Aircraft Structures - 1
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Lecture No -37
Effects of Circular Hole on Stress Distributions in a Plate

Welcome back to aircraft structures 1 course this is professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are at the end lecture of module 7 or the 7th week lectures and title of the lecture remains same effects of circular hole on stress distribution in a plate that is because so far what we have done in this series is we have evaluated the equations or derived I should say the equations of elasticity in polar coordinate system.

So that we can handle the problems problem we have specified. And after that for uniform stress uniaxial stress in plane stress of amplitude is we have solved the problem and in that solution we have also seen that there are different stresses at different points. And those things more about those with those results we will spend some time in this lecture to have some more insight into the problem.

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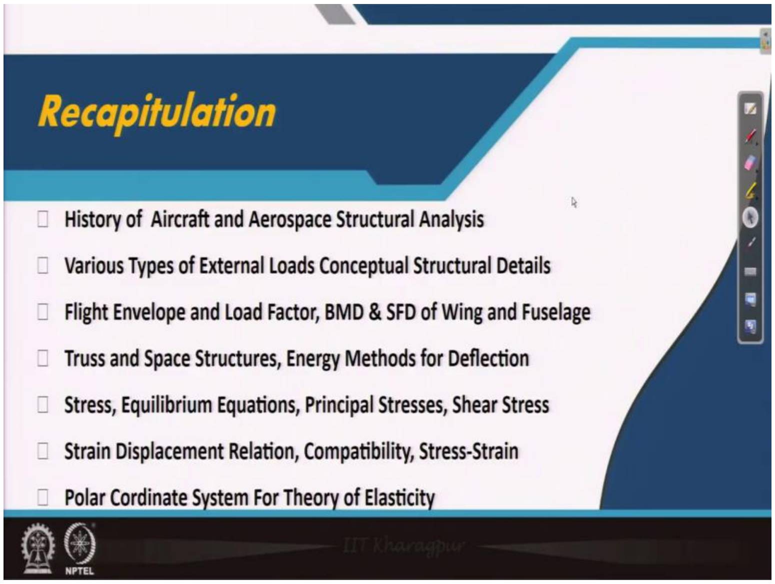

So with that note we move forward and in this what we do is that we do the recapitulation in this recapitulation slide as usual we try to remember the work what we have or the portions. What we have already covered we started with the history of aircraft as well as history of solid mechanics and then we have slowly come to the various types of external loads conceptual structural details of aircraft structures though we have covered.

We have covered in depth the flight envelope and load factor how the flight envelope changes for different types of aircraft that also we have discussed. It is important to find out the shear force bending moment experienced by wing and fuselage. So 1 typical example in that correlation we have solved considering unit load concept considering 1g and some unit loading concept combined with unit load concept.

It is said so that for different g value or inertia load we can find out the bending moment and shear force. We have solved learned to solve three dimensional structures especially trusses. Solved a few problems related to landing gear then we have come across 2 different energy methods predominantly to find out the deflection. So in that method we have learned complementary energy method total potential energy approach unit load method dummy load method Castiglione’s theorem.

And also we have learned one very, very important step in important procedure we must say that is the Rayleigh ridge method. And we have seen how the assumption of initial reflection profile is important to get a good solution or appropriate or close to experimental solutions are necessary. So after that we have gone to the theory of elasticity portion. Theory of elasticity is very, very important in the sense that unless we learn the theory of elasticity.

We are not able to have insight into the problems and this lecture specially is dealing that type of insight already we have discussed to some extent in the last few minutes of the last lecture. Why a hole is analyzing a hole under uniform tension in a plate is important. How more stresses are coming why the cracks open from a certain position and it goes further those things we have learned and a little bit more we will try to discuss in this lecture.

So this lecture is mainly the discussion about the ways it has been it has been implemented or precautions should be taken.

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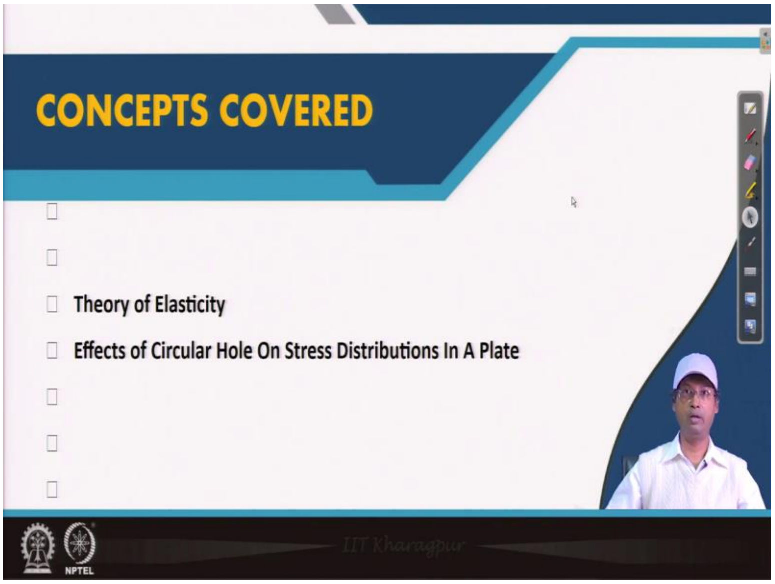

So with that note let us proceed further with the problem what we have solved we have found out the stresses but we need to have a discussion on it.

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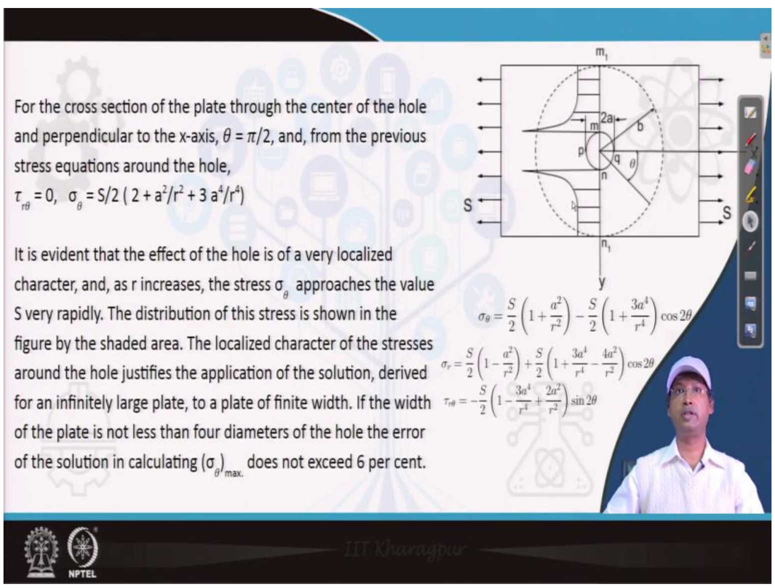

So with that note the same figure we bring let me have a re have a have a description of the problem what we have solved. In this what we have solved is a plate is considered the prime thing is that the plate is loaded uniaxially along the axis x and the intensity of stress is S and then what we have is that there is a hole and we consider that the whole radius is comparatively small in comparison to the width or the other way it has been defined the diameter is less comparatively small in comparison to the width or a is small in comparison to b.

So we are considering that after b the stress remains uniform as if there is no hole in that condition and we have considered found out the stress for that from that part we have not done but you can easily do it simple stress transformation if you do you can find out the stresses for that. And from there we found that it comprises of 2 part 1 is uniform stress of S by 2 and the others are non uniform in nature with it I would like to say why non uniform non uniformity is with respect to theta.

So other part is uniform with respect to theta. So it is axis symmetric case that part we have solved first then the other part we have solved and we have combined equations for sigma theta sigma r as well as tau r theta. So this is what we have got with respect to theta cos 2 theta cos 2 theta sin 2 theta. Also we have discussed that under this particular situation the stress at this particular point is thrice the stress applied to the plate this is 3S and that is also tensile in nature here and as well as here m and n and at p and q this is compressive in nature.

So that is the usual reason why a crack opens up from this and goes further it is three times definitely that is supposed to fail in tension and that position and it is to propagate. Now in this that we have already discussed how do we get to some extent profile also discussed we will discuss again that profile. So let us see what is there today to discuss. For the cross section of the plate through the center of the hole and perpendicular to the x axis theta is equals to pi by 2.

And from the previous stress equations around the whole what we can see is this equations they are here the value is only put for theta and we get that S by 2 2 + S square by r square plus 3a to the power 4 by r to the power 4 it is a at r equals to a it goes to 3S as we have seen. So that holds tau r theta is equals to 0 from here. It is evident that the effective effect of the whole is of a very localized character and as r increases or say in this line if we go further for to find out sigma theta.

The stress sigma theta approaches the value S very rapidly. Since it is 4th order and second order in terms of a and r ratio definitely it is supposed to be very, very quickly to come to 0. So that is what is said mathematically if you plot it gives this type of profile the localized character of the stress sorry the localized character of the stresses around the hole justifies the application of the solution.

So it is said that a means the solution whatever we have got if we try to check the solutions how far the initial assumption what we took that if b is comparatively large with respect to a it that uniform stress remains. So whether that is prevailing or not that we are saying and it is said that it is a little distance apart only it becomes uniform. So the localized character of the stresses around the hole justifies the application of the solution derived from the infinitely large plate to a plate of finite width.

So that finite width is b. If the width of the plate is not less than 4 times the diameter of the whole the error of the solution in calculating sigma theta max does not exceed 6%. So it says that if it is 4 times the diameter then the maximum error involved is 6%. So it is quite clear that means what we can say that this side it is 1 and half diameter this side it is 1 and half diameter because this is 1 diameter.

So is not big if the figure is much more than that so with that concept we say that there is a stress concentration at this point at this point but it does not remain for much more length even if we consider the length approximately 4 times or the width here approximately 4 times the diameter of the circle the error involved is approximately 6% so that is probably negligible. So with that note we proceed further for the discussion.

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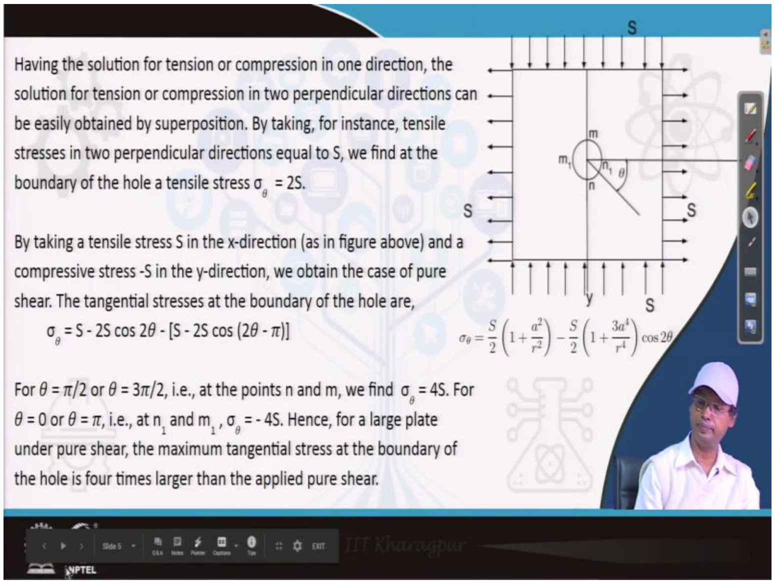

We come across to a different type of problem. In this problem what we see is that instead of putting only stress only in 1 direction we may have stress condition from both the directions so what can be the situation that is what we have tried to find ou? Having the solution for torsion sorry tension or compression in 1 direction the solution for tension or compression in 2 perpendicular directions can be easily obtained by superposition's.

So it is we are assuming that linear superposition holds so if in 1 direction the solution we got have obtained we can easily find out the other direction separately and we can add it. By taking the for instance tensile stresses in 2 perpendicular directions not this case tensile stress in 2 perpendicular directions equals to S. We find at the boundary of the hole a tensile stress of twice S why it is coming like that?

Because if this is S this is also S because if we consider any 1 point there we have for 1 direction is plus 3S but for the other direction it is –S. So it becomes twice sorry twice S. So similarly it is a symmetric condition so in all the corners whatever we have all the diametrically opposite position is having experiencing sigma theta or opening type of stress in this particular point say this point if we talk about sigma theta is acting we are talking about this sigma theta.

So that is what if we apply in both the direction tensile force but if we apply some other type of force which is more critical we need to discuss that is what is shown here it is a critical case. By taking a tensile stress is in x direction better we remove this for in x direction as in the figure above or it is on the right hand side and a compressive stress of -S compressive stress of -S in the y direction. We obtain the case of pure shear this is this is something what we asked you to solve.

See if in this particular case if we consider any element here which is 45 degree angle this experiences a pure shear that means only shear stresses are acting there is no normal stress acting on that particular orientation element. So in the there it is also may be noted that this amplitude becomes S whatever the amplitude of the stress being applied on that that is the reason we call it as a pure shear.

The tangential stress at the boundary of the holes are so in this particular case of pure shear which may be achieved by tensile and compressive stresses as applied here. We can have sigma theta is equals to this minus this it is as usual I do not want to go into deep into it for theta equals to pi by 2 and theta equals to 3 pi by 2 that is at the point n and m we are talking about this point as well as this point what we have we find that sigma theta is equals to 4S.

If you just simply substitute in this formula and see that in that particular portion we have tensile stress of 4S for theta equals to 0 and theta equals to pi that means this position as well as this position we have theta is equals sigma theta is equals to -4S. So this is a particular type of situation where we have very high stress concentration at the diametrical point one is tensile in nature this is tensile in nature this is compressive in nature and those goes to the magnitude of 4 times the stress applied on it.

And in this particular case if we go for a stress transformation we can find out that at 45 degree angle there is a pure shear is oriented pure shear is developed. So hence for a large plate under pure shear the maximum tangential stress at the boundary of the hole is 4 times larger than the applied pure shear. Why it is saying tangential stress because whatever the sigma theta we are talking about here also it is acting this way.

Here also it is acting this way that is the reason we say this is tangential stress. So this is negative compressive in nature these 2 are tensile in nature. So with that node with this critical point of discussion which may be experienced by some part of the structure we will move further to discuss this thing with respect to the application.

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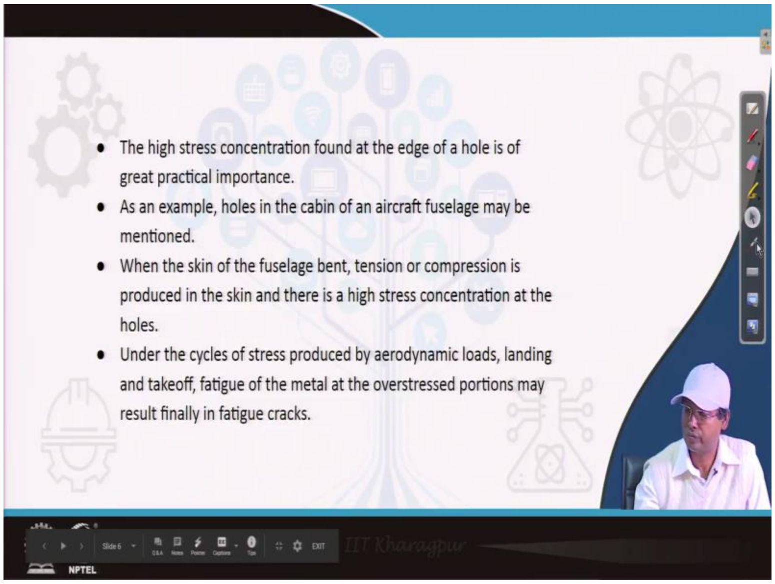

So we have few points to discuss let us try. The high stress concentration found at the edge of a hole is of greater practical importance. So already we have discussed we need to consider this things will come again as an example hole in the cabin of an aircraft fuselage may be mentioned. So this is very, very important case aircraft comet disaster took place they are they used some rectangular type of window if something like this shape corners were round but not serve.

But from there because of the different stress condition the fuselage got tear apart and that was the problem it happened there were many, many catastrophic failures and after that the design got modified people did lot of experiments with strain gauges applied on those areas. Hope you are aware of strain gauges strain gauges are the gauges what we based on the structure put on the structure to find out strains.

So that is very, very important decision people have learned and the crack propagation and fracture has come to a; with a very, very important subject of discussion. So with that what we go further with the skin of the fuselage band tension or compression is produced in the skin and there is a high stress concentration at the holes. Under the cyclic cycles of stresses produced by the aerodynamic loads landing and takeoff fatigue of the metal can be over stressed portions sorry fatigue of the metal at the overstressed portions may result finally in fatty cracks.

So along with that 1 more phenomena is talked about that is the fatigue that means with repeated loading properties of the structure changes capacity to withstand crack changes it becomes prone to failure and those studies again that is a separate area of discussion. So people have studied those areas and seen that if this type of situations are there we need to reinforce the parts surrounding this we generally put another reinforcing ring around surrounding this with rivets.

Because we cannot have a structure where we do not have any hole so once we reinforce it we make it more stress with stress with standing portion then the nature and distribution of stresses change and then we get a structure which is safe to use so with that note we proceed further.

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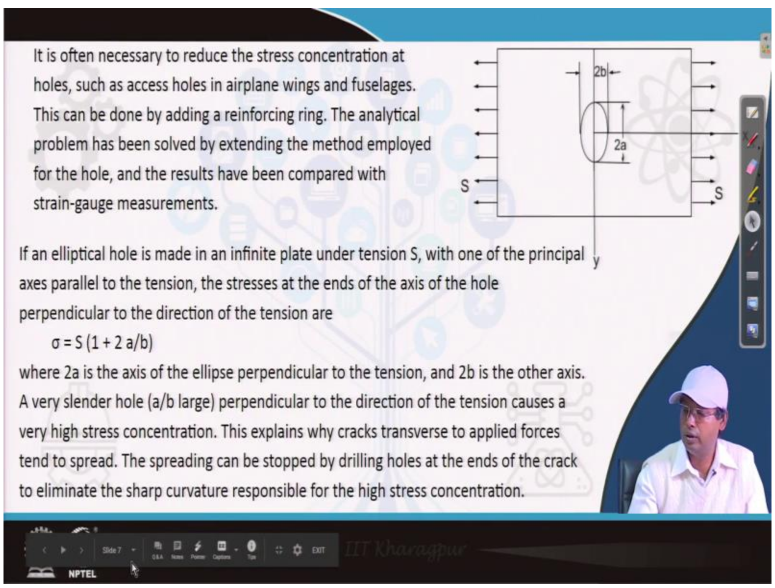

So here we will bring we want to derive the equations but we will try to use the equation for understanding. So let us see it is often necessary to reduce the stress concentration at holes such as access holes in airplane wings and fuselage. There are many access holes not only that we need those holes to reduce weights. We need those holes and areas to say for wing ribs if you look at those there are so many holes.

Because in civil aircraft may predominantly fuel is stored in the wing so unless we have hole how can we store. So the wheel we need to lay the fuel pipes and other avionic other control equipments through those ribs so we need to have holes we cannot have a structure without a hole. But at the same time we need to use those properly so that it does not go to failure. The analytical problem has been solved by extending the method employed for the hole and results have been compared with strain gauge measurement.

So what is done this can be done by adding a reinforcing ring as I have shown in the last slide in general we put a reinforcing ring around that particular hole and that prevents a lot. So with that note we proceed further to this particular type case in an elliptical hole which is oriented this way is made in an infinite plate under tension S with 1 of the principal axis parallel to the tension. The stresses at the ends of the axis of the hole perpendicular to the direction of the tension are sigma is equals to S 1 + 2 a by b at this point.

So where 2a is the axis of the ellipse perpendicular to the tension and 2b is the other axis we are talking about theta definitely. We are talking about stress in this direction so this derivation is little bit lengthy and it takes time. So that is the reason it has been skipped you may find it in any theory of elasticity book. So what we do a very large center slender hole or while a by b is large that means this is a whole something like may be this type of ellipse or maybe may be more perpendicular to the axis of the tension causes a very high stress concentration.

Why in this particular case a by b ratio is high? If a by b ratio is high you see this stress jumps if it is equal it is 3S as the circular case of hole. If it is more it increases so that is very, very good phenomena to increase. So as the hole becomes thin or the ellipse ratio changes a is much more than the b then the stress concentration at this particular point becomes very high. This explains why cracked transverse to the applied force tends to spread.

So if you think about a crack that crack also may be assumed something like a ellipse and but having a and b a different value a is definitely high very high in comparison to b and that initiates and propagates the crack. The spreading can be stopped this is very, very intelligent way of doing it in practical life. The spreading can be stopped by drilling a hole at the ends of the crack to eliminate the sharp curvature responsible for the high stress concentration.

So as soon as if we talk about if we find some crack here and if we put a hole drill a hole around this what will happen? The stress is limited they are up to only 3S it is not more than 3S so that arrests a crack so with that intelligent way of adjusting crack people work in practical situations where it is difficult to prevent the structure by replacing it or by reinforcing it in some other manner.

In some cases this trick is this i should not say trick this intelligent way is applied and with that note on crack propagation stress concentration we would like to come to the end of the lecture.

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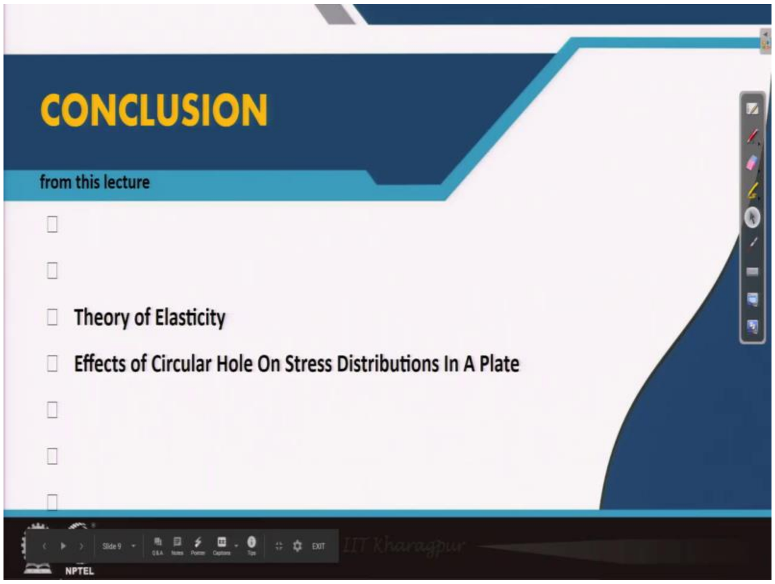

And we come to the standard usual slide of references and with that note we come to the end of the week 7 lecture where we have discussed stress concentration around a whole. And with that note I thank you for attending this lecture we will meet again in the 8th week lecture, thank you.