Aircraft Structures - 1

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Lecture No -32
Semi-Inverse Method of Solution

Welcome back to aircraft structures 1 this is a Professor Anup Ghosh from Aerospace Engineering, IIT, Kharagpur. We are in the lectures of 6th week or the module 6 in sequence this is the 32 lecture. And this is involved with the semi inverse method of solution semi inverse method of solution in terms of theory of elasticity approach to solve problems and in that sequence we will learn how do we solve problem with a typical example.

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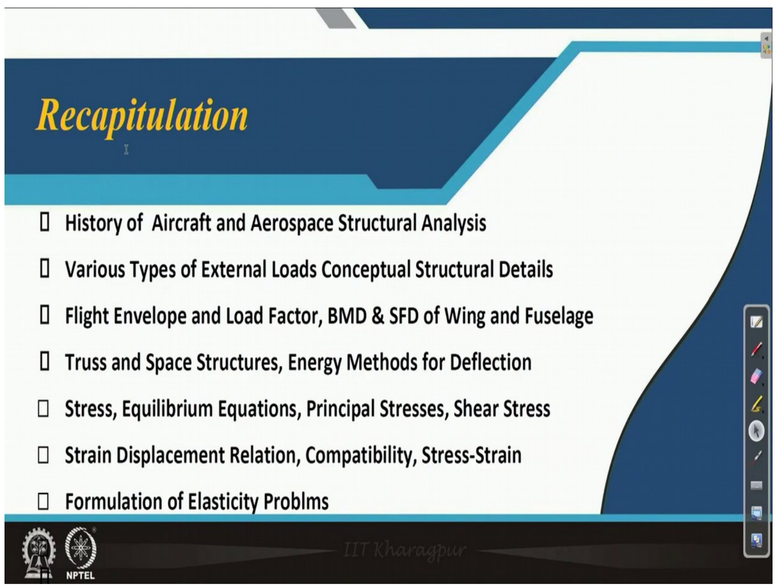

Now in that sequence we always in every lecture i try to cover what we have learnt. So far this time I will try to cover it in more brief as quickly as I can. History aircraft development from a small 1 At Kitty Hawk by Wright brothers to the huge 1 like the AN225 we have learned history of solid mechanics or structural analysis or the way you see. Then various types of loads encountered or experienced by aircraft structure is discussed where how those loads comes.

And then we have learned the inertia loads play a huge role in design and that in that context we have learned the flight envelope and flight envelope also varies with respect to a particular type of aircraft depending upon its service condition. We have learned how can we find out from the overall external load the bending moment and shear force is encountered by wing and fuselage. Then we have learned three dimensional structures we have solved a few problem in three dimensional structures with respect to landing gear problems.

And then we have started the deflections energy methods in that we have learned Castiglione's method total potential energy method complementary energy method and we have learned a very, very good method what I say that probably less the fundamental foundation of future numerical analysis that is the Rayleigh's method. To some extent in that discussion came the variation variational calculus not that way but anyway you are introduced maybe.

So and then the theory of elasticity got introduced to stress then equilibrium with body forces, surface forces, stress transformation, concept of stress transformation our total discussion was predominantly in the Cartesian coordinate system. Then from there the principle stresses how do we find a plane where there is no shear stress. And from there we have discussed that what are the properties of principle stresses.

How invariant is observed? Then we have established strain displacement relation then compatibility, compatibility combination how it is important without that solution is not practical we need to satisfy the compatibility condition or equations. Then stress strain relations we have found out then in the last class last lecture we have covered the way how a problem is formulated and then solved.

In that method we have already solved 2 three problems with respect to the inverse method in the inverse method we directly assume the stress function it is stress function and then we try to put the bound find out we try to find out the stresses components of stresses and then accordingly putting the boundary sorry putting the constants to 0 or modifying the constants we can achieve some problem which represents the stress functions properly.

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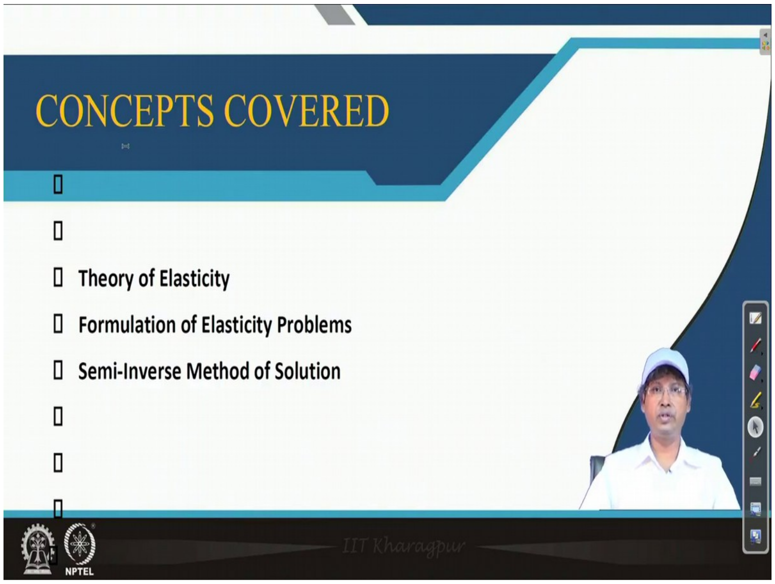

So with that note we will try to solve problems in theory of elasticity with semi inverse method. It is almost similar we will use some conclusions like the pure bending portion what we have learned in the semi inverse method part of the equation that we will be using to solve this particular a particular example.

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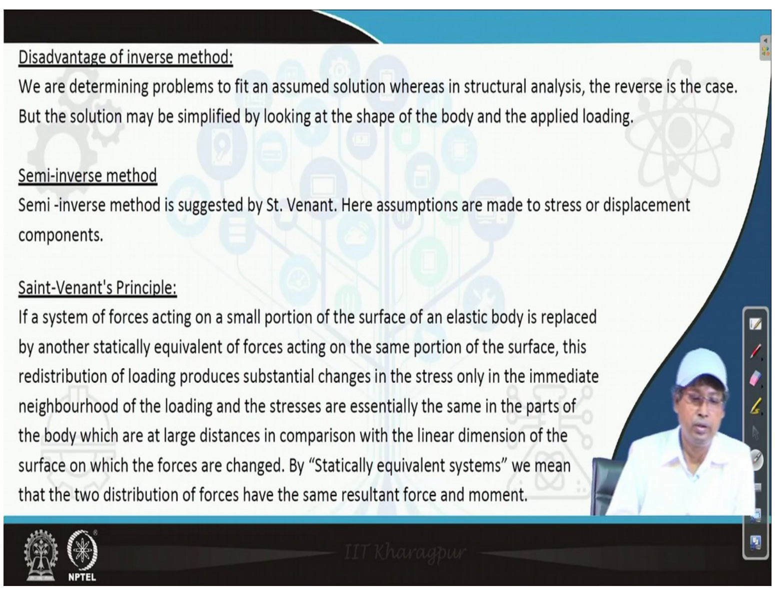

So, before we go for the solution of the particular example let us get introduced to the method. So in this process what we do so disadvantage of inverse method we are determining problems to fit and assume solution whereas in structural analysis the reverse is the case. So that is quite obvious but the solution may be simplified by looking at the shape of the boundary and the applied loading. Semi inverse method is popular in that sense it is suggested by Saint Venant.

Semi inverse method is suggested by St. Venant here assumptions are made to stress or displacement components but before we go since the same benefits St. Venant principle is better to come across once this is important this lines are important you please I will read I will try to explain with 1 example in the next slide I will try to explain. I will tell you very, very simplified words also that would not include all these but those are not totally correct.

Simplification always sacrifices something anyway if a system of forces acting on a small portion of a surface of an elastic body is replaced by another statically equivalent of force acting on the same portion of the surface this redistribution of loading process produces substantial changes in the stress only in the intimate neighborhood. It changes substantially but in the intimate neighborhood of the loading. And the stresses are essentially the same in the parts of the body which are at large distance in comparison with the linear dimension of surface on which the forces are changed, linear dimension of the surface on which the forces are changed.

So it says that if I replace by equivalent or statically equivalent force system at the vicinity it is not the same case but at some distance it is same and that some distance is what it is that distance is governed by this linear dimension of the surface on which the forces are changed that is a principle is really very, very important observation and noted by Saint Venant this helps us a lot in the next page we will see how it helps us.

But without looking into this thing we have considered this in our mind and we have solved problems already in your mechanics in your other say here also whatever we have covered from during problem solving we have assumed this. So by statically equivalent systems we mean that the 2 distribution of forces have the same resultant force and moment. So let us see the example.

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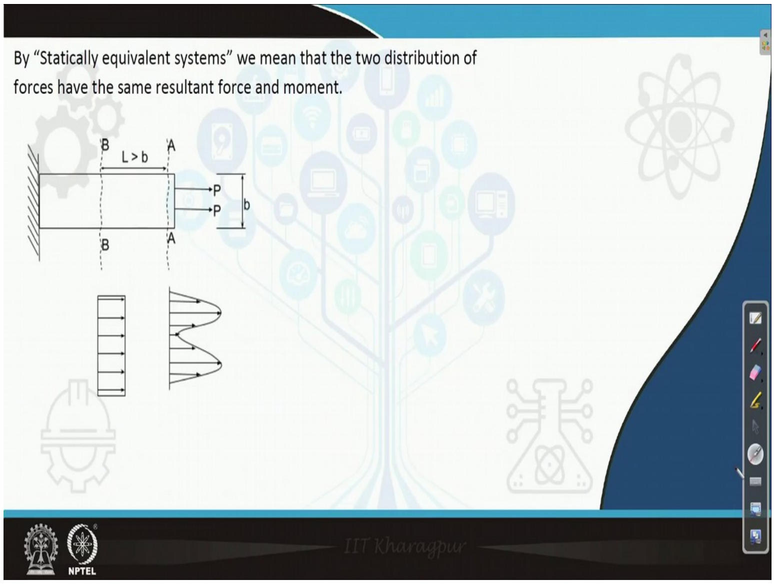

So by static evil equivalent system we mean that the 2 distribution of forces have the same resultant force and moment. So what is shown here that if 2P loads are applied at some section AA this is the distribution of stresses because of this 2 concentrated load at this point. But while the section is l apart which is more than the B it is uniform that is what we you always talk about is not it. We say that load is applied at the tip of the member and the stress is uniform P by A.

So that P by A here it is say the area is A then it is 2P by A so that is what is a Venant said it in a different way it says that if it is replaced by a force system if it is replaced by many other forces summing up to p and statically equivalent then it is same at a distance l where it is more than B. So both the said forces will induce similar type of stresses with this context with this understanding we will move forward to our next slide.

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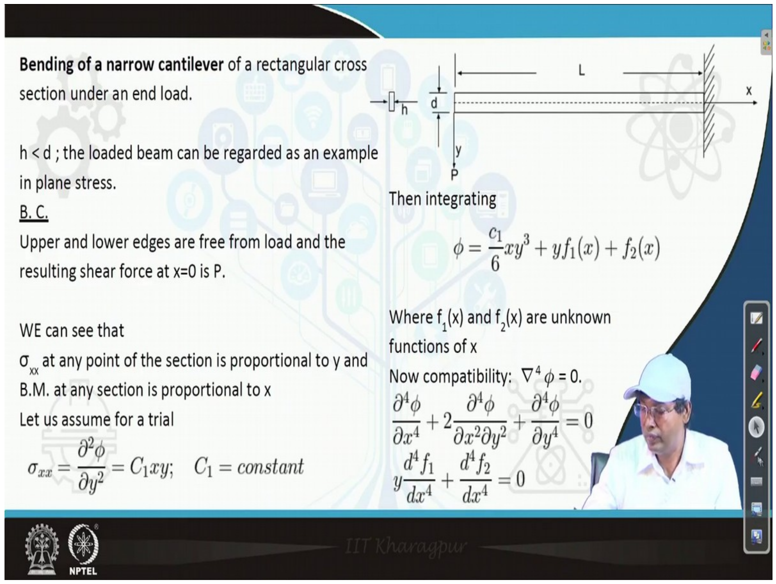

And in next slide we will solve a problem this problem is quite popular from your first day of mechanics probably. Problem is similar only difference is that we have assumed a different coordinate system we have drawn the problem in a different way. The boundary condition or the fixed support is on the right hand side at the center of the beam we have assumed the origin and x is going this way and y is coming downward.

So with this consideration we will do it is h this is d so the section dimension so with this consideration let us try to find out the solution. So solution means stress as well as deflection h is much less than the d the loaded beam can be regarded as an example in plane stress condition. So upper and lower ages are free from load and the resulting shear forces at x equals to 0 is p. So there is no load on the upper and lower edge there is a shear force distributed shear force acting at this edge which results into p this is a boundary condition we will use later.

We can see that sigma xx at any point of the section is proportional to y bending moment at any section is proportional to x. So this sigma x s is proportional to y this part already we have seen in our previous inverse method and that brings us to this c 1 y cube by 6 and then b a bending moment at any section is proportional to x that there is an x factor is coming here and that governs this lines observation to our physical condition gives us the way that this part is something like this.

So, with this note what we can do we can start. Let us assume for a trial sigma xx is equals to c 1 xy where c 1 is constant so this is same c 1 small or capital we are following this I jumped here to give you that observation. But it can have from this type of approach also where following this y and x multiplication also we can assume. And then if we integrate that what do we have we get the same term what we have there.

And we also have 2 functions f 1 x multiplied by y and f 2 x without any multiplication as 2 functions of x because we are integrating the partial differentiation equation with respect to y where f 1 x and f 2 x are known as functions of x. Now the compatibility it is satisfied there is no doubt. But to satisfy this what we get we get 1 more relation in terms of f 1 and f 2. So while we do this if we carry out this term by term I can again explain like the way I have explained earlier.

But better you try once so with application or to satisfy the compatibility condition what we have is that y del 4 f 1 del x 4 + del 4 f 2 d it is a complete derivative because these are only a function of x. So with that note we move forward.

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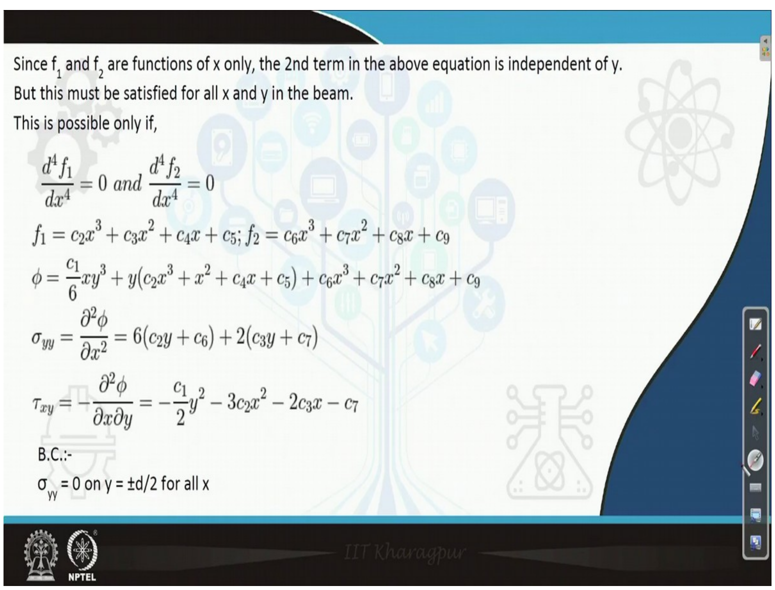

Since f 1 f 2 are functions of x only since f 1 and f 2 are functions of x only the second term in the above equation is independent of y but this must be satisfied for all x and y in the beam. This is possible only if this is equals to 0 this stating this is very easy but it requires a lot of a lot of understanding or on the mathematical equations. If you think deep you will also come to the same conclusion that this individually these 2 functions must be 0 to be satisfied by the grad 4 phi equation.

Now if we integrate all these terms f 1 is equals to what we have is that c 2 c 3 c 4 c 5 and f 2 is equals to 6 7 8 9 it is changing the power I hope it there is no point of explaining it. So the phi changes with the constants starting from c 1 to c 9, 9 constants yes we have to find out. And to find out those constants let us see what do we do we need to put the boundary conditions. But with these constants in place we can find out the expressions for sigma yy and tau xy.

So those expressions are quite obvious as it is shown here and we try to put the boundary condition this boundary condition is interesting. Sigma y is 0 at y equals to + - d by 2 for all x. If we go back to the previous equation previous figure we can easily observe that at this and at this which are + - d by 2 definitely it is stress free and that is the reason we say that this boundary condition holds. So with this node we move forward.

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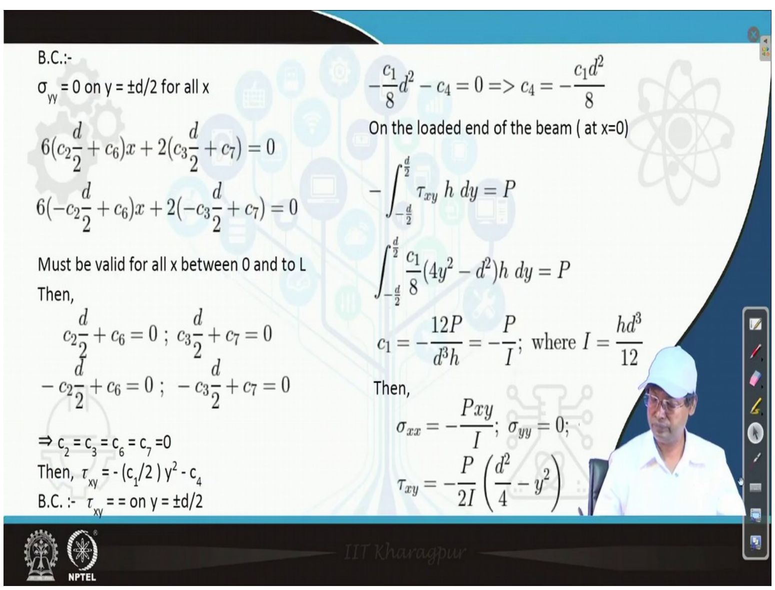

So the boundary condition as it is said in the last page is repeated here and we apply the boundary condition. Once we apply the boundary condition to sig ma yy we have see very, very interesting 1 again minus minus minus minus otherwise it is same and again either I need to mathematically prove or I can simply assume that it holds and we can proceed. These equations must be valid for all x between 0 to l x for 0 to l only when this constants are equals to 0 thats why it is made to 0 this.

So both the equations should hold only in case while these are individually equals to 0 these are proved in maths we would not spend time for that and consequently these 4 conditions this is for this particular nature of the boundary conditions implementation equations we get that c 2 c 3 c 4 c 7 are equals to 0 and then tau xy is equals to we have minus of c 1 by 2 y square minus c 4. We have 1 more boundary condition that is tau xy on y equals to + - d by 2 is also equals to 0 that is it is also shear stress free it is not only the normal stress free the boundary are top and bottom there is no shear stress also.

This is a small correction this is definitely not too equal signs this is equals to 0. So, with that note what we see is that this gives us a relation between c 1 and c 4 and as I mentioned at the beginning while we were defining the problem that distributed shears stresses at x equals to 0 that means if we go back at this point upper and lower edges are free from load and the resulting shear force at x equals to 0 is p. So that is what we will implement.

Now this is implemented here and if we implement that h is multiplied with to make it force. So we with simple integration substitution of the tau xy whatever we have this value and other things and it yields that c 1 is equals to minus of 12 p by d cube h and same since I is already is quite well known equals to minus p by I and at the end all the 9 constants are known. Once we have all the 9 constants known we can have expressions for sigma x as well as sigma y and tau xy.

So the tau xy reduces to the expressions are something like this as we have if after implementation of the constants sigma xx is equals to minus P xy by I sigma yy is definitely 0 everywhere and tau xy is equals dependent on the y square and accordingly we have.

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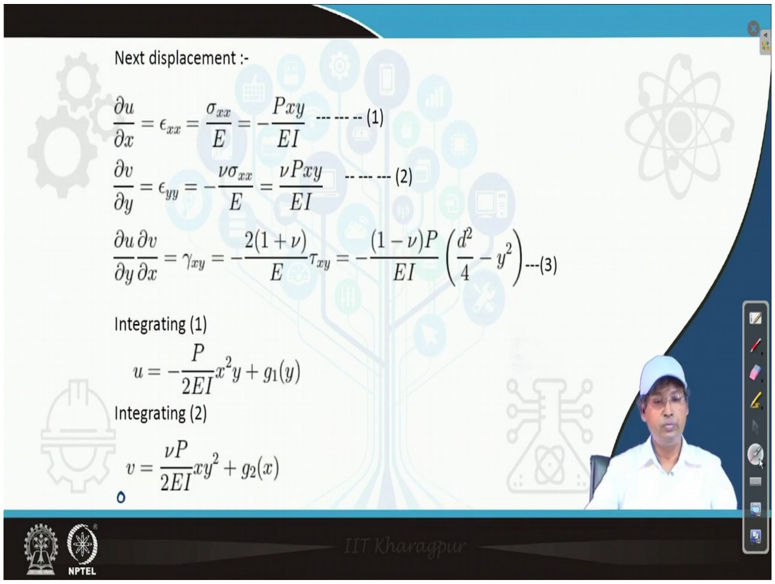

So in the next portion we will try to solve the displacements. Displacements equations are interesting to solve it is simple mathematics now. There is no 1; not much of elasticity or structures only boundary conditions are there. Boundary conditions in terms of mathematics you need to understand and you need to implement. So keeping in mind what we have the strain expression we put from the stress what we have we integrate.

Similarly for epsa y we have this expression and gamma xy we have this expression we name this as this 1 2 3. If we integrate there will be unknowns definitely so we get g 1 y g 2 x. So with that and definitely gamma also will be there. So we can; so we have found out by integration the del u and v here what we see that since it is a partial derivation of x. So integration gives us a function of y and g2 gives us a function of x.

And similarly we will be using this expression so incidentally 1 thing is 1 typographical mistake is noticed you please note that this is not multiplication this is I should do it with red ink blue is not visible this is a positives plus sign. So with this note please I think it is you can also easily put it and in the next slide while we will move we will use this equations we will substitute these values there in the again these values in the third equations and we will proceed further.

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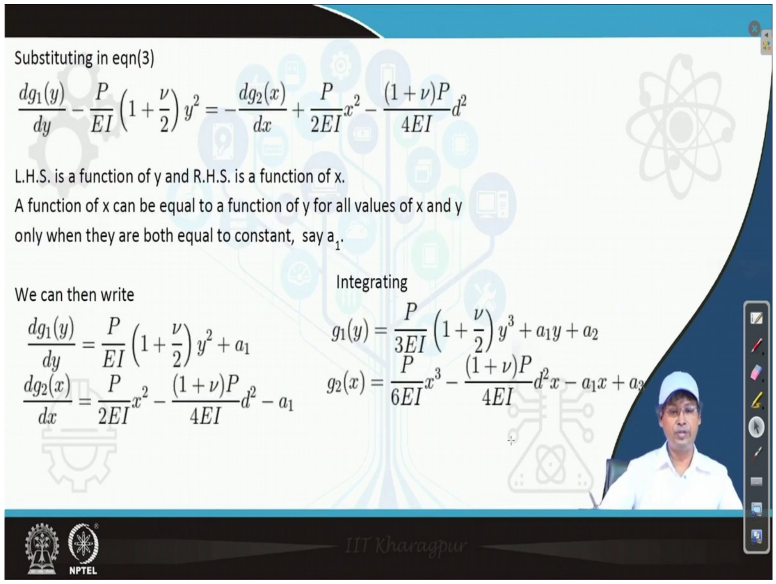

So we go to the next equation. So substituting in three equation three what do we have and little bit rearrangement definitely we have that is the reason we have partial derivative with respect to y and we have also the other term with respect to x and it is rearranged a little bit I hope you can easily carry it out. And then the left hand side is a this rearrangement is done to with keeping in mind with the things that all the y terms are kept on the left hand side and the wall x terms are kept on the right hand side.

So the left hand side is a function of y and right hand side is a function of x. A function of x can be equal to a function of y for all values of x and y only when they are both equal to constant that is quite obvious this phenomena will be using in many part while you will go for the higher stages of say any course say it is if it is CFD or dynamics structural learning's or anywhere. So anyway this is a mathematical phenomena and according to that we assume that it is a constant a ok.

So we put this is equals to a 1 and that is the other side also is also equals to a 2 and again we rearrange once we rearrange and integrate we get g1 and g2 we have 2 more constants because of integration that is a 2 and a 3. So all total we have 3 unknowns again fine. So we need to find out those three unknowns how can we find out those three unknowns we have we are in the process of finding displacement all functions are in terms of u v.

So we need to put boundary condition in terms of u and v. So while we put in terms of that u v the boundary condition.

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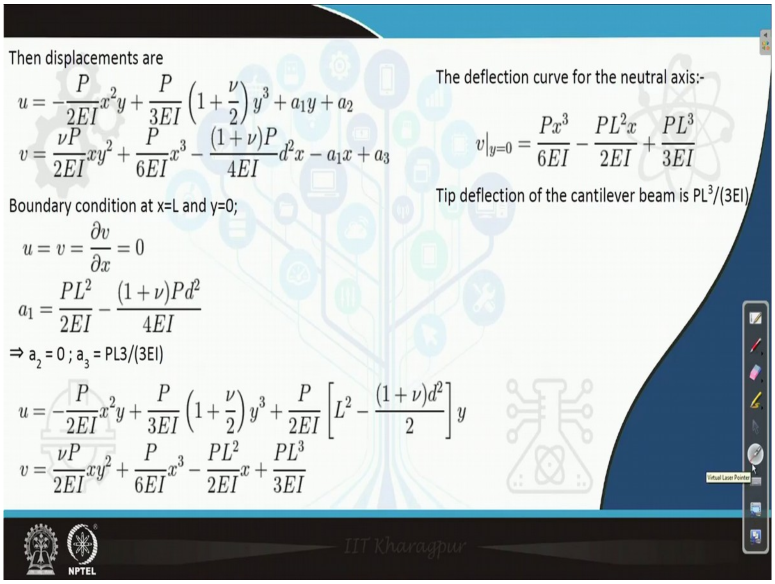

What we can see let us see so then we the displacements are u v as it is put as y a 1 a 2 a 1 a 3 all the constants uv is completely written here and the boundary conditions at x equals to l and y equals to 0 that means at this point as well at this point the other x equals to l sorry not at this point at this point x equals to l y equals to 0 here uv u equals to v equals to del v del x that means the slope is also 0 because the structure is supposed to bend like this.

This slope is always 0 these are 0. So with that implementation of 3 boundary conditions easily we can find out a 1 a 2 and a 3 and if we solve those what do we get is that a 1 is equals to PL square by 2 I z I - 1 + nu P d square by 4 E I a 2 is equals to 0 and a 3 equals to PL cube by thrice EI and if we substitute those values a 2 is 0 so this is a y function is there and we just put it here and in a different way and we also put the v boundary condition.

And the deflection curve for the neutral axis since it is neutral axis which parameter goes to 0 that is y equals to 0 so v expression y equals to put to 0 and we have this expression only. So this is the equation of the deflection line this is the line we are talking about. So this is equation of this line ok and 1 more interesting point probably you have solved using various methods that the tip deflection is always PL cube by 3 EI.

So that if you put x equals to 0 this goes off this is also equals to 0 and we have the desired solution is equals to PL cube by 3 EI. So with this note the preliminary discussion of theory of elasticity ends and in next 2 weeks we will solve specific problems with help of theory of elasticity. We will see how it is important how does it gives insight to a certain problem and accordingly we will learn a lot. So we have come to the end of today's lecture.

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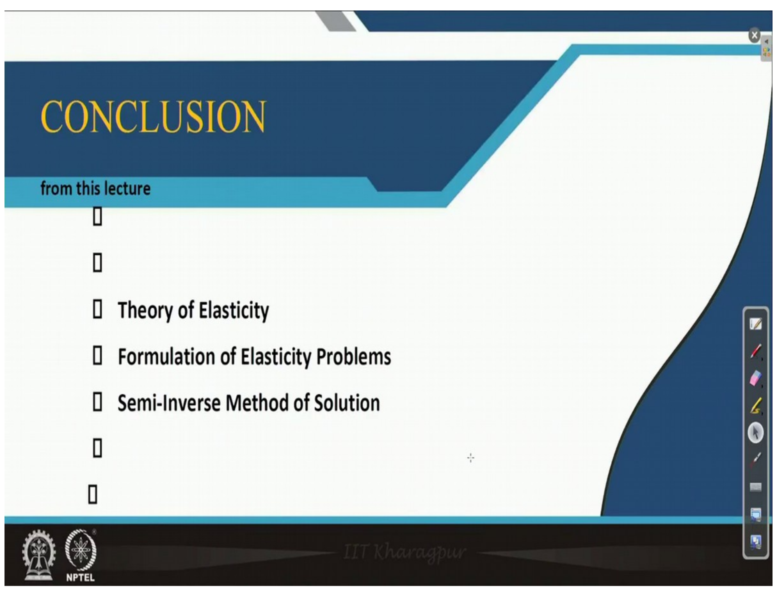

In this process the usual the reference slide comes and also comes the conclusion page where we have learnt that semi inverse method of solution using 1 example of cantilever beam. And with that note I thank you all for attending the course, thank you.