Aircraft Structures - 1
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Lecture No -39
Theory of Elasticity - Torsion Problems (Contd.,)

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are in the continuation of 8th week lecture today is lecture number 39 in the domain of theory of elasticity with an aim to solve a particular type of problem related to torsion of cylindrical bodies we have already covered a little bit we have come across to the displacement equations. We have come across 2 equilibrium equations for that we got introduced to some other topics like warping.

And then we will do some more development or general process of analysis today depending on the stress function. And then in the fourth comic lecture we will solve a specific problem in relation to torsion.

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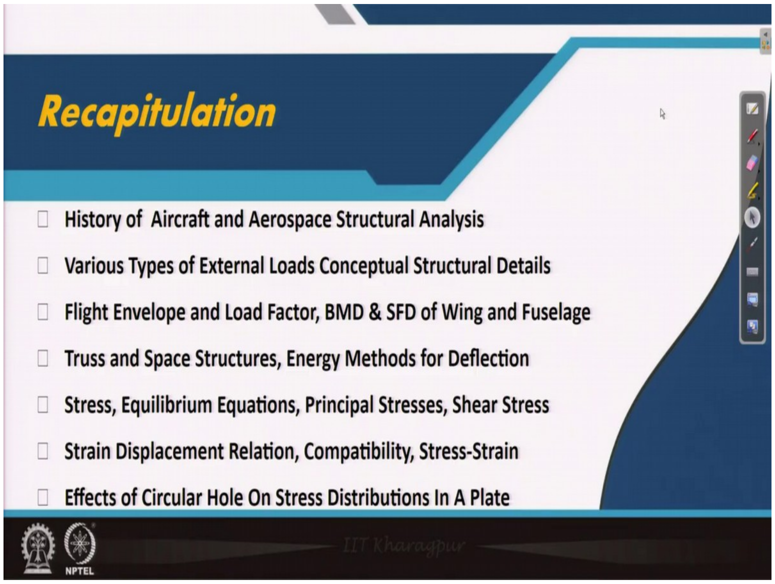

So before we go into the topic today's topic it is time to recapitulate as we have started with the history of solid mechanics or structural analysis the physicist famous physicist started this avenue long ago probably with the experiment of Leonardo da Vinci it started he tried to estimate the strength of and where is a nice experiment we have discussed. Not only that history we have also come across to the development of aircraft from Kitty Hawk flight to present day huge aircrafts including the A380 or Antonov 225.

So then we have come across to various types of loads conceptual structural details various types of loads in the sense while it is in runway or it is airborne what are the types of loads are encountered by an aircraft. And not only that we have gone into detail of how thin wall sections are used to fabricate fuselage and wings different sparse got introduced to different components with names the role of those components also we got introduced.

We also got introduced which part experience what type of loads more and which part is designed for what type of loads those with picture preview we have seen we have come across one agency known as airworthiness agency also which guides the critical uh conditions. They have laid down some critical conditions loading parameters for design. After that we have gone to the flight envelope load factor how it is different for different type of aircraft.

We have seen or carried out examples to find out bending moment diagram and shear force diagram of aircraft wing and fuselage truss is the next structure three dimensional stress also we have used and in this we have solved problems relation related to aircraft landing gear which is generally solved considering three dimensional structure concept or stress concept then we have come across to the deflection determination procedures different types of structures.

Not only determinate, in determinate also we have solved and we have found out indeterminate reactions external reactions as well as deflection at different points. And in this method we have come across with the complementary energy method we have come across to the total potential energy method unit load method dummy load method we have also learned a very, very important method known as Rayleigh method.

Castiglione’s theorem is also covered and then after that we have come or we have started learning the theory of elasticity approach. In theory of elasticity approach we have we have developed several equations to find out stress strain and displacement. In that relation we have seen that there are 15 such unknowns 6 stress components 6 strain components and three displacement components. So we have found out equations corresponding to those.

We have also seen the compatibility equations or a compatibility condition which is very, very important to satisfy while we are we are describing a physical entity in terms of mathematics. So displacement strain and stresses should maintain the compatibility continuity to some extent so that we have seen. And then we have solved a few problems 2 different approaches we have learned one is an inverse method and the other is semi inverse method.

And then we have solved a very, very important problem in theory of elasticity that is the last point here mentioned as the effect of circular hole on stress distribution in a plate. We have seen that even it is uniaxially loaded if there is a hole irrespective of the dimension of the hole the stress tensile stress reaches to thrice the uniform stress experienced by the plate in one direction that is a considerably more stress and that is the reason we have studied it.

So that tensile stress is responsible for opening we have also seen some critical conditions where how maximum stress may be experienced and we have seen that in case of pure shear or while a piece a rectangular segment a plate is under in plane tension and compression it experiences about four times the tension or compression stress. So that is also very, very important point to note. We have also discussed problems with in relation to crack propagation we have not gone into crack propagation in detail.

But a starting point or may be a glimpse of that area we have seen we have considered some elliptical hole and then we have seen that how at the tip the stress becomes very, very high and leads to failure it propagates. We have also seen that it is a very easy way to arrest the crack if we if we are able to drill a hole at the tip of the crack. So that that restricts the stress to the three thrice the stress experienced in general.

So with those nodes we have attempted the cylindrical body of irregular cross section not circular cross section and then we have tried to find out the equilibrium equations and then with the stress function approach we will see how these things can be solved.

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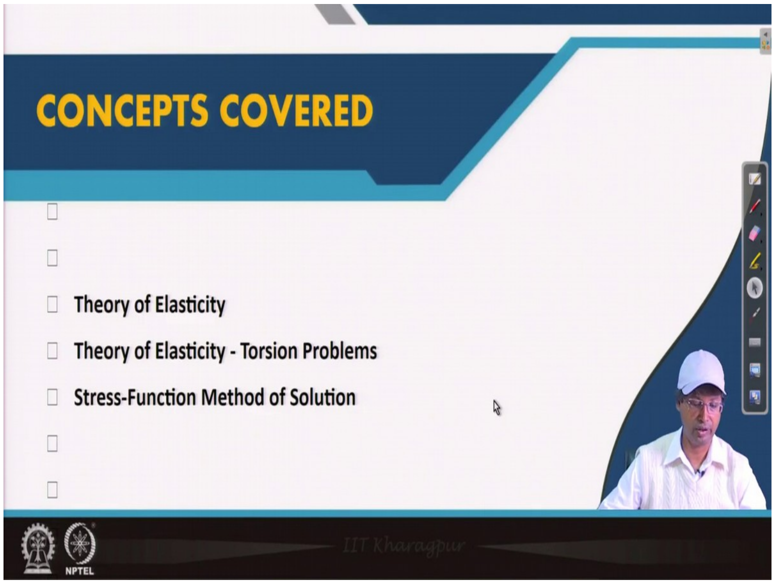

So that is what the stress function approach will be covering today and in this stress function approach how do we get the other equations that we will see.

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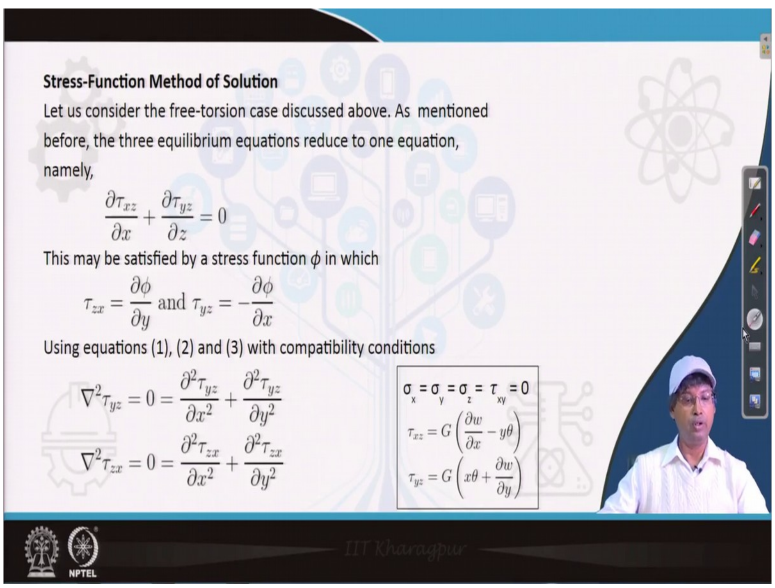

So the stress function methods of solution stress function method of solution let us consider the free torsion case discussed above. So again we are already introduced to the concept of free torsion what is free torsion? We have assumed that no cross section not even the end cross section is restrained to deform out of its plane and that is the reason we are saying it is a free torsion will be solving that type of problem.

And if it is there if it is a non circular cross section we will see it experiences warping. So we have seen with drawing in the last class that how a rectangular bar experiences warping and how it gets deformed in a quick look we can again bring back that if this is the cross section the top surface deforms something like this and it continues for the total. So similarly for all the surfaces it happens like that so this warping is not possible unless it is it is a free torsion.

So considering that we have seen that the equilibrium equations since sigma theta sigma this this this condition holds sigma x not sigma theta sigma r sigma x sigma y sigma z and tau xy, xy is which plane we always consider that this is x this is y and this is z. So there is no tau xy so whatever is there is that tau xz or tau yz so it is the other sense if we say instead of saying xz yz we can better say that tau zx tau zy that means in this plane whatever is acting is there.

And that is acting either in this direction or it is in this direction. So that is the reason this one is we say that tau zx and this one we say that is tau zy. So with this concept what we have seen is that it holds the equilibrium equation as stated here. And one more thing I remember now last class I forgot to mention while we were finding out the displacement u and v. The equations whatever we found out there we are supposed to consider that theta z is very, very small and cos theta z is equals to 1 and sine theta z is equals to theta z.

Unless you consider that we would not get the uv expressions what you we have used in the last lecture. So with that small note let us proceeds excuse me so with that small note let us proceed further.

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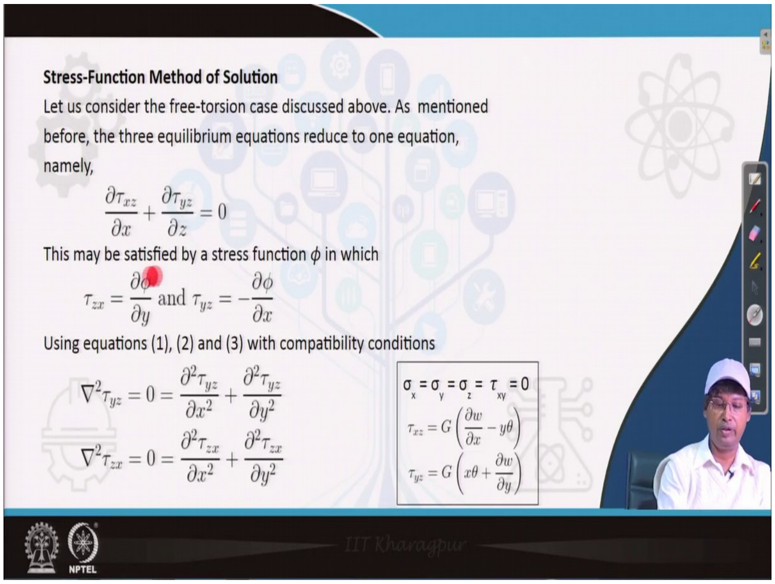

So this may be satisfied by stress function phi this equilibrium equations where the stress function is defined in terms of tau zx and tau zy or yz as it is given one is positive the other is negative and the derivative is also may be noted tau zx is with respect to y tau zy is with respect to x and minus. So this 1 2 3 is nothing but these are the equations I have brought again to that so if we use these equations and these relations easily we can say that the grad square tau yz is equals to 0.

So these 2 relations hold so del 2 tau y z del x 2 del 2 tau y z del y 2 is equals to 0 plus this is equals to 0 and del 2 tau zx del x 2 plus del 2 tau zx del y 2 is also equals to 0. So with this node we proceed to the next slide.

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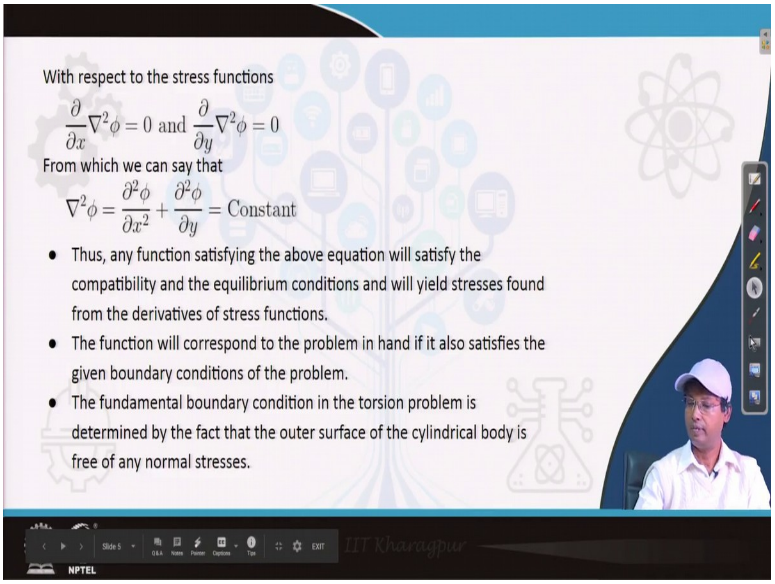

With respect to the stress function if we take back we you can easily see that del del x of grad square phi is equals to 0 and del del y of dr square phi is also equals to 0 from which we can say since this is x derivative and this is y derivative and both are coming to 0 there must be it must be equated plus of these 2 must be equals to some constant value and that is why from mathematical conclusion why we get that we consider that this is a constant value and generally this constant is given by in this particular discussion as F.

So with that note let us see what we have thus any function satisfying the above equation will satisfy the compatibility and equilibrium conditions and will yield stress found from the derivatives of the stress function. So it is quite clear if phi satisfies these and if we know the phi we can find out the stresses the functions will correspond to the problem in hand if it is also satisfying the given boundary conditions of the problem.

So boundary condition has to be satisfied for a particular type of problem and then it will represent that particular problems solution. The fundamental boundary condition in the torsion problem is determined by the fact that the outer surface of the cylindrical body is free of any normal stress. This is quite important we do not assume any force on the outer body and if we do some mathematical jugglery that I have skipped what do we get? We get a nice equation which is there in the next page.

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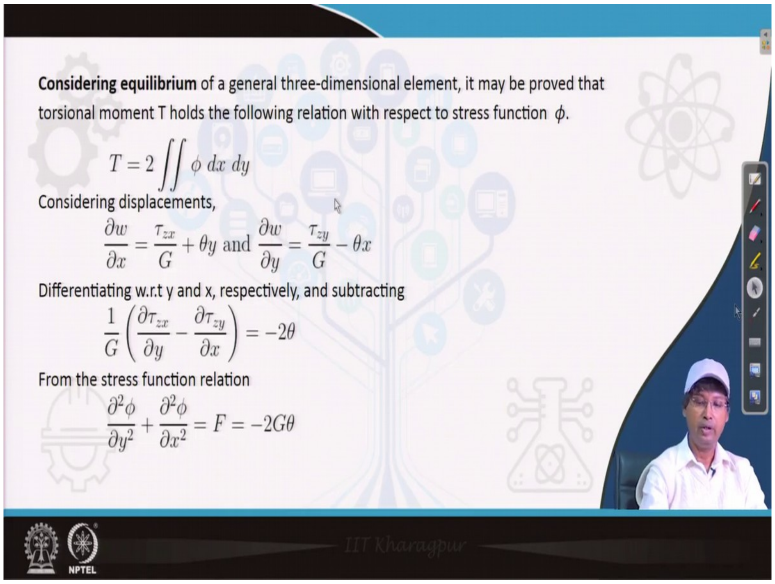

This equation we get and let us see considering equilibrium of the general three dimensional element that is what from the equilibrium concept of external forces whatever we you are taught earlier if you follow those things and if you follow a simple steps as described in all almost all standard books you will find out that it shows that it may be proved that the torsional moment T holds the following relation with respect to the stress function phi.

So this is satisfied so this you may say as the boundary condition with respect to the surface forces. So that is the equation we have we need to satisfy this as a boundary condition for to get solution of any particular problem. Considering the displacement already we have found out these equations are nothing but the displacement equations rearranged in this way in the last class we have seen del w del x tau z x divided by g + theta y and del w del y is equals to tau zy by z - theta x.

Now if we follow this operation that means if we divide it take a derivative with xy and x this is y and this is x and then if we subtract so this left hand side portion will vanish and we will have a relation with respect to this this this and this that is what we have here and since we are taking derivative of this part with respect to y this y also will become one. So only theta remains so one by z del tau zx del y - del tau z y del x is equals to - 2 theta.

Now as we have seen in the previous lecture or maybe in the previous slide if we substitute the value of tau zx and tau zy what do we get we get back the equation where it says that del 2 phi del y square + del 2 phi del x square is equals to a constant or may be defined as F this equation we have come across already here. In this case and so once we get that equation in a different way we say that that F is nothing but the constant minus g 2 theta so that is satisfied.

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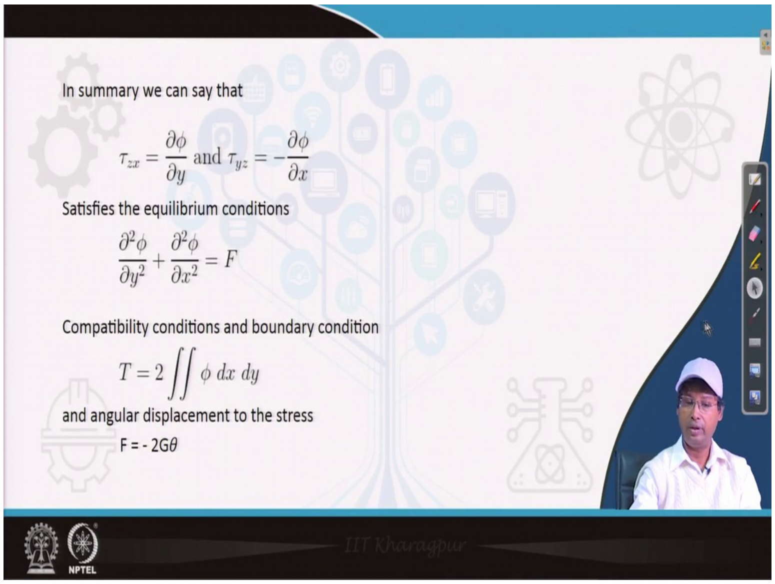

So with this we come to the summary of the stress function approach what is required to solve the problem in torsion. The summary we can say that the we have 2 expressions for tau that is tau zx and tau zy del phi del y - del phi del x we have the equilibrium conditions is equals to F. And F is equals to - 2g theta we also have the boundary conditions which says that T is equals to 2 double integration phi del x del y.

So with that note we will end today's lecture and we will go further proceed to our next problem of finding out solution for a particular type of section that is elliptical section. And in that elliptical section we can we can solve those problems and find out the solution for almost all components up to displacements.

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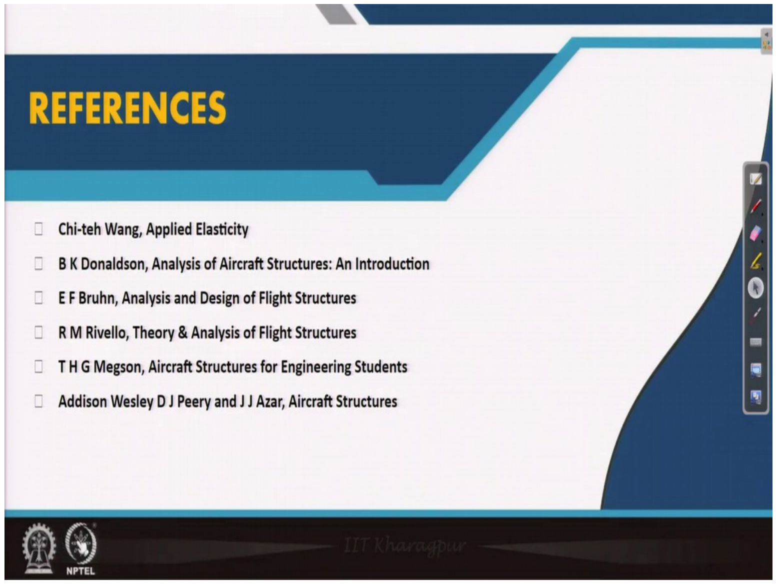

So with that note we come to the slide of references these are the standard references all these problems are solved there and I thank you for attending this course.

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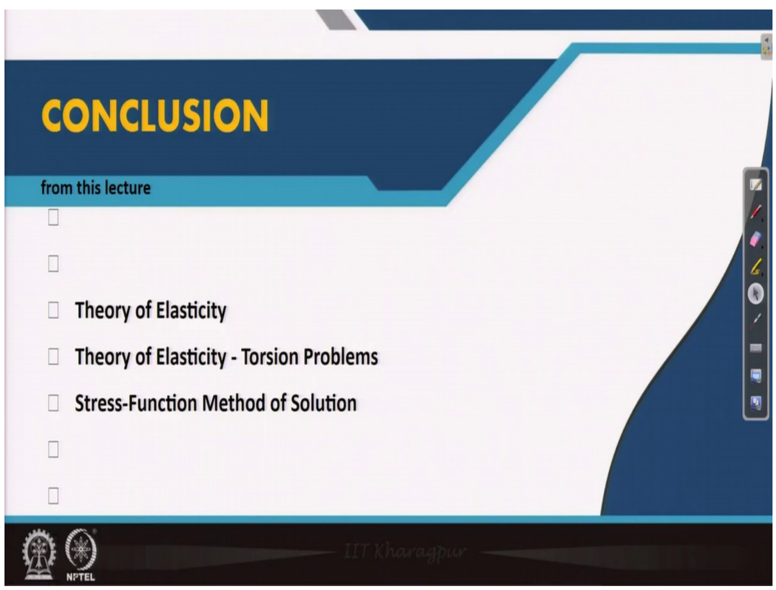

Before I thank you at that we have what we have learned in this class is that the stress function method of solution. And with that note I thank you for attending this lecture and will proceed to solve a very nice problem which gives you insight into the torsional problem where wiping is inevitable unless it is a circular section and we will conclude mathematically whatever we are talking about.

We will see that we will get the equations where if it is a circular section it would not show any warping if it is a non circular section it will show warping. And not only that we will also get introduced with the with the values of the polar moment of the area what we use to find out shear stress as well as the rotation theta probably is not very, very wise to use if it is a non circular section with those things let us end today’s lecture thank you for attending.