Aircraft Structures - 1
Prof. Anup Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

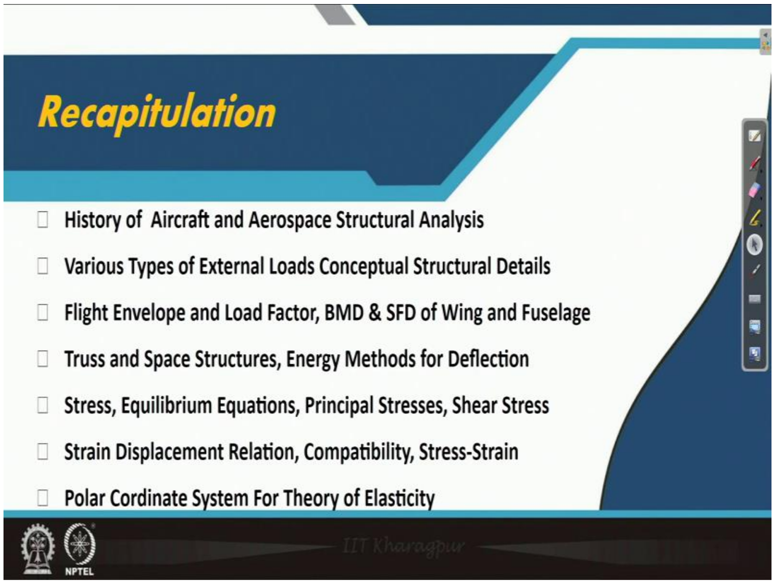
Lecture No -35
Compatibility Condition in Polar Coordinate System

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are in the 7th week lectures or in the module 7 lectures in sequence this is the 35th lecture. Today we will start the effect of circular hole on stress distribution in a plate that is very important it gives us the idea of stress concentration. How stress varies around a hole under a uniform tension.

And to analyze this; what we have done so far is that in the last two lectures we have derived equations of equilibrium as well as compatibility in terms of stresses in polar coordinate system. We have also derived or expressed or changed the coordinate from Cartesian to polar in case of stress components like sigma theta sigma r or tau r theta. Now we will be using those to find out the stress distribution due to a hole in a plate while the plate is under uniform tension.

We will consider initially only the tension maybe later in some lecture we will cover about other cases or critical cases that we will discuss.

(Refer Slide Time: 02:19)



But with this idea let us start but before we go into today's topic it is time to recapitulate or to remember back the things what we have already covered. In this relation we have covered the theory of elasticity or solid mechanics history. We have also seen development of aircraft starting from Wright Brothers Kitty Hawk flight then we have seen various types of external load loads conceptual structural details.

How the ribs sparse skins come in a aircraft wing we have seen how transverse frames are put in aircraft fuselage. How longrons are also put in aircraft fuselage how the deck is put in the aircraft fuselage how it is divided in separate compartments with different structural parts. We have seen landing gear also we have seen other components like tail plane. We have seen later the bending moment shear force distribution for the overall wing and fuselage.

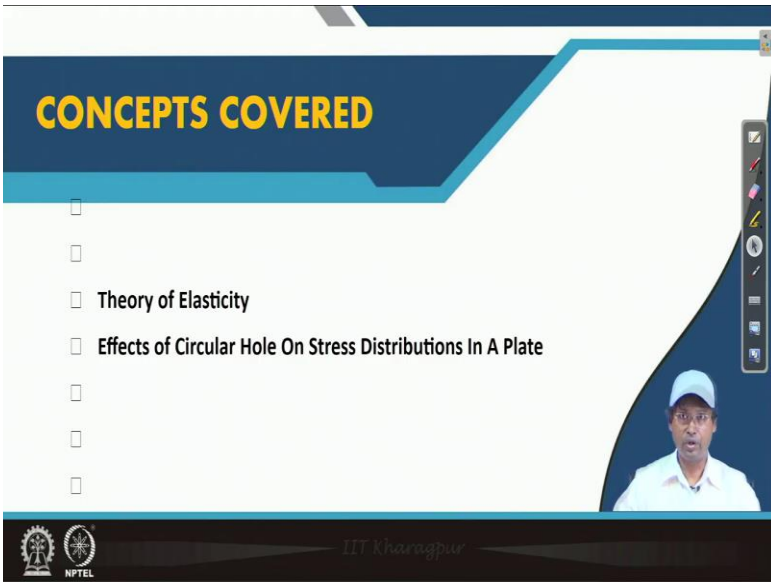
Then landing gear analysis we have done from structures point of view considering those as a three dimensional truss and then we have come to the theory of elasticity. Before we come to the theory of elasticity we have also seen or found out deflections are important in any structure. So deflections to find out deflection we have gone across different methods starting from strain energy complementary strain energy.

And then total potential energy then Castiglione's theorem from their unit load method dummy load method and at the end very, very important method that is the Rayleigh method. So thereafter we have come across with theory of elasticity. Theory of elasticity is important to learn in various ways. If you go further for different understanding in depth the type of problem say we will be solving today or in this week.

That will give us insight in development of stresses around a whole similar way unless we learn theory of elasticity it is difficult to cover all the cases to predict critical conditions. So keeping in mind those things theory of elasticity is taught. So in that theory of elasticity where we need to find out 15 unknowns like 6 component of stresses 6 component of strains and 3 components of displacements uvw.

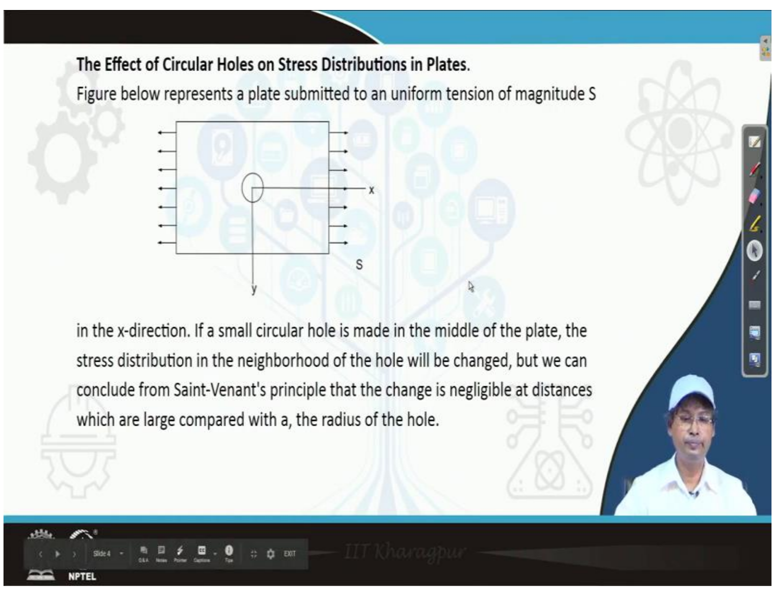
So those say 15 quantities what we need for structural analysis purpose has to have 15 equations those equations we have derived. We have talked about compatibility also compatibility is always important it holds a separate place in the total analysis. So we have also seen conversion of strain compatibility to stress complement compatibility to bi-harmonic equations and through that we have come across to the point where we have some knowledge to analyze effect of a circular hole in a plate which is under in plane tension.

(Refer Slide Time: 06:47)



So with that note let us proceed for today's lecture.

(Refer Slide Time: 06:50)



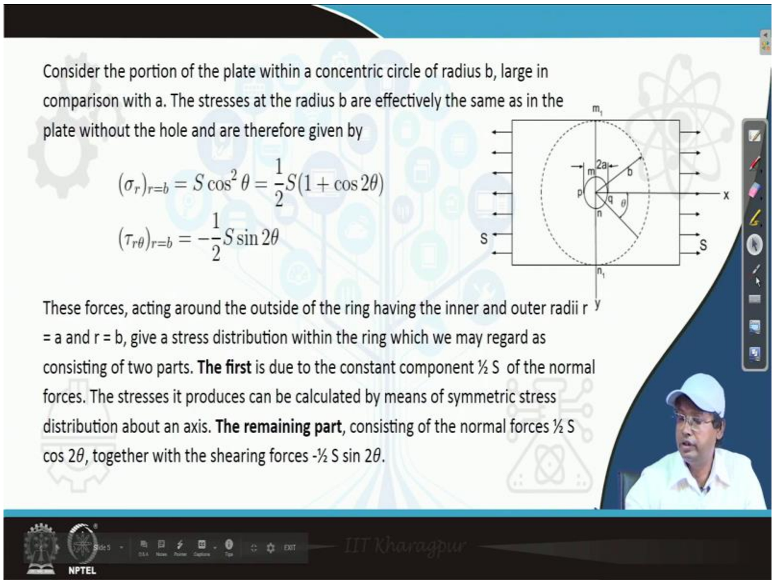
The effect of circular holes on stress distribution in a in plates is interesting problem figure below this is what we idealize this is the way we idealize figure below represents a plate submitted to an uniform tension of magnitude S it is in plane tension in plane means the forces are acting at the plane of the plate and it is of S amplitude magnitude in the x direction. If a small circular hole as it is shown here is made in the middle of the plate the stress distribution in the neighborhood of the hole will be changed.

So how is it going to change we can easily say it is going to change because there is a hole definitely it is not continuous. So it is something will happen if we think the stresses something like on lines parallel lines and then because of the whole the lines will not get discontinuous something will happen it will may disperse in some direction and do so that we are supposed to find out. But we can conclude from Saint Venant's principle that the change is negligible at a distance which are large compared with a, a is the radius of the hole.

The radius of the hole so its Saint Venants principle already we have learned we have not gone through the proof but it can be proved in higher elasticity classes, theory of elasticity classes that is not proved. So with that respect we are considering that if it is the dimension position is more than a few times a then it remains same. So with that concept so the changes whatever will be in the vicinity of the circle may be around in this region.

What is the change it is coming so while we consider this is as a. So let us see what are the other changes may come up and we can continue.

(Refer Slide Time: 09:32)



So with this we do some idealization of the problem and that problem let us see how do you idealize it is same stress stresses as is shown here. We are imagining that the dimension of the hole is twice a as it is shown. And we also imagine some circle which is of diameter b sorry radius b and b is considerably large in comparison to a. We name the whole points this is n this is m.

We also name the other diametrical point as p q we consider if any point which is at angle theta. So x is acting this way y is acting in this direction. So with that concept consider the portion of the plate with in a within a concentric circle of radius b large in comparison with a that is what we said this dimension b is large in comparison to a. The stresses at the radius b are effectively the same as in the plate without the hole and r therefore given by this.

So what we are considering here with this expression that if there is a plate without a hole what are the stresses to be? We are saying that the stresses to be the sigma r in the radial direction is half of S cos 2 theta component. So it is something like if we consider a plane here and if we consider this as the sigma r and if it is theta degree in angle instead of considering there if we consider here it will be easier to imagine.

If this is sigma r theta is the angle then we say this is equals to half of S + S cos t 2 theta by 2 this you can easily get from the two dimensional stress stress transformation what we have learned if you put the values only the sigma x you will get this value of sigma r there. And the shear stress tau r theta in this particular plane the shear stress tau r theta is minus of half sine 2 theta. So it is said that beyond this circle whatever the circle we see the stresses are in this form sigma r and tau r theta.

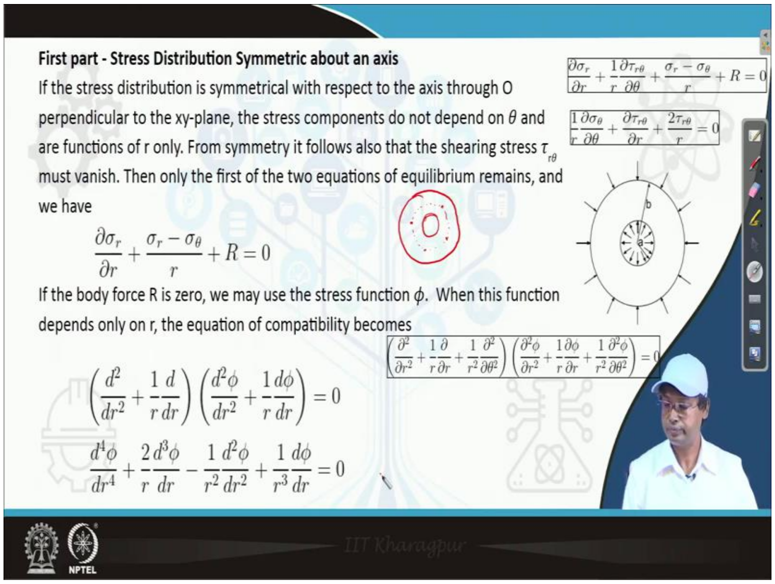
So with that note let us proceed these forces acting around the outside of the ring having the inner and outer radius r equals to a and r equals to b give a stress distribution within the ring which we may regard as consisting of two parts. So what is coming down now? Now instead of considering these stresses what we are considering that this annular ring is under this stress distribution stress distribution.

So if it is under this stress distribution what we can again subdivide the problem in this way the first part the first is due to the constant component of half S this is the half S of normal forces. The normal forces the stress stresses it produce can be calculated by means of symmetric stress distribution about an axis. So what it says that if this is the case then if we can imagine some problem where stresses are acting in this form.

And this stress value is S by 2 then we can and if we are able to find out the distribution of stresses inside this body that is what the problem we want to solve. So, one part of the problem is this. The stresses it produce can be calculated by means of symmetric stress division about an axis so this is symmetric about an axis means the axis perpendicular to this or the z axis.

The remaining part consisting of the normal forces half S cos two theta that means this and this multiplied together with the shearing stress minus half sine two theta will have to also find and will have to add it up considering linear superposition of the stresses developed by two different cases. So the problem gets divided in two parts. So let us see that is the way we will solve the problem in one part we will consider this and the other part we will consider that there is a sigma r consisting of this part and everywhere there is a tau also with this formula.

(Refer Slide Time: 15:59)



So what do we have? The first part as i said the access symmetric case where axis of symmetry is z sigma r we are considering as S by 2 that part we will try to solve. Now the stress distribution symmetric about an axis if the stress distribution. If the stress distribution is symmetrical with respect to the axis through O perpendicular to the xy plane the stress components do not depend on theta and are functions of r only.

So it is quite obvious that if it is symmetric it is not dependent on theta it becomes a function of r only. So we consider that we think that if this is the case we are talking about then whatever the stress here if it is some distance same distance apart from the center it is same here it is same at this place or at this place. So it is only dependent on r not on theta for from symmetry it follows also that the shear shearing stress tau r theta must vanish.

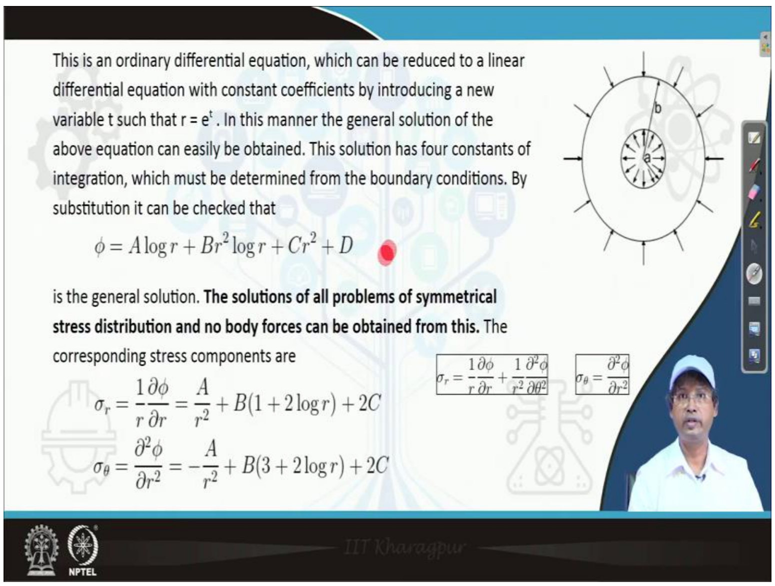
Since it is a symmetric force so there would be any tau r theta then only the first of the 2 equations of the equilibrium remains and we have these are the 2 equilibrium equations just to remember those I have put back again. So since as it is says it is independent of theta this term is not there and since tau r theta is not present this equation completely vanish. So the only remaining equation that to where we do not have any theta dependent term.

If we discard that part that remains this part this part and this part that is what is written here. If the body force R is 0 we may use the stress function phi when this function depends only on r.

The equations of compatibility becomes this is the original equations of compatibility what in polar coordinate we have derived already but since as we have already mentioned theta dependent terms are not there it reduces back to this.

And phi what we have considered as a stress function if we put that it becomes the compatibility equations after this multiplication and derivation accordingly done this becomes the equations of compatibility in terms of stress or in terms of stress function.

(Refer Slide Time: 19:43)



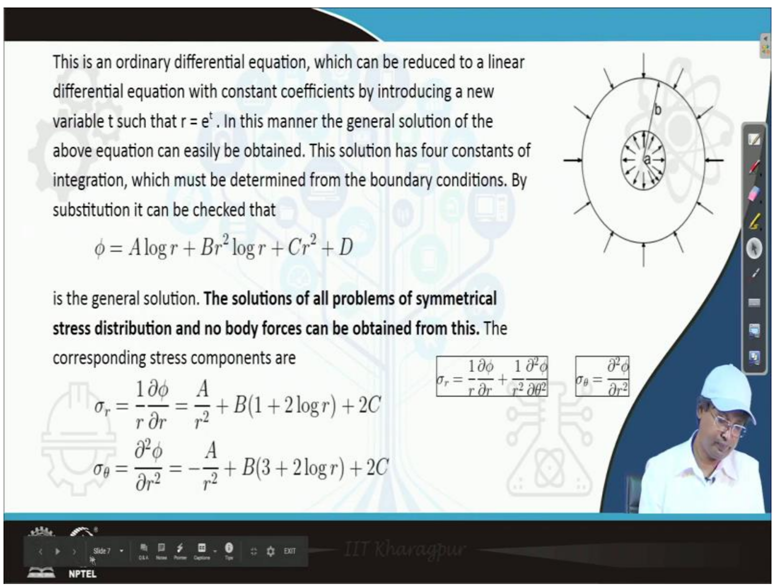
This is an ordinary differential equation which can be reduced to a linear differential equation with constant coefficients by introducing a new variable t such that r is equals to e to the power t. In this manner the general solution of the above equation can easily be obtained this solution has 4 constants of integration which must be determined from the boundary conditions by substituting it can be checked that phi is equals to this is a general solutions.

This requires lot of experience to find out this you need mathematics knowledge of good mathematical knowledge to do it we have skipped that part. So we are considering that this holds this satisfies the previous bi harmonic equation in terms of polar coordinate applicable for a symmetric axis symmetric stress distribution and we have checked people have checked scientists physicists have checked and said that holds.

The solution of all problems of symmetrical stress distribution and no body forces can be obtained from this. This corresponding stress distributions are what we can say that since already we have derived the stress functions we have converted the stress functions and stress relations which are sigma r equals to 1 by r del phi del r +1 by r square del 2 phi del theta square sigma theta is equals to del 2 phi del r 2.

So if we substitute this value in this form of equations what we have we have these two components. So once we have these 2 components these 2 equations what we can think of we can think of putting boundary conditions. So to find out the constants A B C and D is vanished because it is not having any component with respect to r or theta. So let us move to the next slide.

(Refer Slide Time: 22:00)



So it is repeated back those two slides those two equations and if there is no hole at the origin of the coordinates constants A and B vanish since otherwise the stress components become infinite when r is equals to 0. So if you look back here if r is becoming 0 this component is definitely going to in finite and that is the reason we need to discard that part and for the case while we do not have any hole at the center.

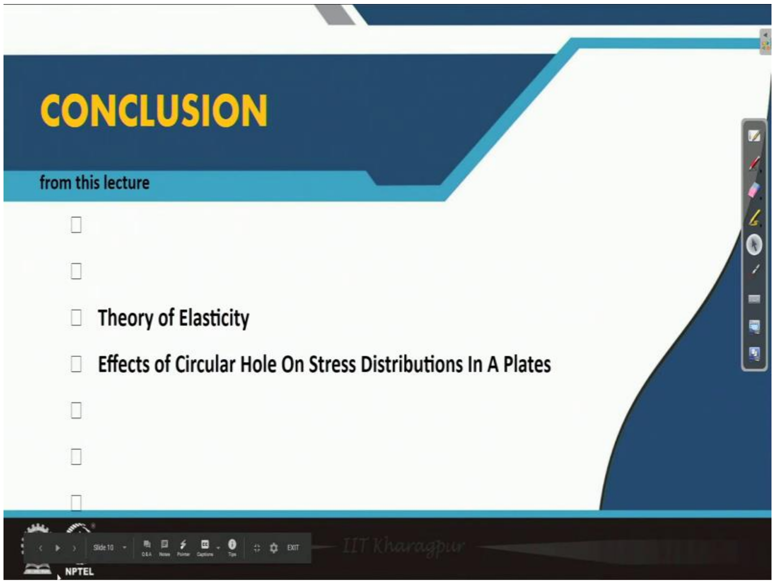
But it is a axis symmetric crest which satisfies this equation so we hence for a plate without a hole at the origin and with no body forces only one case of stress distribution symmetrical with respect to the axis may exist namely that when sigma r is equals to sigma theta and is equals to constant so both a b vanishes as we said if we look at it carefully then only it is equals to constant or equals to two c and the plate is in a condition of uniform tension or uniform comp compression in all direction in its plane.

So either it is in the; depending on the type of stress acted upon on that particular case which is not this case but a case where we do not have any hole. So in this particular case what we can see is that if there are stresses something like this in that case it is sigma r is equals to sigma theta and it is equals to some constant here it is 2C. So it if it changes it accordingly we need to find out that value. So with that note we would like to come to the end of today's this lecture.

(Refer Slide Time: 24:36)

Particular lecture and we will use these equations in the next lecture with boundary condition otherwise it becomes a bit discontinuous portion that is the result we have taken this way and we will find out the stresses in the hole. So with that note we come to the end of this.

(Refer Slide Time: 25:02)



And we will proceed further in the next lecture thank you for attending this lecture.