Aircraft Structures - 1
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Lecture No -36
Effects of Circular Hole on Stress Distributions in a Plate

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department IIT, Kharagpur. We are in the series of the lectures where we are at the seventh week the sequence is 36th lecture. We were solving stress around a hole how for uniform in plane tension those stresses vary on the circumference of a hole. So we need to we have solved to some extent the problem.

We have solved the problem with no hole and with uniform stress condition access symmetric stress condition and this lecture we will solve the remaining portion the most important portion of the stress distribution. Here we will consider a hole we will put boundary condition accordingly and with those boundary condition we will see how we can solve the problem.

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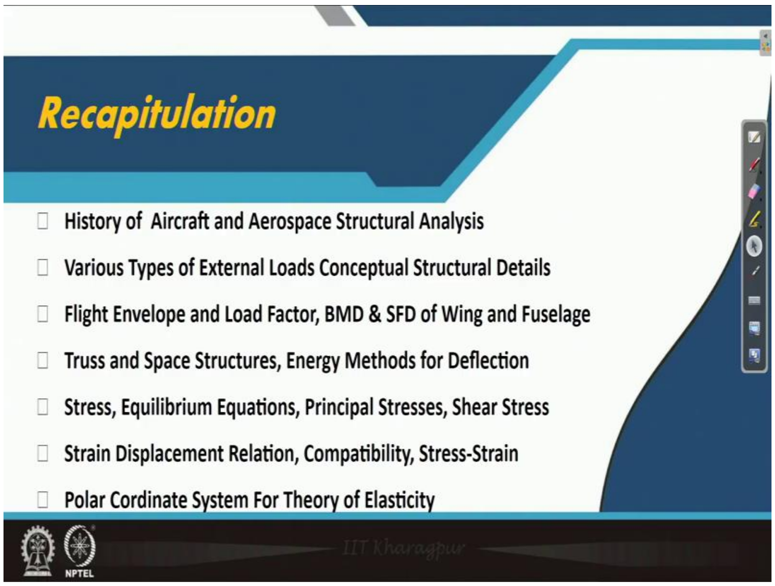

So with that note we will proceed further the recapitulation slide comes. Every lecture we simply try want to remember again what we have learned this course may be long for somebody maybe not very long but anyway whatever we have learnt and where we are it is better to come get a reminder for that. So history of aircraft and solid mechanics or structural analysis we have done.

We have done various types of external loads conceptual structural details. We have done flight envelope and load factor bending moment and shear force diagram of wing and fuselage.

Truss and space structures we have done. Solved problems related to landing gear. We have used different energy methods to find deflection it is for determinate as as well as indeterminate structures external internal both we have done. We have used different methods like dummy load method unit load method Castiglione's theorem. We have also learned a very, very important method like Rayleigh ridge method.

We have come across the theory of elasticity next and then there we have learn different equations required to solve problems and in that process we have solved problems in inverse and semi inverse method. We have solved problems for a cantilever beam loaded at tip that's a very good solution we get and then we have we have discussed a part of of the problem where there is a hole in a plate and the plate is loaded on its plane.

And because of the hole how the stress varies on the sun circumference of the hole we need to study and we will go into that problem.

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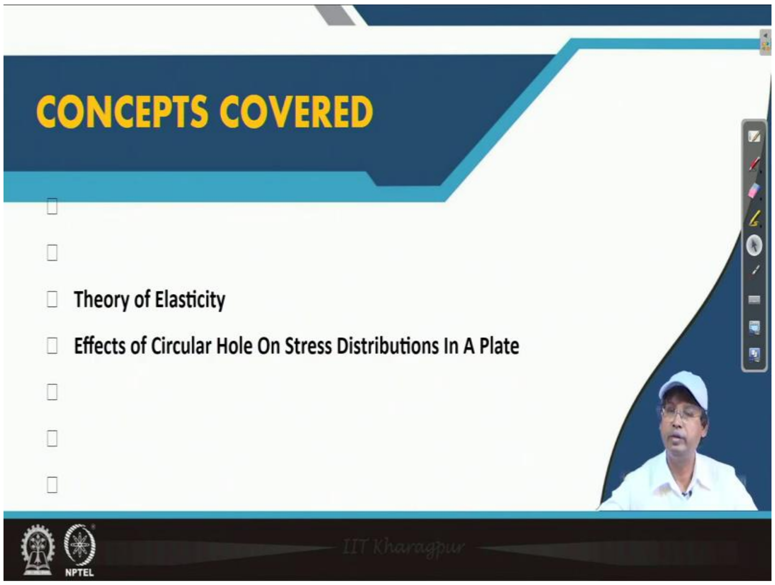

So we have solved a part of it in last 2, 3 lectures and we will continue with that without any introduction.

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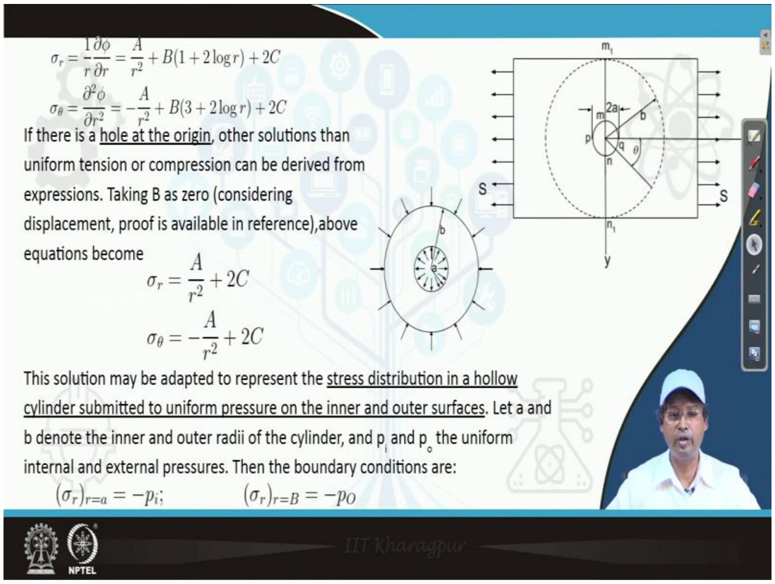

So what we have consider here is that we are there is a hole of diameter twice a uniform tension is acting of amplitude S we are considering one more circle b which is considerably large in red in radius in comparison to the whole radius. And we are assuming that the stress distribution beyond this hole beyond this circle is a uniform, uniform in the sense as if there is no hole in the structure or the plate.

So considering that we got that there are 2 part one is sigma r and the other is tau r theta. Sigma r is also divided in 2 parts one is because of the uniform compression or tension S by 2 other is theta dependent component tau r theta is completely theta dependent component. So the first part considering the first part that is access symmetric case S by 2 that we have solved to some extent and we have discussed that if there is no hole in this particular case AB goes to 0.

And it results in to the sigma r or sigma sigma r or sigma theta as constant but we are discussing if the problem with hole that is the reason after repeating the equations what we have derived in the last class. We are considering the case with a hole. If there is a hole at the origin other solution then uniform tension and compression can be derived from the expressions from these expressions only. Taking b as 0 this comes may appear bit arbitrary but we do not have the scope to prove this let us consider that b becomes 0 considering the displacement considerations.

The above equation becomes sigma r equals to A by r square + 2C and sigma theta theta is equals to minus A by r square + 2C. So if this is the case and if we imagine that it is under pressure from inside as well as from outside it may look like a cylinder this solution may be adapted to represent the stress distribution in a hollow cylinder submitted to uniform pressure on the inner and outer surface.

Let A and B denote the inner and outer radial of the cylinder and p i and p o the uniform internal and external pressure. So what it says that this is p o and this stress distribution whatever we see that is p i ok and we are considering some problem where we have a cylinder something like this and it is under pressure from outside as well as from inside also there are pressures as it is shown here.

So this is xyz it is not the same way it is given here it is upside down may be considered for Cartesian right hand rule system it has to be xyz in this way otherwise we need to draw it in a different way anyway so here xy is in acting in a different direction. So as it is says that both are compression that is the reason it is minus this is also minus acting here. So we need to put the boundary condition we need to check what are the constants we get?

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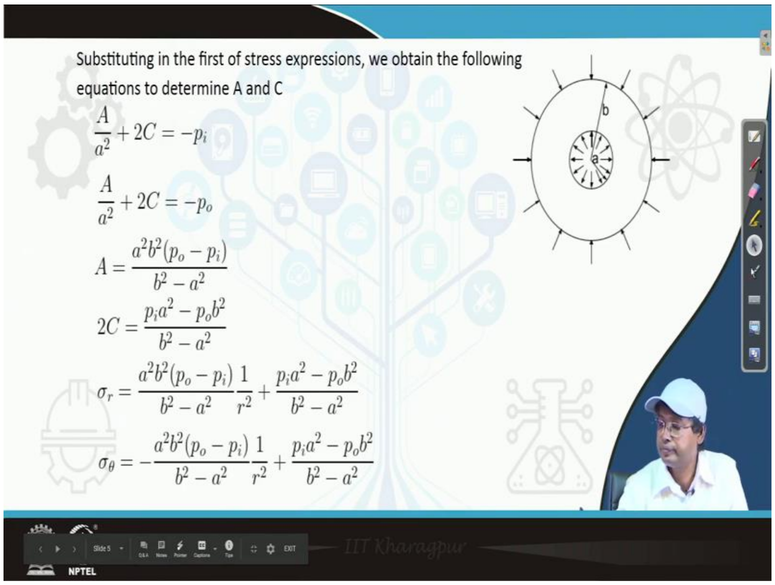

Substituting the first of stress expression we obtain the following equation to determine A and C. So in sigma r if we substitute we have this is equals to p i and this is also a this is say mistake this should be b. So A by b square is equals to + 2C so with that if we go for solving this A and C we get the equation this as well as 2C is equals to this and from there what we can if you directly substitute this to the values we have 1 by r square remains and 2C is this value.

So there is nothing more to discuss and similarly sigma theta is having this expression. So we now know for a case where it is in compressive stress from inside as well as from outside a cylinder how the stress distribution is it is dependent on again only on r not on theta because it is axis symmetric case again.

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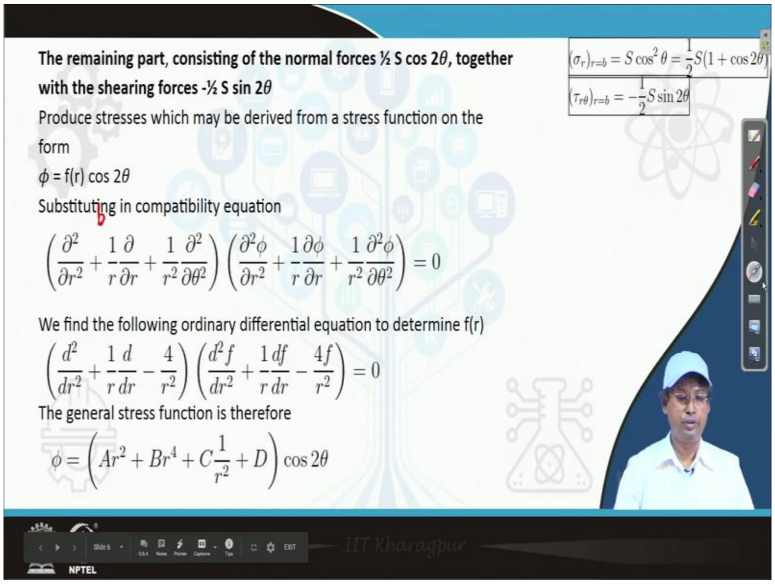

So the remaining part consisting of now we come to the remaining part remaining part that is the reason I have brought back those 2 equations why we are saying remaining part? Because sigma r and C tau r theta what we have seen is that is the expression beyond the b or while there is no hole. So this part we have considered so far with a hole the remaining part means that this S cos 2 theta by 2 and tau equals to - S sin 2 theta by 2 those 2 parts we need to consider and find out the stresses.

The remaining part consisting of normal stresses half S cos 2 theta together with the shearing stress minus half S sine 2 theta produce stresses from which may be derived from stress function of the form phi is equals to f r cos 2 theta. So substituting in the compatibility equations what we can see is that it is substituted here we have if f is a function of 4 f this is completely I think you can get the is there any need to explain I do not find any need to explain it is some simply substituted here.

And accordingly we get this ok del 2 phi del 2 r square is simply theta is coming out in all the cases cos 2 theta comes out that is the reason only f component is present there. So del 2 del 2 f del r 2 then 1 by r del f del r - 4f r square and this is the other parameter. So because it is grad 4 form so phi is equals to the general stress function is therefore maybe as this phi is equals to Ar square + Br to the power 4 + C1 by r square + D again how do we get to this it is simply a mathematical portion we are not going to discuss it. So with this we proceed to the next part.

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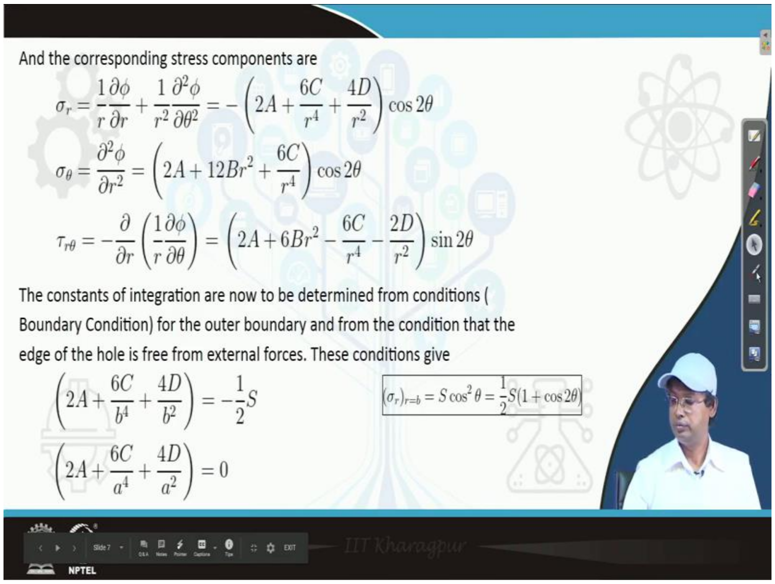

So what we need to do is that and the corresponding stress components sigma r since we have got that stress functions with constants A B C D if we use the stress function expressions with to find out the normal sigma r and sigma theta and tau r theta we get these components as minus of 2A + 6C by r 4 to the power 4 4D by r square cos 2 theta sigma theta is in this function these are the fr portions and then we have this form also.

And what do we have after that we need to substitute the boundary conditions so if we go into substitution of the boundary conditions. The constants of integrations are now to be determined from boundary conditions for the outer boundary and from the conditions that the edge of the hole is free from external forces this condition gives that the considering this part only what we have is this minus has come in the other side.

And for the outer boundary at this is at b and it is 0 sigma r is zero and the inner boundary and we put that is equals to 0 considering this equations we have these 2 boundary conditions and if we go for the other equations with tau r theta if we put that again we have 2 equations.

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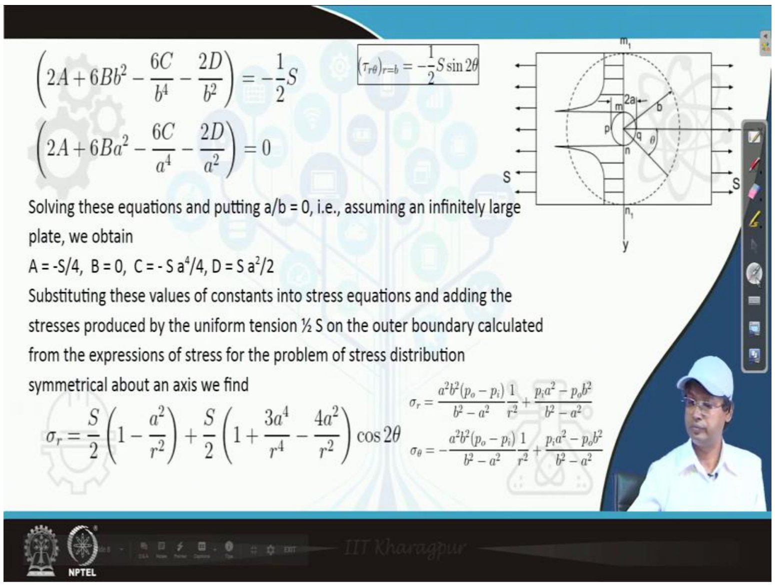

So total we have 4 equations and if we solve those 4 equations this is the other boundary condition just show shown here solving these equations and putting a by b equals to 0 that is assuming in finitely large plate we obtain that A is equals to minus S by 4 B is equals to 0 C is equals to minus S a to the power 4 by 4 and D is equals to S a square by 2 substituting these values of constant into the stress equations.

And adding the stresses produced by the uniform tension half S on the outer boundary calculated from the expressions of the stress for the problem of stress distribution symmetrical about an axis we find this is for stress distribution symmetrical about an axis and here we are supposed to they are p and p forces were compressive here it is intense type is tensile. So keeping in mind that the sign change we need to consider we also need to consider that a by b is equals to 0.

So if we take common from this side the b, b gets cancelled and accordingly we get we remain the a the r square remains this portion becomes 0 no not this portion becomes 0 this is also p p inner is equals to 0 p i is 0 here also p i is equals to 0 and p outer is supposed to be minus of S by 2 putting those values what do we get that sigma r is equals to S by 2 1 - S square by r square + S by 2 1 + 3S square + r square – 4S square by r square cos 2 theta.

So this is because of the axis symmetric part what we have solved earlier and this part is from this up to this process whatever we have solved. So this becomes a linear combination of both the cases and then we have an expressions for sigma r.

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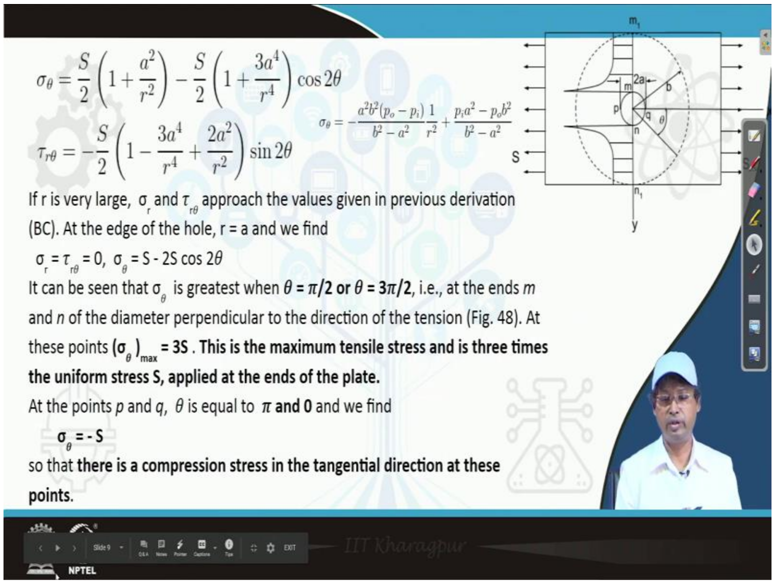

Similarly if we put the values and we know the constants for sigma theta if we consider we have the value for sigma theta and also the value for tau r theta. So finally we have all the expressions for sigma theta tau r theta and sigma r sorry sigma r sigma theta and tau r theta around this hole. And what does this what does this drawing what is shown here represents that we will try to explain now and see how it is do we get that portion

So if we look into this if r is very large sigma r and tau r theta approaches the value given in the previous derivation or in the boundary condition at the edge of the hole r equals to a we find that sigma r and tau r theta is equals to 0. If we substitute the value r equals to a in the sigma r expression which is not brought here we can easily see that so that becomes that sigma r and tau r theta equals to is 0.

But sigma theta is not 0 sigma theta makes an expression something like this S – 2S cos 2 theta and that is a very, very significant equations and this gives us a distribution something like this. So it can be seen that sigma theta is greatest when theta is equals to pi by 2 or theta is equals to 3 pi by 2. So where it is this is at this point and this is or the other way we are measuring theta in this way so if I say this is theta 1 this is position 1.

If we say this is theta 2 this is position 2 so with this we see that these are the 2 points where sigma theta is maximum which way sigma theta is acting in this particular case if we consider this portion consider some element here sigma r is acting in this directions sigma theta is acting sigma r this is and sigma theta is acting in this direction. So if this is acting in this direction what is the value in this particular point at the end m m and n of the diameter perpendicular to the direction of the tension.

This is maybe is not for may be darkened please note that this portion to be but how to do excuse me **(FL: 21:10 to 21:42)** so these are at this point as we have seen the stresses sigma r and sigma theta is acting this way this is sigma theta and that value what we get is that this is become minus we go becomes plus in this particular value. And we have this is equals to 3 S that means whatever the stress applied to the plate this particular portion the element whatever we see this element is experiencing sigma theta is equals to 3S 3 times the S.

So and it is in tensile nature it is in opening nature if you look at it if we consider this portion it is opening in nature. So this is the maximum tensile stress and is in is 3 times the uniform stress S applied at the ends of the plate at the point p and q theta is equals to pi and 0 this if we look at instead we have –S. So if we consider some element here if we consider say here some element in this particular case this is sigma theta and this is sigma r and this sigma theta is equals to –S.

So this part is m n is in compression tension m and n in tension tensile stress and p and q in compressive stress. So this is very, very important point 2 note you please also note and now if we apply the values in sigma theta and if we increase the other values increase the value of r say not equals to a say little bit more than a if we continue putting we along this direction along this line we have the sigma theta distribution following this parameter this profile.

So this is a very good point to note though it is of 3S it reduces very quickly as we have learned following the Saint Venants principle there is a and that distribution is something like this. So if it is under tension with s stress at these 2 points it is experiencing tensile stress of magnitude 3S and at this 2 point it is experiencing compressive stress of magnitude S. So with this note with some considerations or a few discussions we will come to the next lecture.

We will see how things are improved for service and accordingly we will go to the next problem to solve torsion related things. So with that note we come to the end of the lecture today.

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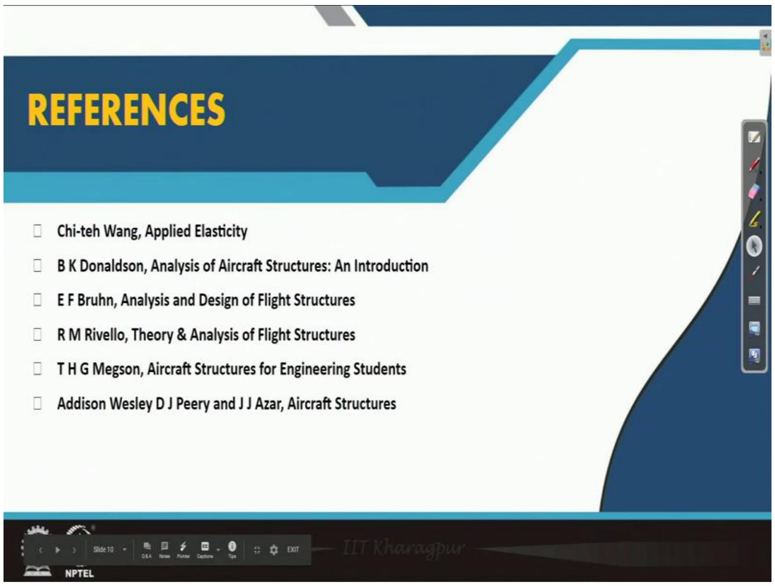

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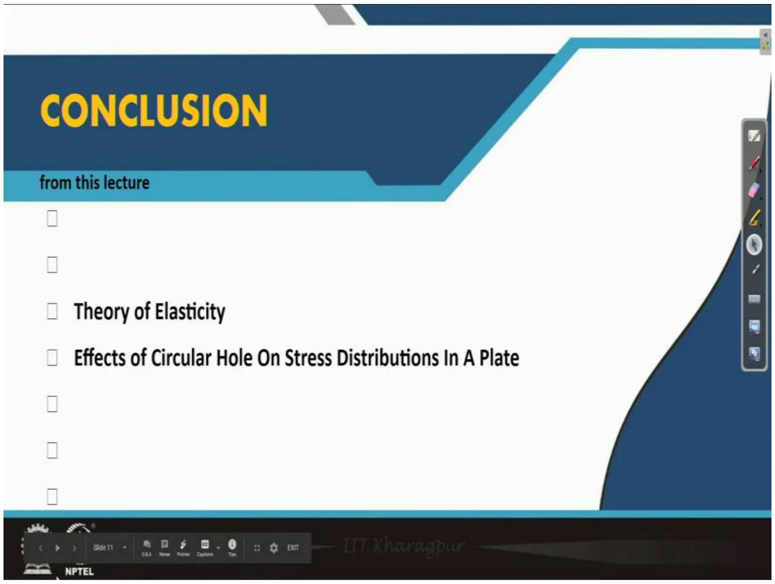

Reference is a standard reference so what we have learnt that the distribution of stress, stress distribution on the circumference of circular hole and in this process very, very important phenomena with inside how the stress varies we have learnt and with that note we come to the end of today's lecture. So thank you for attending this lecture we will meet again with some more lectures, thank you.