Aircraft Structures - 1
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Lecture No -31
Inverse Method of Solution

Welcome back to aircraft structures one this is Professor Anup Ghosh from Aerospace Engineering Department, IIT, Kharagpur. We are in the lectures of sixth week or module 6 in that sequence this is lecture number 31 and in this already we have covered to some extent how the problems to be formulated those from the unknowns fifteen15 unknowns and 15 equations already we have come up come across. So, using those; how to how to solve problem that is the predominant approach and importance in this lecture.

We will start with there are basically two types of approach in that also in solution method also. So, in that solution method we will see we will learn one process in this lecture series and the other process we will learn in the next lecture. So with this introduction we let us start but before we start anything I prefer always to come back to the recapitulation slide.

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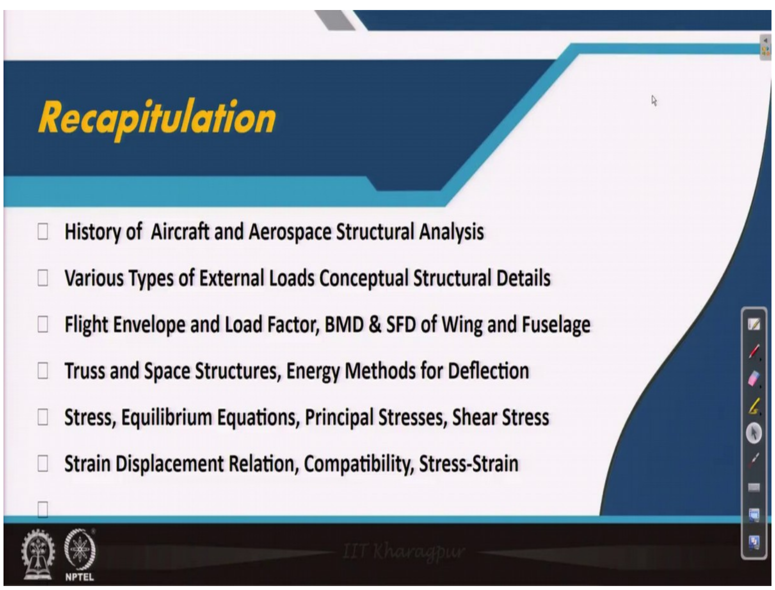

Recapitulation slide really helps us to learn what we have done. So far we have covered history brief history of aircraft development of aircraft from Wright brothers and then to the present days the huge Al225 or A380 or and also you have learned aerospace structural analysis or solid mechanics from the starting point. How physicist approached the problem how the first experiment or documented experiment done by Leonardo da Vinci all those things we have learned. We have also said that in the recent years few years back also there are good famous theories by Quieter Sanders with respect to the shells.

Shells are very good or applicable structure for aero industry aerospace structures always things are curved in most of the cases. So from there we went to the various types of external loads encountered by an aircraft and then we have come across to the conceptual detail of those structures aircraft structures. Those conceptual details to some extent we have seen with different diagrams internals of ribs, frames all those things spar.

We have seen how do they look like then what are the load type may be encountered while it is on ground or it is airborne why? What is the concept of light envelope? How the inertia row plays a huge role in case of aircraft structure? How the load factors take care of that and fix a regime of design for different type of aircrafts. From there we have seen how different parts of aircraft like wing landing gear, tail plane tail assembly experience different type of loads.

Fuselage experiences different type of load and from there also we have learned with typical example of shear force bending moment diagram of whole aircraft we have considered in two part. One the wing as separately how the bending moment shear force is coming on it considering unit load method and then we have seen for fuselage also. Then three dimensional structures are important in our aircraft industry.

It is not only used in landing gear it that is also used in different other parts of the structure also like the tail wing or the internal fuselage construction. If we look at it may be analyzed as considering a plane frame but it is probably better with invention of modern tools may be considered as three dimensional one. With modern tools all structures are nowadays done with as three dimensional structures and it is analyzed unless it is very, very computationally expensive or it requires huge resource.

So from their introduction of loads and other things we have gone to the deflection. How energy methods help us to find out deflection. Deflections are interesting because in any structure especially in aircraft structures deflections are interesting because deflection governs many things. Many things in the sense one good example is that as we have discussed earlier also then again to there is no harm in discussing again.

While at the position of takeoff say it is taxing in that case the aircraft wing is loaded with fuel the maximum amount of well in general whatever is possible is stored inside. And because of that it bends down how much the tip bends down how much the engine comes close to the ground that becomes important. So we need to find out deflection not only that because of that the aerodynamics also changes because of that say the deflection of wing changes.

And if the deflection of wing changes it also changes the lift. If you go into more deep the aero elastic phenomena that this I have not talked about earlier that also changes because inertia is changing deflection pattern is changing many way it changes in that way. Up to that we have talked about more on aircraft structures but after that what we have started is that a theory of elasticity part which is the basic of solid mechanics structural analysis whatever the way we think that way.

And we have learned and redefined we must say we have learnt how the stress is defined and from there we have found out equations of equilibrium. We have found out principal stresses shear stresses strain displacement relations compatibility equations have redefined strain and then we have come across about 15 unknowns what we were talking about few minutes back and with that concept let us try to see how do we solve problems.

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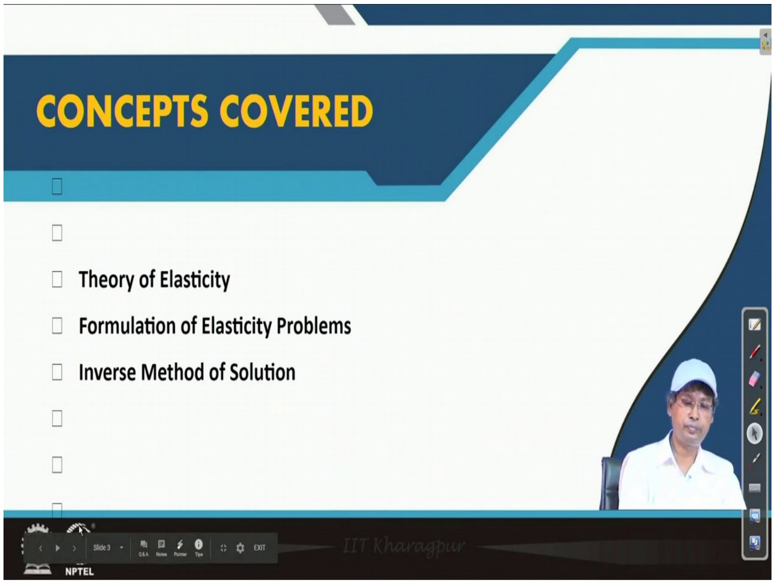

So in this we will be mainly concentrating on the inverse method of solution and let us see how do we do that.

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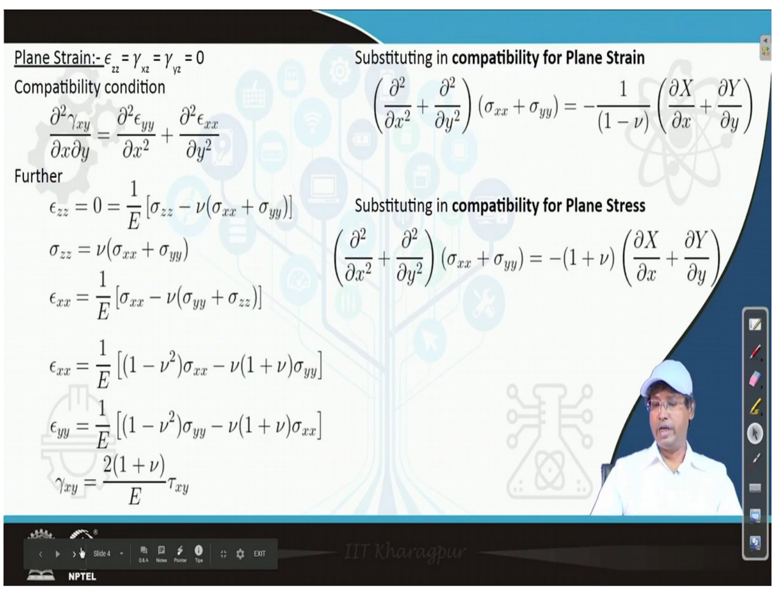

But before that some part was truncated in the last lecture. We have done in the last lecture the compatibility equation in terms of stress for solution what we generally satisfy to solve problem instead of satisfying the displacements in terms of strain. We generally try to do in terms of strain and that is the approach we try to do because stress it is easier. So in that context we have already learned that for plane stress condition how the compatibility equation in terms of stress gets modified.

So we have so grad square sigma x + sigma y and the right hand side x and y are the surface forces acting for unit area and this is -1 1 + nu but same way this is for plane stress condition that means what we have done is that sigma zz tau in this case all these are equals to 0 so considering that we have got this equation. Now if we try to find out similar one while we do for the go for the plane strain.

So let us see how do we do in plane strain already it is it is described many times that. It is described many times that it is similar to the stress but only the third direction component of strain is restrained it is considered as 0. So compatibility condition we need to satisfy in terms of stress that is the reason compatibility is written first at the beginning. Now what we can do the previous method if you not all the steps are repeated here but it is easy you can easily do it this strain components are put considering these values for the strain expressions are put back to this equation.

So we get the equations in terms of stress. Once we get the equations in terms of stress then a little bit modification of the equilibrium equation with respect to the surface forces we can easily do and we can find out the equation what is listed here as the plane strain condition. So I repeat what is done this epsa z is equals to 0 epsa xx epsilon yy and epsa zz all these terms are put back in this equations and then what do we do is we do a little bit modification of the equation and go for 2 and we find out the equations for compatibility in terms of stress in planes in plane strain condition.

One point you must note that here that the left hand side this is same even this part is also same only this constant is changing. So with that note let us move forward.

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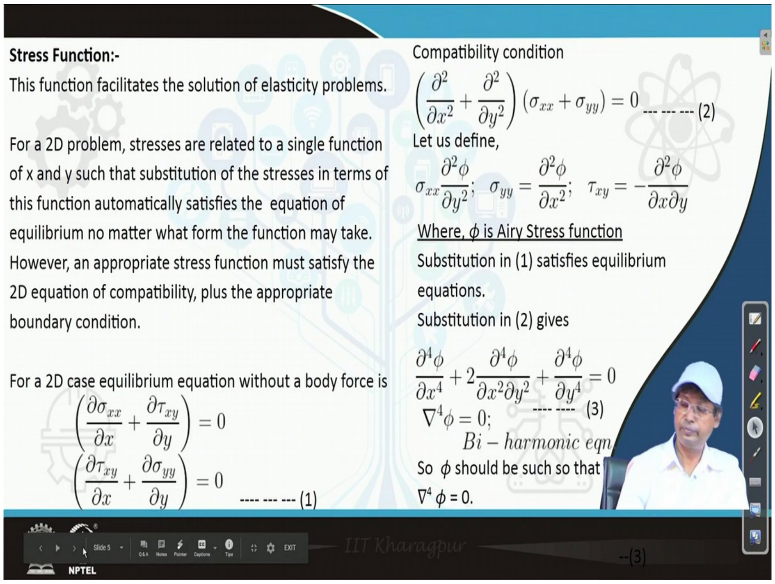

So we here it comes the stress function concept as we have already introduced that instead of compatibility in terms of displacement or strain we are coming to the stress function or in terms of stress we try to solve in terms of stress. So in that process what we will do this function facilitates the solution of elasticity problem but the function is how the function is what are the things to satisfy that let us see.

So in that case for a 2D problem stresses are related to single function of xy such that substitutions of these stresses in terms of this function automatically satisfy the equation of equilibrium. No matter what form of function may take however an appropriate stress function must satisfy the 2d equations of compatibility plus appropriate boundary condition. So this has to be done and if we have seen that instead of the boundary instead of the forces surface forces capital X and capital Y the compatibility equations in terms of stress what we have seen reduces to this or we can say that the compatibility condition in terms of stress is this one.

And we also have in for a 2D case equilibrium equation without a body force is this when we do not have any force like that. So considering that what we can go forward that here comes whatever conditions we have said here about the function that function is denoted by phi we say that we need to define such a way that that sigma x here again a small mistake is there typographical mistake definitely this is equals to sigma xx equals to del 2 phi del y square sigma yy or sigma y is equals to del 2 phi del x square and tau xy is equals to delta phi del x del y.

So if we see we also define or bring one more persons name famous physicist name that is Airy Airy's stress function phi is known as where phi is Airy test stress function substituting in one in this satisfies the equilibrium equation. So this can directly be substituted here and this we say that this takes the form of Bi harmonic equation grad 4 phi is equals to 0. So if this z is satisfied we say that the equilibrium condition is satisfied.

So with this note and concept of phi we need to consider a phi i should not say imagine we need to consider we need to need to find out a function phi which represents the stress for a particular problem. And in that particular problem the stresses components in 2 dimensional is sigma xx axis this sigma yy is this and tau xy is this. So with that concept let us move forward to solve problems.

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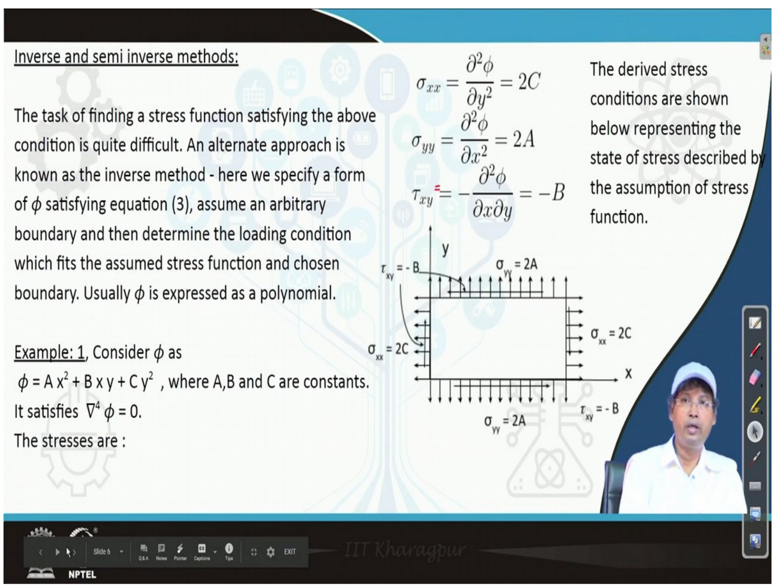

Here comes the two ways of solving problem inverse and semi inverse method as it is mentioned we will first learn the inverse method and in the next lecture we will go through the semi inverse method. So to go through the methods in brief the task of finding a stress function satisfying the above condition is quite difficult as just now I was telling you, I was hesitating to define so you just may think of how difficult it is to find out to formulate the phi so that it represents a particular stressed body with its stresses.

So it is really difficult it needs lot of experience it needs maybe while they did all these things probably they did a lot of experiments and then from the experiments they try to do all these things to find out the stresses. So with this note let us go forward and alternate approach is known as the inverse method. Here we specify a form of phi satisfying equation 3 what is equation 3? This is equation 3 ok so that is the equilibrium equation to satisfy it.

And then what we can do is that assume an arbitrary boundary and then determine the loading condition which fits the assumed stress function and choose boundary. So this is the reason it is said inverse so we are finding out the first stresses and boundaries conditions and then we are put saying that it is applicable for this type of case. Usually phi is expressed as polynomial that is the good way of doing it because the nature if you look at of grad 4 phi it is better to follow a polynomial.

You may use other things say whatever the experience you have you may try and share with us. So first let us consider as an example one in the inverse method let us consider that phi is equals to Ax square + B xy + Cy square so where A B and C are constant it satisfies that grad 4 phi. So that means the fourth derivative of x if we consider this is definitely goes to 0 that is y definitely will go partial derivative we will see and the second two consecutive second partial derivative the second term if we talk about that if we consider for this that also is 0.

Here also it becomes 0 here also it becomes 0 and the third one which is third part of the grad 4 that is the partial derivative with respect to y in 4 order that also makes it 0. So it is satisfied once it is satisfied the stresses are stresses are listed here it is double derivative so we have 2C it is also double derivative with respect to x we have 2A and this tau - del 2 phi del x del y that leads to –B. Now let us try to see the let us try to draw what is the condition we are getting if we if we represent all this in this particular element sigma x is 2C on this side as well as it is 2C in this side.

And acting uniformly in the element in the x direction y is also 2A acting in the y direction sigma yy and we have a shear stress where it is of -B that is the reason the sign is shown in the opposite direction and with this notation we say that it represents a good plane stress condition.

And this plane stress conditions we may use for solution of any problem the derived stress conditions are shown below representing the state of stress described by the assumption of stress function.

So this is what this assumption describes this problem so this is inverse method. Let us see 1 or 2 more problems in the inverse approach.

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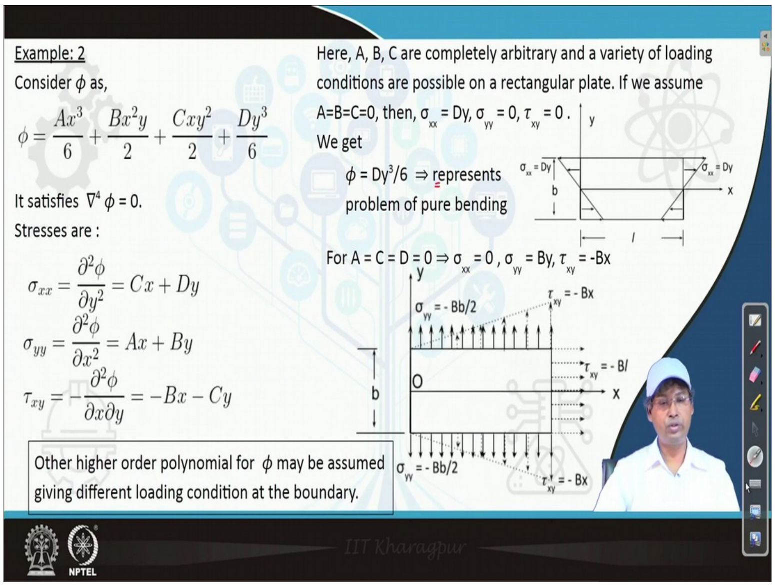

So in this approach what we see is that example 2 we again assume one polynomial in this case the polynomial what we have assumed is is Ax cube by 6 Bx square y by 2 Cx by square by 2 and Dy cube by 6. So this also satisfies this by harmonic equation or grad 4 phi is equals to 0 that you can easily check I would suggest you check I have not worked out these things here verbally I have described in the last but it is you may do try this.

So similar following the similar approach since this is the first part of assumption we can find out the stresses. What do we have in the stresses that the sigma x is equals to Cx + Dy Cx + Dy right and the sigma y is similar function is very very symmetric one so definitely it is Ax + By the sigma y and tau xy what do we have we have as -Bx – Cy. Now this gives us a certain type of stress conditions.

But probably that does not represents a problem what we are looking for. So let us see if we consider these constants A B C D in such a manner or that it represents some practical problem so the first case we will see that is the problem where we are assuming here ABC are completely arbitrary and a variety of loading conditions are possible on a rectangular plate. If we assume ABC is equals to 0 that means ABC is equals to 0 what is happening sigma x is equals to only dy sigma y is becoming 0 oh here it is written sigma x is equals to dy sigma y is equals to 0 and tau xy is also equals to 0.

So what do we get we get a stress condition something like this D is varying if this is the element we are talking about half above half below in that case what is happening with respect to D since it is changing it is representing stress here is distension and in this part it is compression where do we get this type of problem this type of stresses we get it in pure bending pure bending means there is no change of bending in along the length of the structure as well as there is no shear force is acting.

So that is the reason it is it is to some extent concluded and we will be using this letter this understanding will use later that Dy cube by 6 this term of this inverse approach shows its it this inverse approach solves or shows the bending problem. Now with this approach we will see this bending is as I told you this is one good example probably for pure bending a cantilever beam and at tip there is a load p if this length is l the projection length then this beam is actually under pl moment.

And that moment is a pure bending because there is no shear force acting in the transverse direction of the beam. So the beam load in the beam is pure bending and that stress distribution in this is the only normal stress and there is axial one more addition is there p we can easily have that that is not represented here. But it is similar to that there must be some additional load of the p.

So to nullify that we can easily put one more load here as p then that axial load is also missing but there is a bending moment. So with this condition we can have a pure bending but the next portion with assumption of other variables, let us see how it represents. So while we assume that

A C D equals to 0. What do we have we have sigma x is equals to 0 sigma y is equals to By and tau xy is equals to -By and that particular stress condition is shown here.

In this stress condition if you look at the sigma y is By at any distance B this amp amplitude if we talk about this is how much then if this is B that means capital B multiplied by small b by 2. So this is what and the other way this side it is minus but please note that the tau is having varying with x and here also along this boundary it is varying in here there is no stress acting in this boundary.

So in this boundary axis value is 0 and sigma y is not having any component of x so that is the reason you please note that the shear stress is varying this way here similarly it is varying this way here since it is like this. So the shear stress varies like that and it is constant here at this end. So with representation of A C and D equals to 0 considering the inverse approach considering the stress functions the stress function as shown as the polynomial we can have this type of problem.

And with this introduction to the inverse method we will consider the semi inverse method in our next lecture. So we have come to the end of this lecture.

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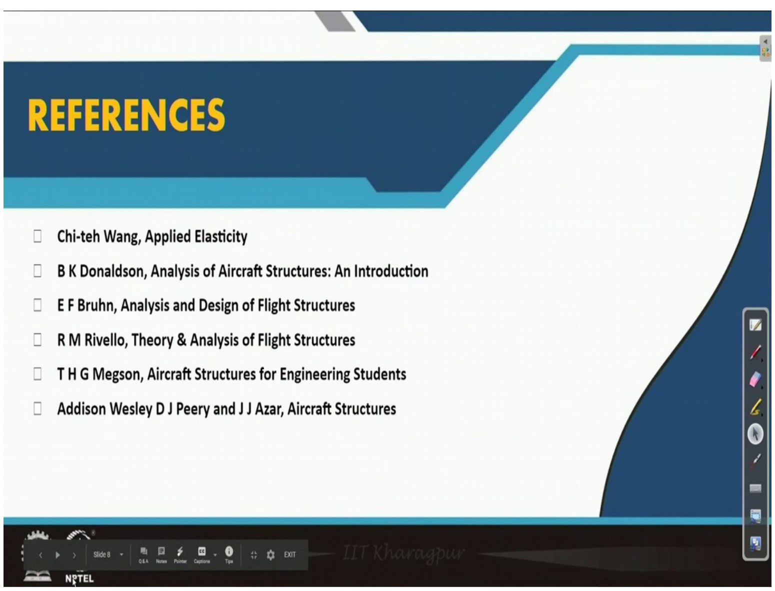

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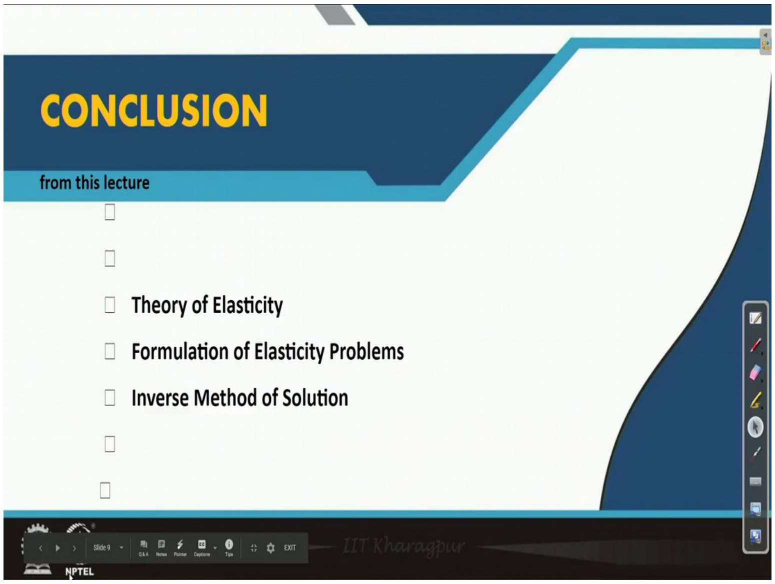

The standard reference is shown here. So please try to follow in case of problem or if you have more query about those. So the inverse method we have learnt and in that consequence I would like to thank you for attending this lecture and we will move forward to learn the next lecture, thank you.