Aircraft Structures - 1
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Lecture No -34
Compatibility Condition in Polar Coordinate System

Welcome back to aircraft structures 1 this is Professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are in the middle of the 7th week lectures or the module 7 in sequence it this is 34 lectures. In the last class we have learned in polar coordinate system the equilibrium equation in this lecture we will be covering the compatibility condition in polar coordinate system and we will discuss a little bit of transformation of those stress function expressions first in terms of to find out stresses.

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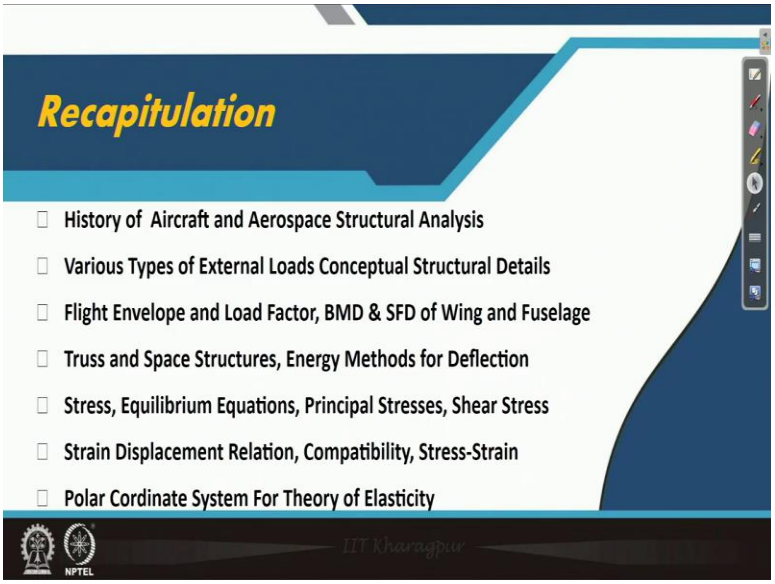

So with that note let us come to the recapitulation slide among the recapitulation slide in the last lecture we have we have recapitulated history of aircraft various types of external loads in detail flight envelope to some extent we have detail we have recapitulated. Flight envelope let us starts with that again flight envelope. Flight envelope is the envelope within which a particular type of aircraft is supposed to maintain its flight.

And flight envelope is important for structures or design because whenever it flies it experiences inertia loads more than 1g more than acceleration involved 1g. So since it is very useful by any aircraft to crush 2.5 or something about come to about 3g forces so it is better to maintain a an envelope or prescribe an envelop within which the design should persist or should remain. So keeping in mind those parameters what we have discussed is that flight envelope and the load factor.

Load factor is how many times of g forces g means the gravitation force of earth is encountered by any structure that we need to discuss and we have seen in which condition which load factor becomes more. And in this correlation we have discussed problems related to bending moment and shear force diagram of overall wing and fuselage. We have come across the unit load method unit load method is nothing but considering unit g not only sometimes unit g sometimes unit force is also considered.

So that any multiplication of force or any multiplication of g factor can easily give us the desired forces or bending moment and shear force encountered by wing and fuselage. So but it is important to analyze bending moment and shear force of wing and fuselage that is the reason we have learned about those wing, wing and phase large bending moment and shear force diagram. We have learned three dimensional structures three dimensional structures are space structures are important in terms of aircraft structures there are many applications 1 of the most prominent application is landing gear.

And in case of landing gear we use in general the truss concept or axially loaded member concept. So considering a landing gear as three dimensional truss what are the deflection and what are the loads coming to the components of a truss we have solved we have solved specific problems we have seen how it is solved and done. We are also introduced to some extent components of trusses like the oleostart member or torsion links.

So after that we have gone to the displacements or energy methods energy methods and displacements where it is mentioned energy methods involving complementary energy method involving Castiglione’s theorem involving unit load method dummy load method. So unit load method dummy load method Castiglione’s theorem are how those are similar to each other and how efficiently we can use to solve problems.

How can we even solve indeterminate problems using those methods that we have seen not only that we are introduced to a fundamental process of approximate analysis based on energy principles that is the Rayleigh method? We got introduced how approximation is considered there and how approximate good it is depending on the initial assumptions of the displacement profile that we have seen with examples.

And after that we have come across to the stresses or theory of elasticity approach. Theory of elasticity approach is important in the sense because we specifically encounter different types of types of problems which leads to fatal accident 1 of the important thing is stress concentration around the whole that we are in the process of discussion. And we can see from internet that there was an aircraft designed with almost rectangular windows.

And that rectangular window led to catastrophic failure of the fuselage. So we need to see why those are important to study. So unless we learn the theory of elasticity approach unless we look into the in depth about the behavior of stress development and strain development and displacement it is difficult to predict problems so we that is keeping in mind those things.

Not only those things we need the fundamental development of any numerical method what we are popular like the finite element method is based on this elasticity theory of elasticity and energy methods so unless we learn all these methods very, very efficient way we will not be able to learn the further topics that is the reason theory of elasticity is introduced. And in last two weeks we have we have considered Cartesian coordinate system.

We have found out stresses equilibrium equations principle structures shear stresses stress strain strain displacement relation compatibility strain stress strain relations all those and the last lecture we got introduced to the polar coordinate system for theory of elasticity because we our aim in this week is to discuss the stress distribution around a hole due to a simple tension in a plate.

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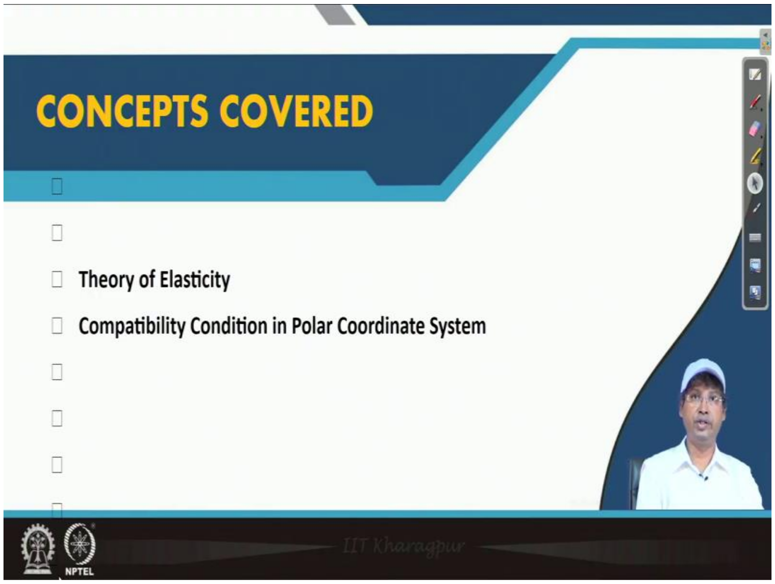

So in following that line what we will do we will learn in this lecture the compatibility condition in polar coordinate.

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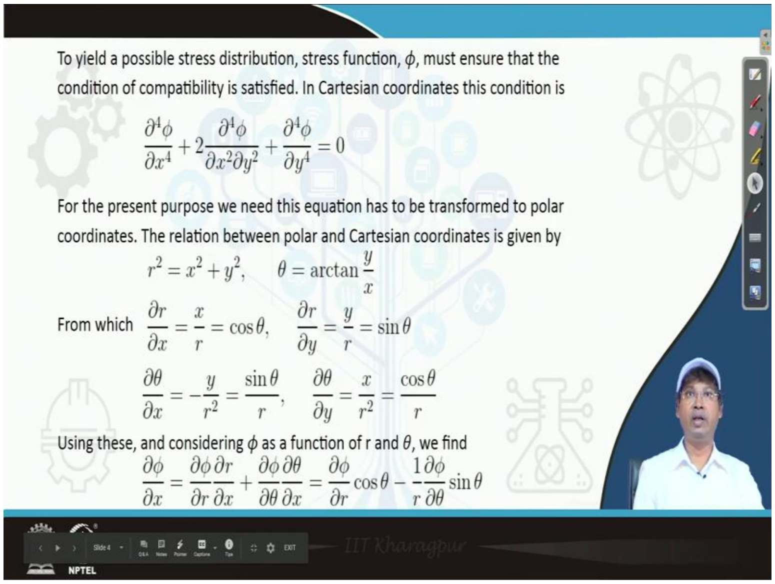

This is again a kind of simple mathematical derivation it there you do not have much concept of elasticity or structures its simple mathematical approach. Let us see how it is done. So to yield a possible stress distribution stress function phi must ensure that the condition of compatibility is satisfied this is the condition of compatibility in Cartesian coordinates this condition is as stated here and sometimes we write this as grad 4 phi is equals to 0.

So with this we proceed further for present purpose we need this equation to be transferred to polar coordinates the relation between polar and Cartesian coordinate is given by r square equals to x square + y square its quite obvious and theta is equals to tan inverse y by x or r tan y by x this is quite a I think you can easily do it. It is not big issue this is r so that is the way it is done for which we have del r del x just simple from this what we have is x by r.

And if we that is equals to cos theta del r del y is equals to sin theta. Now del theta del x if we use this 1 we have minus y by r square and that can easily be stated as sin theta by r. And similar way del theta I think a minus is missing here so please put that del theta del y is equals to x by r square cos theta by r using this and considering phi as a function of r and theta we find that it is del t phi del x is equals to simple series way it is done del phi del r multiplied by del r del x.

Similarly del phi del theta multiplied by del theta del x and then we substitute these values we have del phi del r cos theta - del phi 1 by r del phi del theta sin theta. So minus is because of this it is missing here please note that. So with that we move forward to the next slide.

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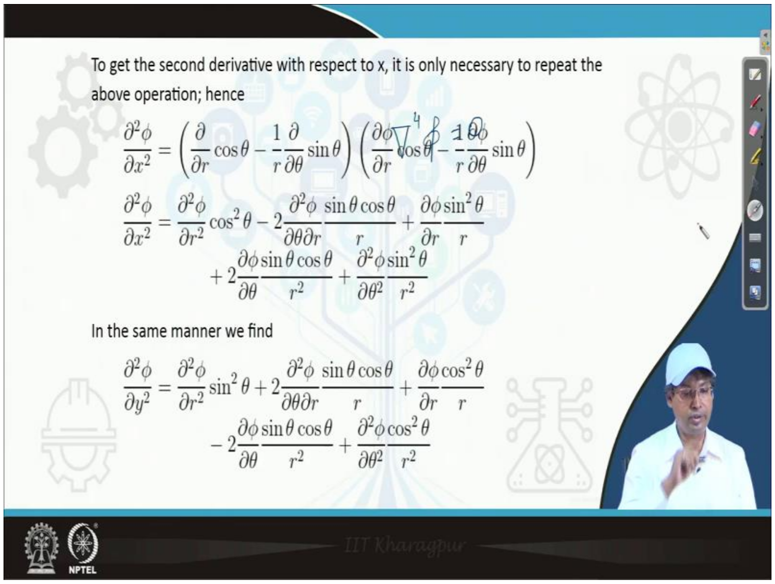

In this slide what we have to get the second derivative with respect to x it is only necessary to repeat I think I need to clean more to get the second derivative with respect to x it is only necessary to repeat the above operation hence del 2 phi del x square is nothing but multiplication of those two. Here please note in mathematical way how it is written and if we carry out that multiplication we have how many terms 1 2 3 4 5 terms and those 5 terms are because of this rr is involved here those and other terms will come and we need to simplify those things. I would suggest you carry out that.

So it is finally we get del 2 phi del r square cos square theta - 2 del 2 phi del theta del r sin theta cos theta by r del phi del r sin square theta by r + 2 del phi del theta sin theta cos theta by r del 2 phi del theta square sin square theta r square. So these are the similar term and the similar way if we if we find out del two phi del y square we get a similar expression with five terms. But please note that this term and this term is simply opposite in sign this term and this term are also opposite in sign.

These two if it is added will man is the theta component similarly this two if we add will vanish this theta component sin square theta plus cos square theta is becoming equals to 1 it is similar to these two terminals these two term this two term. So finally what we have we have this 1 by r this and 1 by r square.

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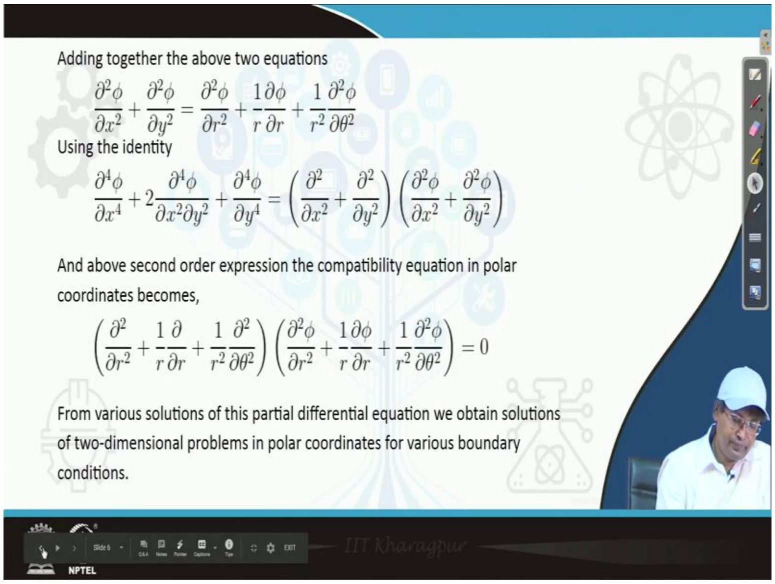

Those three terms as it is mentioned we have grad square phi but we need the 4th 1 knows by harmonic equation we need. So to in that sense using the identity it is nothing but this is equals to this multiplied by this where this is the derivative multiplication the way it is written. And it is not carried out it is better not to carried out at this position because if we carry it out it becomes lengthy it becomes difficult to handle and we probably do not need that way that is the reason it is not carried out and simple way this term is written here as the bionic form in polar coordinate system.

And above second order expression the compatibility equation in polar coordinate system becomes this, this is here and the other as parameter. Form it is written from various solutions of this partial differential equation we obtain solutions of two dimensional problems in polar coordinates for various boundary conditions. So equilibrium equations and compatibility equation in terms of stresses compatibility equations in terms of stresses or stress function is evaluated.

So we need with using this we can attempt to solve problems and in this week we will try to solve problem with respect to a problem which is a whole circular hole in a plate. We will see that problem how the problem simplified way we can discuss in this slide to some extent. A plate if we consider this way and if it is under uniform stress is and if we consider 1 circular hole at the middle what is the distribution of stress at this point what is the distribution of stress at this point.

Whether those points are having same amount of stress experienced or not that is a very interesting point interesting problem to discuss. So with that note we will move to the next slide where we will again find out the stress function quantity in a different form in a different way.

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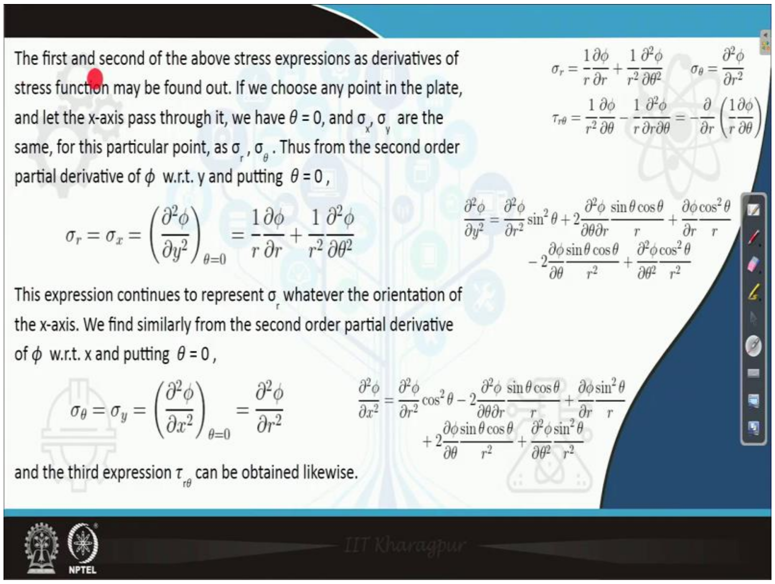

So the second the first and second of the above stress expressions as derivatives of stress function may be found out now because already now we have the coordinates transformation equations from Cartesian to the polar we have. So we can easily check those. If we choose any point in the plate and let the x axis passes through it we have theta is equals to 0 and sigma x and sigma y are the same.

For this particular point as sigma r and sigma theta it is also similar thus for the second order partial derivative of phi with respect to y and putting theta equals to 0 we can have an expression of sigma r and sigma theta and that is what is done here sigma r is equals to sigma x at theta equals to 0. So that is was del 2 phi del y square what is we have learnt in ins area stress function definition if you remember but this transformation was not done earlier in the last lecture that is the reason it was not given.

We simply stated this so this equation what we can do now we are putting using this value and putting that theta is equals to 0. So if we put theta equals to 0 what will happen this is 0 this is 0 this is 0 but this is 1 this is 1 right so we will have only these two components that is what 1 by r del phi del r and 1 by r square del two phi del theta two is present here similarly this x expression continues to represent sigma r whatever the orientation of x axis.

We find similarly from the second order partial derivative of phi with respect to x and putting theta equals to 0 the expression for sigma theta and in that expression its similar way theta if we put it its similar way what we get is that this this expression only because all other terms are involving sin, sin means it is leading to 0 so we have finally this. So with this small derivation note we will proceed further and if we follow similar approach we can have the expression for tau r theta tau r theta is this as it is written here we can easily find out.

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So with as I said with this note of derivation we come to the end of today's lecture this is the standard slide of reference.

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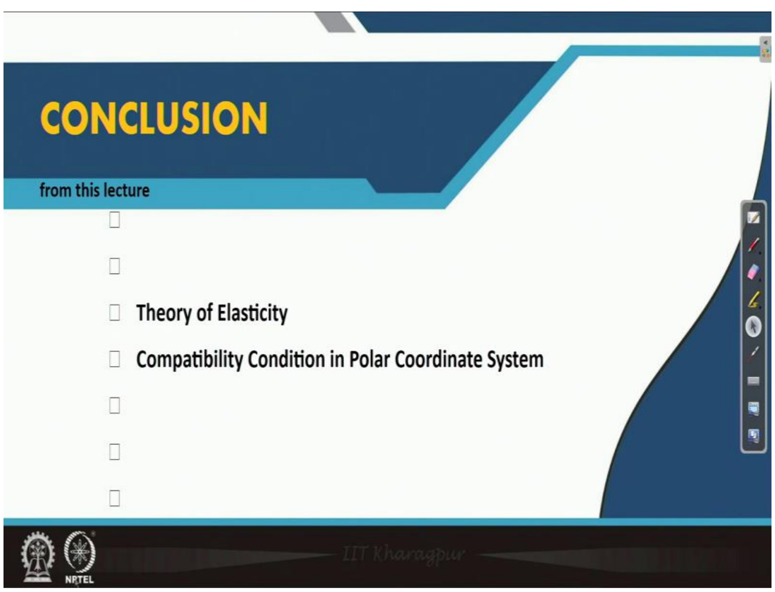

Conclusion is that we have learned compatibility condition in polar coordinate system and we will learn further polar coordinate to consider problems in the sense of finding out stress distribution around a hole and with that node we come to the end of today's lecture. Next lecture we will start the problem of a circular hole in a plate which is under uniform tension, thank you.