Aircraft Structures - 1
Prof. Anup Ghosh
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture No -29
Introduction to Strain - Equations of Compatibility

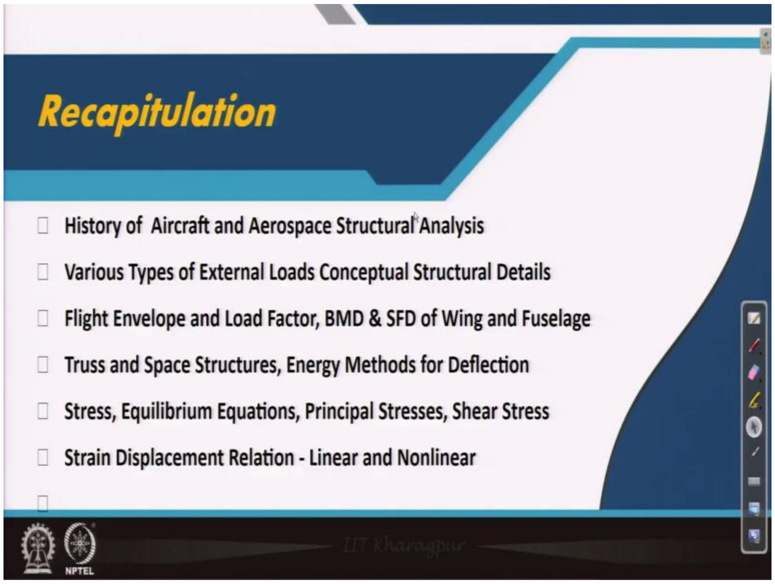
Welcome back to aircraft structures 1 this is Professor Anup Ghosh from Aerospace Engineering IIT Kharagpur. We are in continuation of the 6th week lectures this is in that sequence the 29th. In the last class we are already introduced with the derivation of strain more popularly that is the terminology use derivation of strain. But actually that is also known in the words like a strain displacement relation.

So with respect to displacement how strain is defined that we have done we have done considering a mathematical approach not considering a visual approach like many other books have presented may be considering 2 dimensional then going to 3 dimensional. What we have done we have considered a vector approach or beta to say tensor approach and then using the tonsorial notations a tensor calculus we have found out the expression for strain.

In that process it is good that we have got the expression including higher order terms that is the non-linear part of the strain displacement relation. Non-linear part of the strain displacement relation is in general not required probably for discussion of the stage you are going through. But it is better always to get introduced with the non-linear part also because we should not keep in mind that all relations of strain and displacement are always linear.

There are cases for large deflection especially we need to consider the nonlinear part. So in those cases where we need we need to consider but the discussions on theory of elasticity or the type of problem what we generally solve in this stage of study is concerning about linear part of strength. So with that definition of linear part of strain which is also ij is equals to half of del u, i del j we will come to that better.

(Refer Slide Time: 03:13)



Before that we go into the recapitulation slide this helps a lot about what we have already covered in the course. We have covered history of aircraft as well as solid mechanics or structural analysis all these things what we are covering in the last during the last week and also this week those things how those got developed by 1 after another big physicist that we have to some extent in a very brief way we have discussed.

Then we have come across the various types of external loads encountered by an aircraft and then we have come across to the conceptual structural details how it is fabricated why thin structure where how is it done why asymmetric structures come all those things are discussed. And then we have discussed about the loads, load factors flight envelop those points we have discussed. We have come across to a good method of unit load consideration for analyzing structure whole structure.

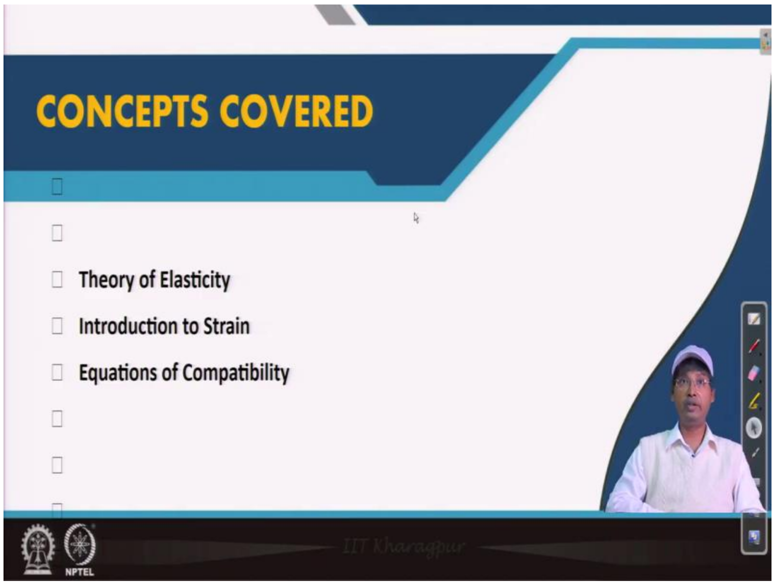
In the sense we have we have divided the whole structure in wing as well as in fuselage and then wing and fuse larger is separately analyzed for bending moment and shear force distribution. We have drawn those shear force and binding movement distribution considering a typical case. Then we have come across with to the cases of space structures solid we have solved landing gear problems.

Then in last week we got introduced to the theory of elasticity definition of a stress and in the last lecture 2 definition of strain. It is not that theory of elasticity has not been introduced to you earlier but this portion of the course what we are trying, is to cover the theory of elasticity from 3 dimensional point of view or from the visible possible orientation or dimensions of any structures and its loading and the stresses what it encounters.

So in that sequel we have defined stress we have come across with equilibrium equation. Equilibrium equations with respect to the body force with respect to the surface forces. Then we got introduced with the stress transformation how stress is transformed along with little bit of a coordinate transformation. So and then we found that there is a plane on which no shear stress act and not only 1 plane there are 2 more orthogonal planes to the parent planes so there are 3 planes.

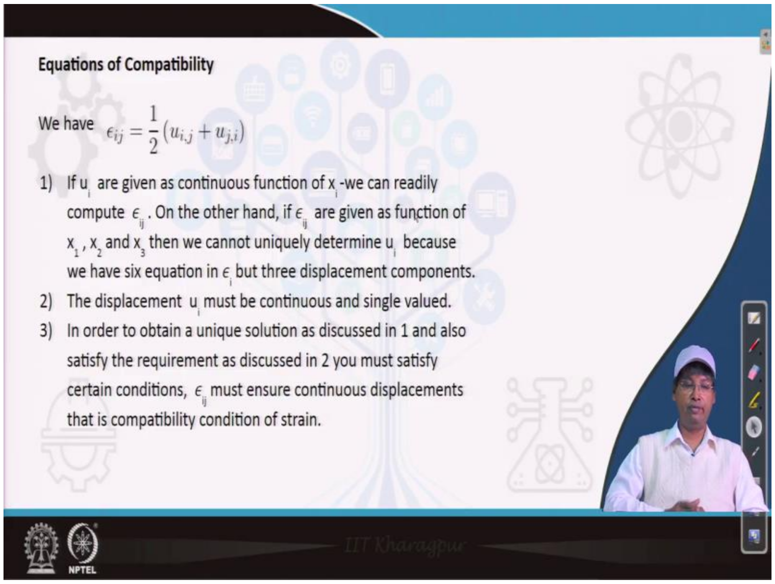
Where only normal stresses act and no shear stresses act on that those planes and those stresses are defined as the principal stresses along with that we also found the shear stresses maximum shear stresses we found. And then got introduced to the strain in last lecture and today we will go further with the definition of strain.

(Refer Slide Time: 07:21)



What we need to do is today's main topic of discussion will cover the compatibility equation. Compatibility equation is really very, very interesting one.

(Refer Slide Time: 07:36)



We will have many things written with help of those written scripts and other things we will try to have why compatibility equations are required and what is compatibility we are talking about? What type of compatibility we are talking about? That we need to discuss that we need to see so as I was defining I thought of saying orally but it is very difficult to say orally the expression of strain with respect to displacement.

So epsa ij is equals to half of u i, j u j, i that is u i with respect to t to the partial derivation of j and then u j with respect to i where i j is varying for 1 2 and 3. So that is generally written but sometimes it is not because our purview of analysis is within that domain only. So in that sense it may not be mentioned always we may always consider that i and j is equals to 1 2 and 3 so we get 6 components of strain epsilon x x epsilon y y.

So all the 6 components we can easily get sometimes to keep a conformity sometimes this is also written as x y z with respect to the Cartesian system. So um if u i are given as continuous function of x i we can readily compute epsilon i j. So that is the reason sometimes x i it is written sometimes instead of x y z this is also said as x 1 x 2 x 3 that is the reason it is said x i so with res respect to to x i we can readily compute the epsilon i j because this we can easily put in this expression we can we can find out what is what are the 6 components of strains we can easily find it out.

On the other hand epsa ij or epsilon ij are given as function of x 1 x 2 x 3 then we cannot uniquely determine ui because we have 6 equations in epsilon ij but there are displacement but 3 displacement components. This 3 displacement components 6 this so that is the reason we cannot find it out in unique manner. So we need to have some relation between the this components to satisfy maybe in terms of in terms of strains, strain components.

So that it becomes unique that is the point what we say from mathematical point of view while it got derived it was a from mathematical point of view only it got derived so that was the first reason to say that it should hold some this is the basic reason to hold some. But later the physical representation physical continuity and those things are talked about later let us see. The displacement u i must be continuous and single valued it has to be that is the other requirement to keep this reverse relation satisfying.

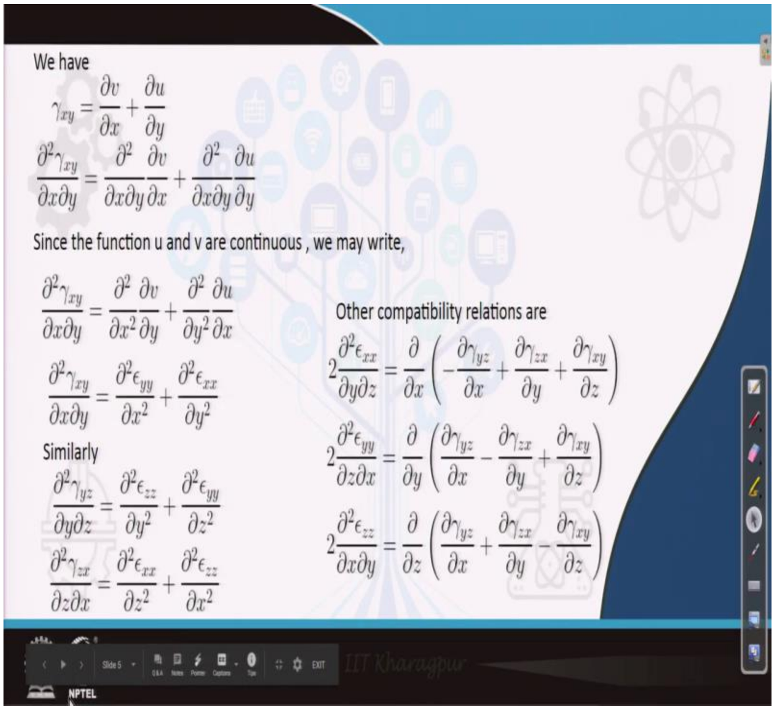
That single value unless it is single valued we may have a strain while derived from the same expression considering some part will give some value or maybe some other value considering some other parts. So that may create anomalies it would not be conforming to the existing or physical system so this has to be continuous and single valued. In order to obtain a unique solution as discussed in 1 and also satisfying the requirement as discussed in 2 you must satisfy certain conditions those are the compatibility.

Epsa ij must ensure continuous displacement that is compatibility condition of strength continuous displacements continuous for how much that is in which way what is happening you please try to observe say we better always we consider a simple structures example. Say I have a cantilever beam and it is loaded at tip by p what will happen this structure will bend like this. Now the displacement slope these parameters are always continuous along this length of the beam.

But while we define mathematically that may not be; so if it is not that will create the problem. So these are some points what mathematically says that that continuity must hold and that continuity to hold we must satisfy a set of equations known as compatibility to keep it unique the solution or relation between x 1 x 2 x 3 and the 6 strain components that is where the compatibility equations come.

But in general we do not need those equations to discuss much because the assumption or type of solutions problems we solved those are already solved by many people and these things are already satisfied so we need not to think much about these things. So with this idea of equations of compatibility or compatibility condition let us move forward.

(Refer Slide Time: 15:26)



All equations this derivation I would say is available in almost all books. It is too mathematical there here also it is too mathematical you only need to be an expert of partial derivation and algebraic equation what I say observing algebraic equation and rearranging those equations to the desired form or desired expression way. So in that form we would like to have establish the relations why these are the relations that is rarely proved in a book.

I will also try to skip those that part in this context that probably comes in very higher mathematics or continuum mechanics domain of analysis or in mathematics. So those things we would not say but what we would say that the compatibility equations what we see this 1 2 3 4 5

6 how do we get 1 or 2 we will discuss and we will say that the other may be obtained following a similar way.

If you are not able to obtain there are good books available very popular good books or almost all books it is available I would suggest you refer those to going to much beyond let us start with the expression of shear strain gamma xy. Gamma xy as usual with respect to that if we do half goes with the if you compare with the previous 1 that is for epsilon we have already established a relation that epsilon is equals to half gamma.

So from there half gets cancelled and we get this expression del v del x plus del u del y. Now what we do we do twice the partial derivative with respect to x and y and the right hand side gets modified this way. This line from mathematical point of view is very, very important since the function u we are continuous we may write we are changing the derivative sequence of derivation. So this thing usually comes in higher mathematics or you have probably covered in your higher mathematics course.

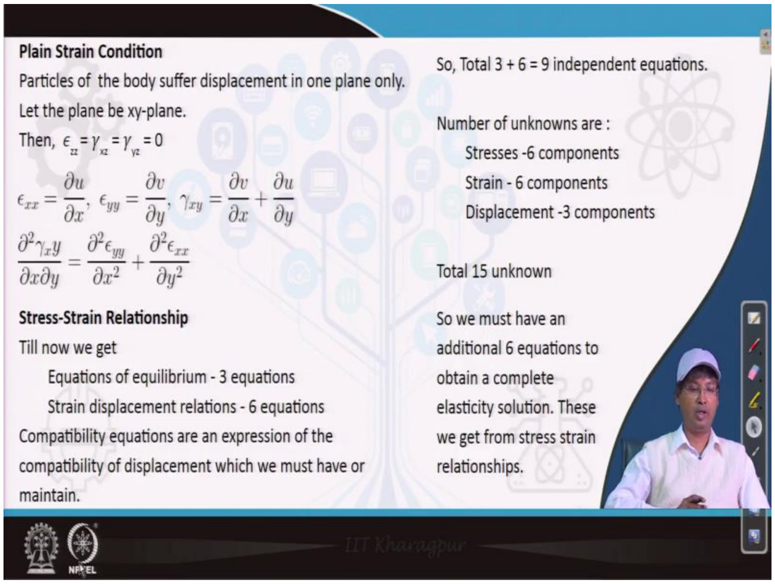
So what we get is that the right hand side gets a little bit modified del 2 del x 2 equals to del v del y + del 2 del y 2 equal multiplied by del u del x so this we can easily say already defined as epsa yy or epsilon y y and this is epsa xx. So we have a relation between the gamma and 2 epsilon. So this is the first compatibility equations. So following this if we start with this gamma y z easily you can come to this if we if we start with this easily you can come to this it is not a big issue to solve it.

Similar way if you start with this expression or epsilon x x is equals to del u del x and then do these 2 see it is the other 2 y z if we do this derivation and substitute all these x these expressions are the other expressions whatever you have got and rearrange those you will easily get the expressions for this. So please look at the compatibility conditions compatibility equations these 3 equations establish a relation between the shear and the normal shear strain and the normal strain.

And this set of equations on the left hand side with the normal strain to the shear strain. So once this equations are satisfied we have unique solution for the strain with the assumption of displacement function that is the reason it is important and we need to satisfy this condition whenever we are solving any problem following elasticity approach not only in elasticity approach say when we go for the process of solution of numerical methods.

Do not think there it is not satisfied numerical methods are fundamentally rooted to these theory of elasticity and they are whatever the displacement functions are chosen those satisfy this condition. And from there the derivation is done and those are approximate because it is considered piecewise satisfaction of these equations and that is the reason as a whole while we go it gives us approximate solution.

(Refer Slide Time: 21:19)



So we come to a specific case known as plane strain condition plane stress you are already introduced. It is similar to plane stress in that case say z components of sigma was 0 here z component of epsilon r 0 that is what we mathematically say. It is easy to remember that way that is why we say it frequently but problem wise it is a different type of problem. In case of plane stress problem we say that the surfaces which are free those are the z direction.

And in case of of plane strain where the surfaces we are confined or restricted to expand or contract those surfaces or that direction is considered epsa zz. So with that we let us see how the equations are modified. Equations modifications are not much to do only these things we are supposed to substitute we are supposed to substitute in the previous relations whatever we have got for the strain displacement relation as well as the compatibility condition and we get these values.

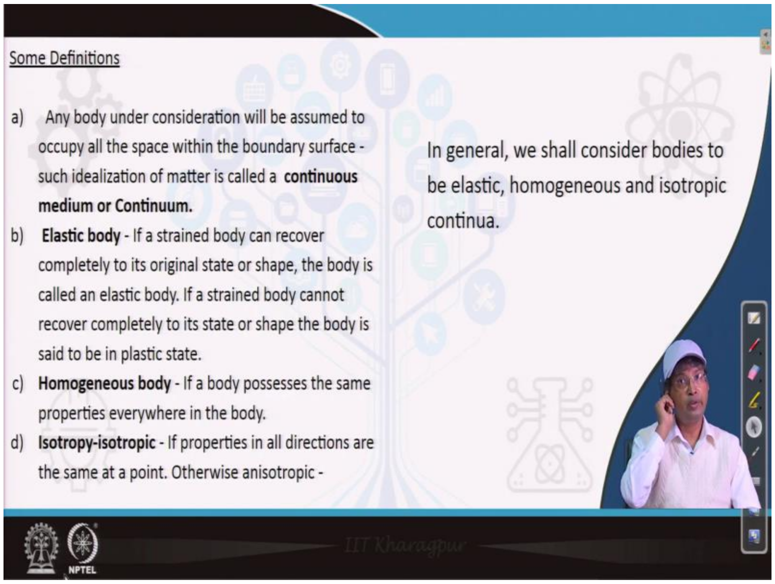
So the particles of the body suffer displacement in 1 plane only. Let the plane be x y we say that it is a plane strain problem. The other way I said I told you that in the z dimension it is restricted or restrained to strain so no expansion or contraction is allowed in that direction. So accordingly the compatibility equations we get. Now this is a good discussion whether we have got many equations is not it.

I have tried to give you small less number of equations but up to some limit I cannot restrict myself I have to give. So anyway let us see what we have and what we need more. So the stress strain relationship if we talk about why do we need the stress strain relationship that is what is briefed here in this things. Till now we get equations of equilibrium 3 strain displacement relations 6 equations compatibility equations are an expression of compatibility of displacement which we must have or maintain.

So those are not something relating in relation to the unknown and the known things and number of equations so that cannot be used that is what we said. So, similarly if we consider that so total we have 3 + 6 independent equations. So number of unknowns what we have described using those equations are 6 components of stresses 6 components of strain and displacement 3 components.

So there is a shortcut so 6 more relations we need that 6 more relations will come from the stress strain relationship. So we must have an additional 6 equation to obtain a complete elasticity solution this we get from the stress strain relations. So we move forward to stress strain relation that is a very easy stuff I would not spend much time on that.

(Refer Slide Time: 25:31)



But before we go for before we go for the stress strain relation it is time to have some definitions. Probably you are aware of this definition for better to remind those. So anybody under consideration will be assumed to occupy all the space within the boundary surface such idealization of matter is called the continuous media or continuum. This may be visualized something like that.

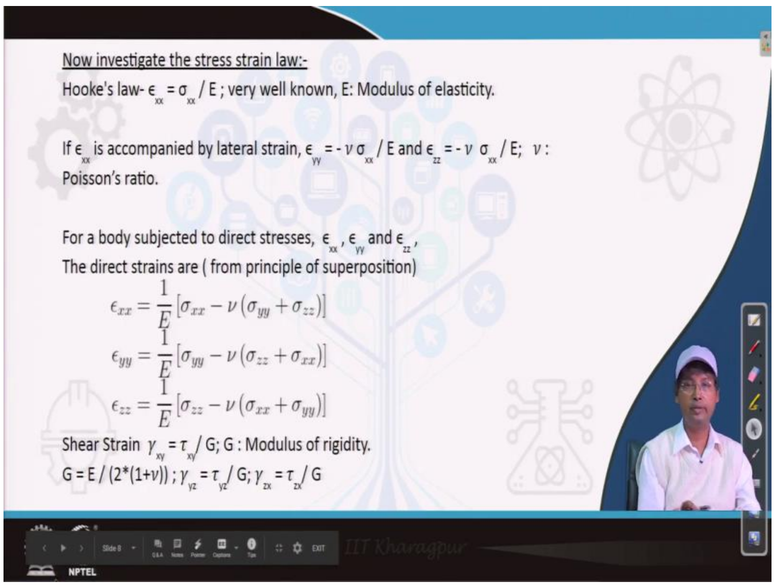
Say I visualize this way say you have a bubble inside a rubber or eraser what you usually use and if you stretch it many times you see that while it is in normal condition you do not see that bubble but while you stretch it you observe that bubble that makes it not a continuous media. So something like that if it is there are discontinuities so this continuous assumption is not holding.

Elastic body it is a if a strained body can recover completely to its original state of shape the body is called an elastic body. Please note its original state or shape if a strained body cannot recover completely to its state or shape the body is said to be plastic state. So that is the basic definition. So this may relations we will talk about in next slide. Homogeneous body if a body possesses the same property everywhere in the body that is a homogeneous body.

That means any sample we take that shows the same property that is why we say it homogeneous. And isotropic; isotropic is the properties in all directions are the same at a point. All directions we have we have talked about only 3 directions. So in the all the 3 possible directions or maybe any if you rotate whatever the way you like the axis system in any direction the properties are same so we call that as isotropic is if not we call it anisotropic. Orthotropic there are many others categories we would not discuss those things those are generally discussed in other subjects in more detail.

In general we shall consider bodies to be elastic homogeneous and isotropic continuous that is what we our all elasticity problems what we are discussing is based on. So on these uh definitions we will define the stress strain relation.

(Refer Slide Time: 28:54)



So the stress strain relation we come across is this is what is new in this right why did I discuss the previous slide. The previous slide is the basis previous definitions are the basis of these assumptions or say I should say it is not assumption this property Hooke's law epsilon xx is equals to sigma xx divided by E very well known E is modulus of elasticity. See here one more thing you must note in the previous one we talked about elastic that means which recovers.

But it may recover in a linear manner or in a non-linear manner so whether E is constant during the process or not that is a big concern. Our discussion is again within the limit where we have E is a straight line that this is sigma this is epsilon and the slope is E. So our per view or discussion is in that region. So epsilon xx accompanied by lateral strain epsilon yyy equals to -nu sigma x by epsilon this is easy to write sometimes I visualize in the way that it is very easy if we I used to play in my still I play with eraser is a very soft piece of rubber many things are easy to visualize there.

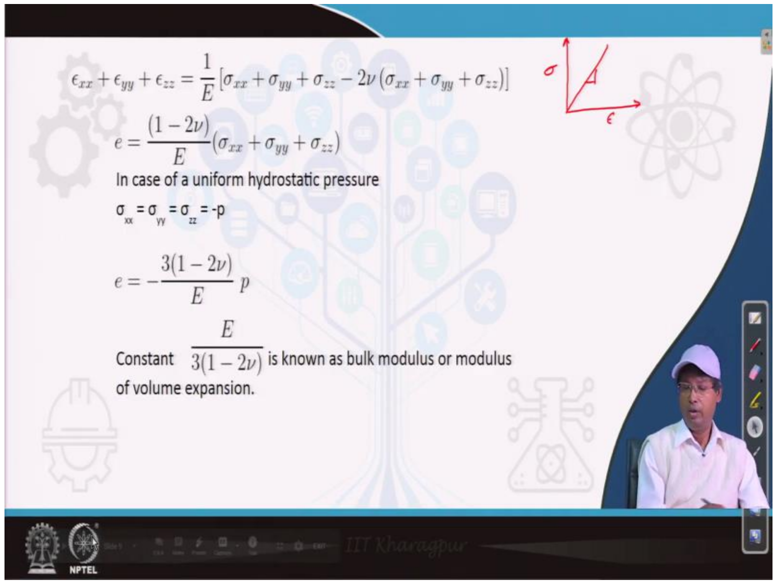
So you can easily observe that if you pull the eraser or the rubber block it contracts from the transverse 2 directions why it contracts? That is because of the Poisons ratio and that is the reason the minus is there that ratio of contraction is given by this Poisons ratio these are already introduced to you so better not to spend much time but I like to bring that example that is why I am talking about that.

So with this thing with this scenario we can easily write this equations because if a body is under stress of sigma x sigma y and sigma z or sigma xx sigma yy or and sigma zz we can easily say that 1 by E multiplied by sigma x - mu into sigma y by + sigma zz and similarly we can write the other 2 normal strain components and the shear strain components we can easily write considering G as the modulus of rigidity as gamma xy is equals to tau xy by G and so on.

These are easy equation probably in mechanics many times people introduce this without giving the background and other things I think no harm in that so we are repeating the same equation. We have come across to some relation in the previous slide E and G relations are written I did not say it very clearly this relation you can you can easily as I said I have skipped that part you can easily prove that following some 2 dimensional case of strain where a block is tilted because of shear strain and then put those values and easily you can find it out.

So I do not want to spend much time G is equals to E by 2 into 1 + nu. So, one more constant is there that we can easily discuss in relation to that and with that will conclude today's lecture.

(Refer Slide Time: 33:03)

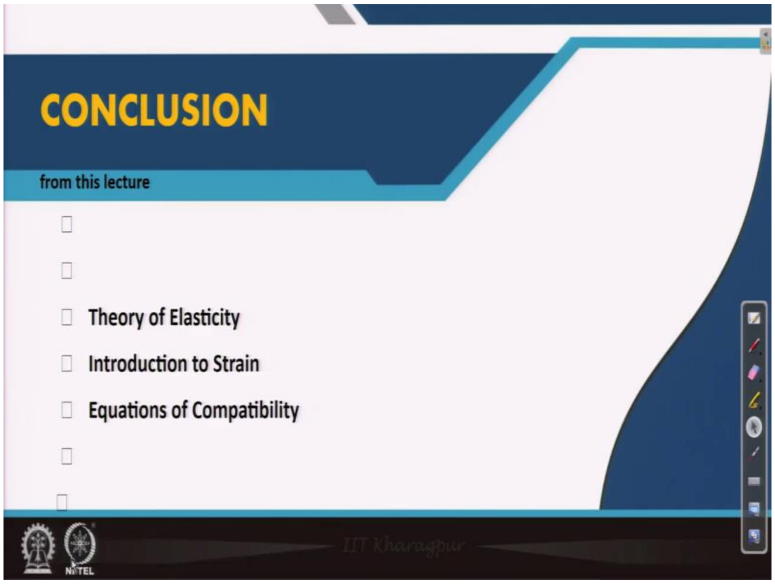


So what we can do if we sum all the components of normal stresses we get the equations as we see here. And if we take it out uh this portion sigma xx sigma yy and sigma zz then we can have an expression something like that e will define what is e? e is equals to 1 - 2 nu by e. And in this particular case if we consider that all these components are equal and that is -p or its a compression from all size side or it is a hydrostatic pressure hydrostatic pressure is easy to visualize again while we keep something under water.

So from all the surfaces the body if it is considerably small in size with respect to the depth of the water it experiences hydrostatic pressure or equal pressure from all the sides or you may need to do something else to visualize and we can have those type of environment for experiments. But anyway if we put those we get the relation something like this and this constant in relation to e is known as the bulk modulus or modulus of volume expansion. So with this small definition we would like to conclude the lecture with compatibility.

(Refer Slide Time: 34:36)

(Refer Slide Time: 34:39)



And the standard references come and whatever we have learned that is what is it are reiterated here please keep a note of that and then with that I thank you for attending today's lecture we will meet again with some more concept of problem solving, thank you.