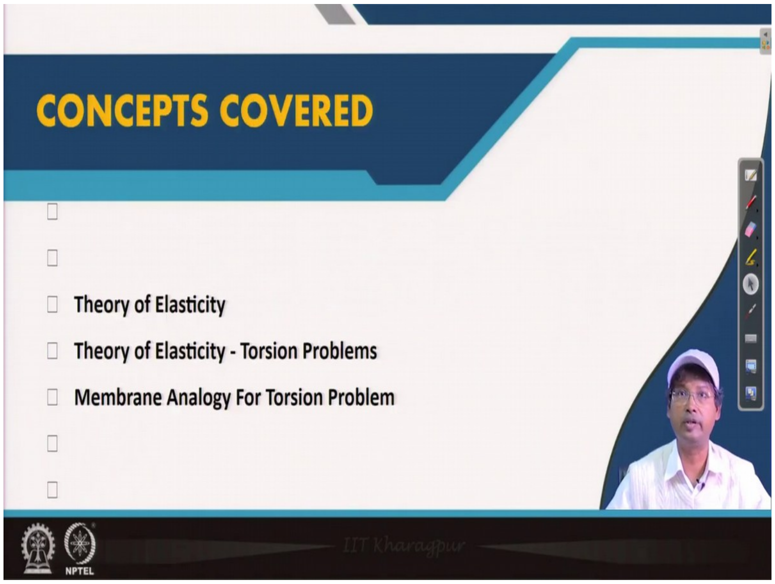

So with that note let us start that discussion. So we will be discussing membrane analogy for torsion problem what is membrane analogy for torsion problem?

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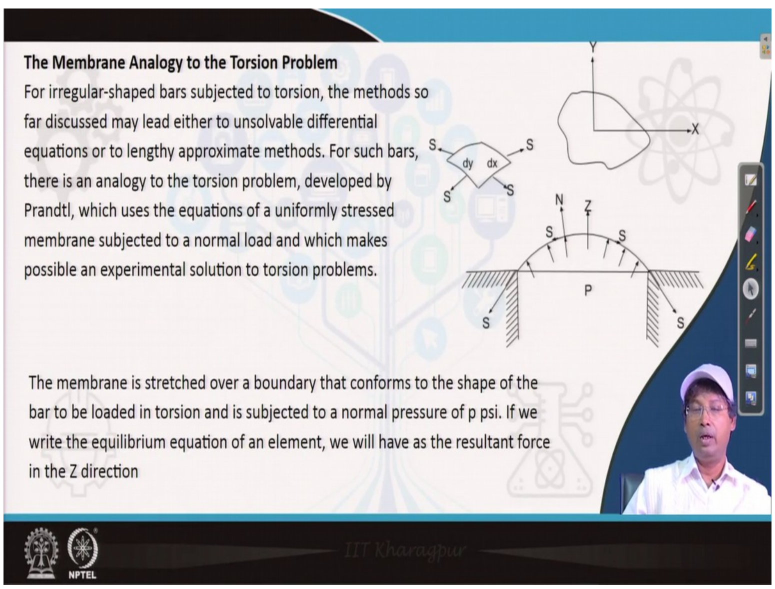

If we think about the process first and then if we go to the description it will be much better what is done that for a regular section maybe a rectangle maybe a circle may be a cylindrical, annular cylindrical section or member we can find out torsional rotation and maximum shear stresses to design. But if it is a irregular section say it is a C section a hat section or a z section which are very, very popular in aircraft structures.

It is difficult to find out the J effective or the polar moment of inertia effective to find out the torsional maximum shear stress as well as the rotation theta. So to do that Prandtl introduced some experimental method. What he introduced that if we if we put a film on the cross section and if we are able to measure the internal pressure of the film and if we are able to measure the profile of the film then we can correlate that film properties to the torsional properties that is what the membrane analogy.

I am talking about film but I am it is written there the membrane that is because the film is considered as membrane. So in experiment what is done is in general say if it is a C section if we just think of a C section we consider the C section similar to this and we put a this section is generally prepared as hollow there is nothing is put on this section it is very, very thin wall section generally prepared first.

And then one rubber membrane is put on top of it and internal pressure is produced. So because of that if I look from this side what will happen it will there will be a profile of the rubber sheet something like this here. So if we are able to measure this profile and we are able to measure the P internal pressure then we can have some correlation. So since a membrane is used generally for experiment people used to use these things.

Nowadays it is not much used but if it is used this way then we can find out the maximum shear stress or resultant shear stress at any section as well as we can find out the rotation theta. So with that concept let us try to have a discussion or have the equations derived. The membrane analogy to the torsion problem for irregular separate bars subjected to torsion the methods show discussed may lead either to unsolvable differential equation or to lengthy approximate methods.

For such bars there is an analogy to the torsion problem developed by Prandtl which uses the equations of uniformly stressed membrane subjected to a normal load and which makes possible an experimental solution to the torsion problem. The experimental procedure just now I have described that is what is said here. But unless we have the correlation between the slope between the slope as well as the pressure and the profile we would not be able to have a correlation.

But before that mathematically we need to pre have the correlation that is what we will discuss. So what we are trying to say that there is an element we are considering this is the irregular separate section and is a part of it as dx dy is considered. And it is may be x direction it may be considered as the y direction z remains same. So it is there is some internal pressures and because of that international pressure in sorry because of that internal pressure the membrane inflates like this way.

So these are the corners edges of this member this is the edge this is the edge and that edge is this edge. The membrane is stretched over a boundary that conforms to the shape of the bar to be loaded in torsion and is subjected to a normal pressure of p psi. If we write the equilibrium equation of an element we will have as the resultant force in the z direction. So let us see how do we get it.

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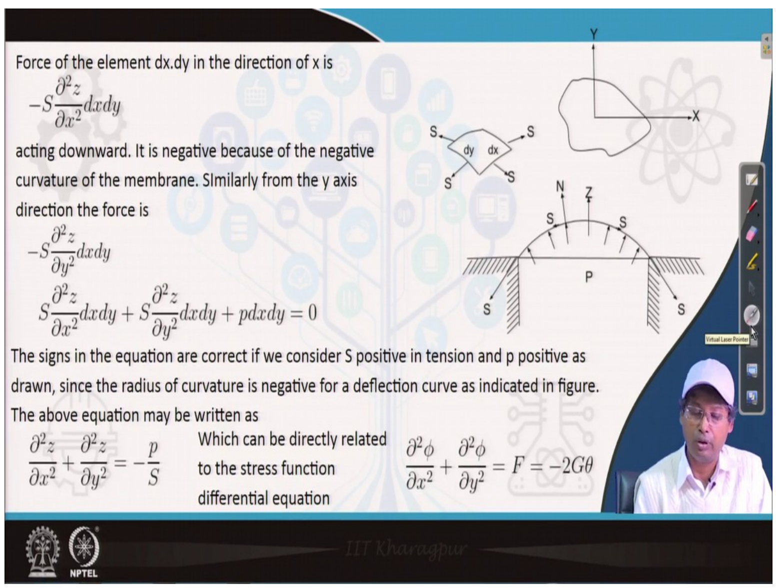

So this derivation portion I have skipped this portion of derivation this is similar derivation with respect to the plate bending there are many more books which describes the plate bending. So you can easily have this type of expression considering in plane load So we can easily see that but let us see what the concept is for the element dx dy in the direction of x we are considering in this direction.

This figure may be considered here in that case this is the x direction the load experience because of the stretching is -S del 2 z del x 2 dx dy acting outward acting downward this way it is acting. It is negative because of the negative curvature of the membrane. Similarly for the y direction the force acting is -S delta z del y 2 dx dy and if we sum it these are in this direction p is in the opposite direction we get the equation as it is given.

And from there we can easily come to this solution but it is reiterated the sign in the equations are correct if we consider as positive intention and p positive as drawn its going upward in the direction of z. Since the radius of curvature is negative for the reflection curve as indicated in figure the above equation may be written as it is divided and we get a second order differential equation partial differential equation del 2 z del x 2 + del 2 z del y 2 is equals to minus of P by S.

So we can have a simple analogy with the equations what we have already derived which can be directly related to the stress function differ the differential equation this is where the analogy starts. So from this analogy we can easily conclude that P by S is equals to 2G theta or P by S equals to –F. So with it this conclusion will use this property later let us proceed further.

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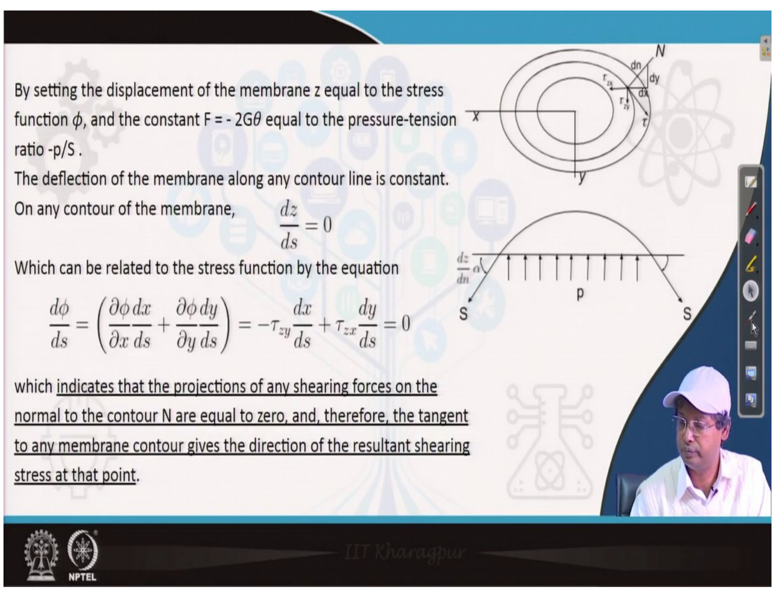

So by setting the displacement of the membrane z equal to the stress function phi and constant F equals to -2G theta equal to the pressure tension ratio - P by S the deflection of the membrane along any contour line is as is constant. So what is happening if we think of a contour line the irregular shape what we considered that would give this type of contour line. This type of contour line is generally obtained for a body which is may be elliptical in section and then it converges to an ellipse here otherwise there will be such changes of curve and the contour would be something very symmetric like this.

So but we can imagine a even elliptical section and we can go for the understanding the deflection of the membrane along any contour line is constant. So here at particular height maybe this height if we consider this it is z is constant so that is the reason along this line the del z del s dz ds is equals to 0. So which can be related to the stress function so we will be substituting z to phi and then we are rewriting the equation in a split manner so del phi del x del x del s del phi del x dx ds.

We are doing plus del phi del y dy ds we can write in this way and then as we have seen earlier this is nothing but minus of T zy and dx ds is there and this is nothing but tau z x this is dy ds and this is equals to 0. So while we get mathematically this expression this gives us a conclusion in physical manner. What is that conclusion from the stress point of view, indicates that the projection of any shearing force on the normal to the contour N this is the N to the contour r equals to 0 so if we consider components of tau zx or tau zy in this direction that is equals to 0.

And therefore the tangent to any membrane contour gives the direction of the resultant shearing stress at that point. So this the tangent at that point gives us the resultant shearing stress. So with this conclusion we let us see what more we can have analogy with this problem and let us see.

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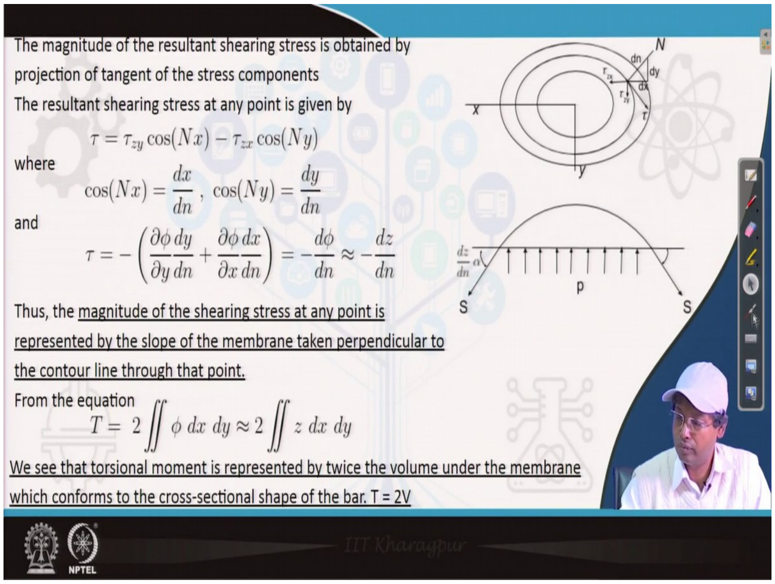

So the magnitude of the resultant shearing stress is obtained by projection of the tangent of the stress components. The resultant shearing stress at any point is given by tau is equals to tau zx cos N x - tau z tau zy cos N x - tau z x cos N y. So if you look at it first component is zy this is the N x component here actually the N x is this but a simple correlation if we geometrically if we think this becomes the component.

So accordingly we get that that tau is equals to this value and we get this equation and from there what we can conclude is that we can conclude that cos N x so not conclusion we can observe that cos N x is equals to dx dn where in dn is the unit normal in this direction and this cos N y is equal to dy dn and then if we substitute those values what we can say that this becomes minus of rearrangement from the conclusion that this is equals to minus d phi dn and which is since phi and z is correlated this way d this is dz dn.

Now from there what do we conclude magnitude of the shearing stress at any point is represented by the slope of the membrane that means if we correspond to this, this slope that is this, so of the membrane taken perpendicular to the contour line through that points. So the from the other equations what we have already derived T is equals to twice double integration phi dx dy and that can be easily approximated to the volume twice the volume.

And we have the correlation we conclude that we see that the torsional moment is represented by twice the volume under the membrane which conforms to the cross section shape of the bar so this is another very, very important equation we are going to use. So with this note we will come again in the next lecture to find out relations for theta two relations we will find already one we have got with respect to theta.

But one more we will find out considering the equilibrium in s direction we come to the conclusion of this slide.

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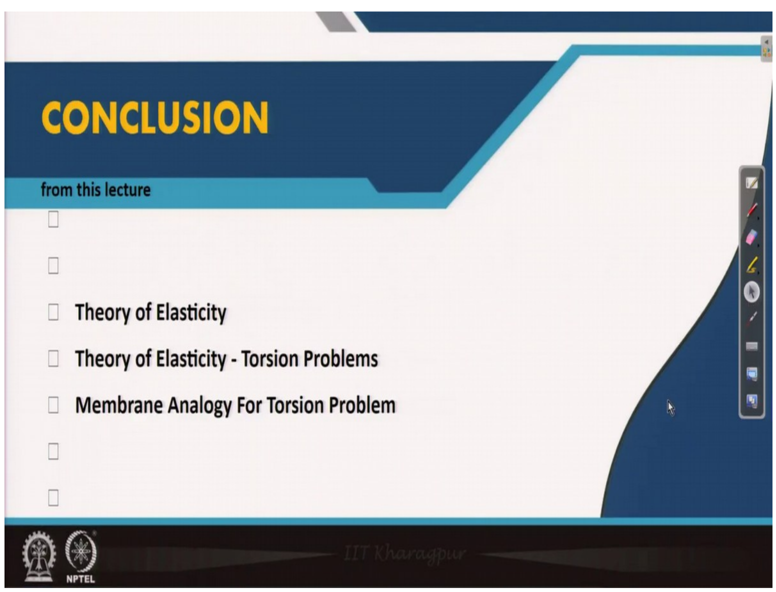

This is the standard reference slide and we proceed further to the last slide where it says that we have learnt to some extent the membrane analogy method. And at the end I thank you for attending this lecture we will meet again with the next lecture, thank you.