Aircraft Structures - 1
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Lecture No -27
Shear Stresses

So welcome back to aircraft structures 1 course this is Professor Anup Ghosh from Aerospace Engineering Department, IIT Kharagpur. We are in the 5th week lecture series this is in sequence 27th lecture. We will be learning on shear stresses we need to learn in many times shear stress governs the failure theories we want to discuss. In this scope it is discussed to some extent in a small way probably in mechanics course in the previous courses.

Or it may be discussed with respect to the subjects whatever is generally considered for specific requirement that way.

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So, with that note we move forward to the recapitulation slide. We have done a brief history of aircraft as well as a solid mechanics or structural analysis related to aerospace engineering. We have learned various types of external loads and how do they act where do they act how those are different in landing condition or also in while it is flying. And how the load factor plays a big role and how the flight envelope is also important.

Why we do need to keep a monitoring eye on the flight envelope. And after that we have learned that the whole fuselage and wing bending moment shear force diagram with a typical condition of loading we have learned to find out we have got we are introduced with the unit load analysis. And then we have gone to the a specific type of truss structure truss is coming so that is 3 dimensional trusses which is important in aerospace vehicles.

So that we have learnt with some example of landing gear systems and then in this week we got introduced to the theory of elasticity. Theory of elasticity you are introduced already with the course of mechanics. You are already introduced with what is stress what is strain different forms of stress and strain, transformations equilibrium but we need to learn for advanced studies for development of advanced programs or analysis.

We need to learn what people have already done and how they have approached those problem initially and recent days numerical things are not at all discussed here those are discussed in the respective courses along in advance to these topics what we are discussing now. So, in that encompassing the definition of stress equilibrium equations principal stresses already we have covered. We have also covered the transformation of stresses and 1 important thing is introduced to you that is the truss stress transformation or notation of stress system in terms of tensorial or index notation.

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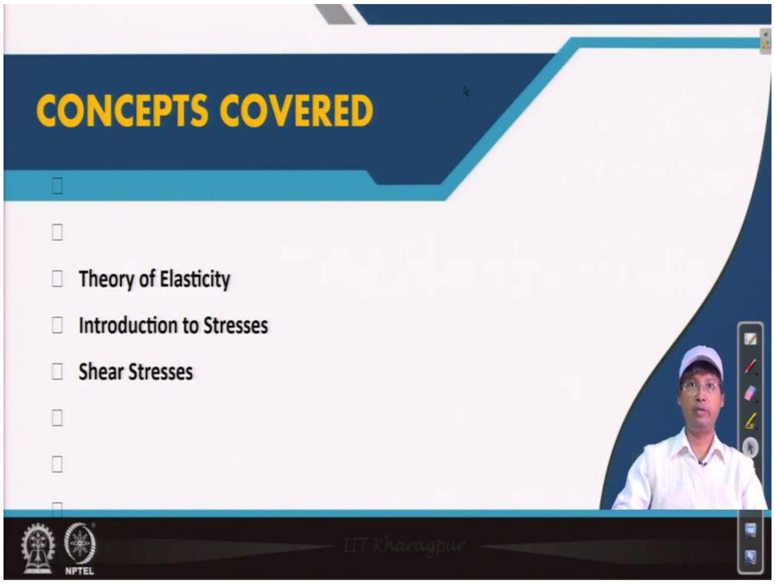

With that that sequel will come to the shear stress this lecture will consist of shear stress. Let us see how do we find out shear stress and how do they represent in specific cases famous cases also will be learning with example.

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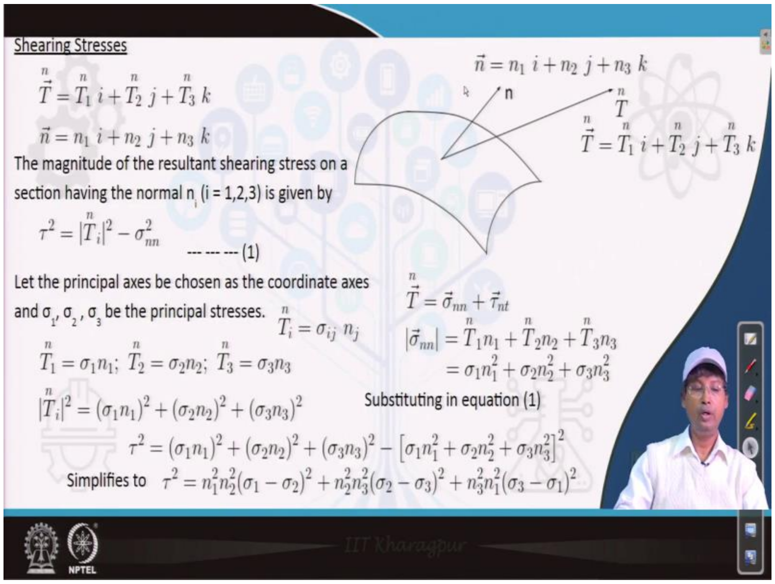

So, shearing stress what I we see is to consider that we are considering any surface as it is given on the right hand side figure. This figure where what we have is that that plane that surface is denoted by the normal n and this normal n vector is denoted as represented above with direction cosines n 1 n 2 and n 3 unit direction cosines. So, with this we see that we also hold one more important equation that is the traction or external loads what is acting on that, that may be divided in components T 1 T 2 T 3.

And the magnitude of the resultant shearing stress on a section having the normal n i is given by this, This n is sometimes given as n i also in index notation in 1 2 3 that is the reason it is stated that way. So, if the normal stress acting on that particular plane is sigma n and the shear stress acting on that particular plane is tau and then easily we can say that it is that tau square is equals to T i square plus sigma n square.

So that in a vectorial way we can amplitude vectorial amp which amplitude we can easily find out and then let the principal axis be chosen as the coordinate axis in this, this is we are considering a special case. This is the general type of case and sigma 1 sigma 2 sigma 3 be the principal stresses. So, it becomes easy to understand because already we are introduced to sigma 1 sigma 2 sigma 3.

So because we are introduced to the coordinate transformation or transformation of traces stresses also along with that so anything even if it is sigma ij acting on the system or experienced by the system that can easily be transferred to sigma 1 sigma 2 sigma 3. And accordingly we can if we choose accordingly we can write that this is sigma ij as it is mentioned here this is in general but if it is in 1 2 3 form we can write that T 1 is equals to sigma 1 n 1 sigma T 2 equals to sigma 2 n 2 and T 3 is equals to sigma 3 n 3.

Now what we can see is that the square of this is nothing but the square of this because those are orthogonal in direction. So, that gives us this value. Now if we come to the previous equation what is written this is already in vector way we can easily say if this is T so we can write that this may be the sigma n n and this may be the tau nt so we can easily vector definitely here the vector is not completely represented with the length.

It is only the direction is represented here. So, following that what we can say that the sigma n n can have be also found out with multiplication of the unit direction cosines and that component if we take all the 10 components will give us the normal direction sigma n n. So, we get the sigma n n is equals to this. Now if we substitute back to this equation what we have is that tau square is equals to this minus this, this one square.

And few steps are jumped we have jumped here i would suggest you do yourself and try to find out that tau square is equals to n 1 square + n 2 square multiplied by sigma 1 - sigma 2 whole square + n 2 square n 3 square multiplied by sigma 2 - sigma 3 whole square n 3 square n 1 square multiplied by sigma 3 sigma 1 whole square so this is a good expression for the shear stress acting on a surface while the surface force surface boundary force are denoted by T and the normal stress on that particular plane is given by sigma n n.

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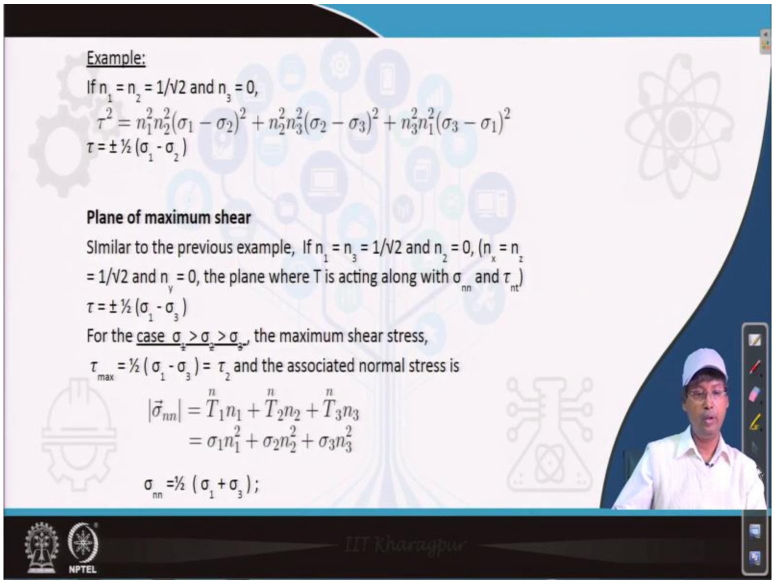

So we move forward to discuss the situation discuss the shear stress conditions we will consider typical examples it is not a very general usual example but it is it is having some significant representation. So, let us see, first example we are considering that n 1 and n 2 is equals to 1 by root 2 that is it is 45 degree in angle with the sigma 1 and sigma 2 axis whatever we have chosen from the general stack system sigma i j we have found out sigma 1 sigma 2 and those corresponding directions.

And we are considering that the say n 3 is equals to 0. So, if it is like that we may imagine a cuboid where the n 3 is a matching or the third direction is matching with the sigma 3 whereas n 1 and n 2 the other 2 directions of the qr plane of the cuboid is at 45 degree angle with respect to their 1 and 2 axis. Now if we substitute this value in the shear stress equation what we have derived in the previous slide what we get is that the tau in that particular plane is equals to half of sigma 1 minus sigma 2.

So this we can easily visualize that is why the plane of maximum stresses is given here this is one of the maximum stresses what we find in that this particular case. So, similar to the previous example if we consider this is changed to n 1 and n 3 just to visualize it properly otherwise it is a similar case whether the axis system is rotated 90 degree with respect to any one axis hardly matters to the stress system.

So just for visualization we have changed the axis system n 1 and n 3 we have considered 1 by root 2 that is 45 degree and n 2 is co-linear or in the same direction to the sigma 2. And sometimes it is it is easy to imagine with respect to x y z that is the reason this x y z is also given and considered that 1 and is corresponding to x 3 corresponding to z and the 2 is corresponding to y. The plane where T is acting along the sigma n and sigma nt along with sigma n n and sigma nt.

So what we have similar way sigma 1 minus sigma 3 we are getting for the case where sigma 1 is greater than these are not cut please consider that it is a typographical error sigma 1 is greater than sigma 2, sigma 2 is greater than sigma 3 the maximum shear stress we get is sigma 2 and the sorry tau 2 and the associated normal stress is sigma n n is given by putting the values in this equation we can find out that sigma n n equals to half of sigma 1 + sigma 3.

So, we have in those particular planes plane we have I have said this it is difficult to imagine we will come to the next slide and see but what is the stress condition and that particular plane. In that particular plane the stress condition is this is the truss shear stress and this is the normal stress. So, we have 2 planes 2 orthogonal planes 1 one and n 3 such that where this same amplitude of stresses are acting. And in n 2 we are on that plane we are not sure what is acting there.

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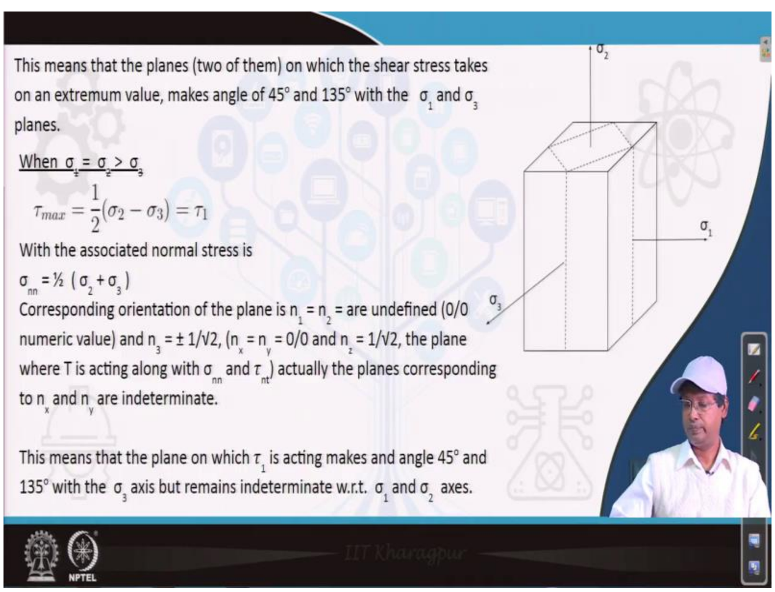

So, if we move forward sorry here we see the representation of that particular problem what we have discussed in the previous one. Here what we see is that 1 and 3 that is the reason for the drawing sake and for understanding with the type of coordinate system so far we have followed we have followed this. As we have said that 2 is along the same direction whereas 1 and 3 those planes are making 45 degree angle.

So this plane and this plane I should say that this plane this plane is making 45 degree angle with sigma 1 as well as sigma 3 and additionally if we look at this plane is also making 45 degree with the sigma 3 and negative of sigma 1. So, that is what is said these are the planes we are talking about this means that the planes 2 of them on which the shear stress takes on takes on an extreme value makes angle of 45 degree or 135 degree with the sigma 1 and sigma 3.

So corresponding we have 2 more that is the reason 2 both the angles are mentioned. So, accordingly we find those cases. We have one more typical case where we assume that sigma 1 is equals to sigma 2 2 and they are greater than sigma 3. So, let us see try to check what happens so tau max becomes T 1 following the similar way putting the substituting the values as we have done earlier and sigma n n is equals to half of sigma 2 + sigma 3.

Corresponding orientation of the plane is n 1 equals to n 2 and since sigma 1 sigma 2 are same and it is undefined. It is undefined because those are same and it is mathematically speaking we cannot find out the orientation as it is here we can find out the orient orientation but there we cannot find out the orientation. And n 3 which is equals to plus minus 1 by root 2 may have some orientation, so it is something we can say that this cuboid only but it is rotating with respect to this axis.

So it may have different position because the other 2 are undefined that is the reason. Keeping in this axis sigma 3 oriented in this direction or in this direction it may have any angle on the other side so that is the reason it says undefined. So, in terms of x and y and z it has been said again. Actually the planes corresponding to n x and n y are indeterminate this means that the plane on which tau 1 is acting makes an angle 45 degree or 135 degree with sigma 3 axis but remains indeterminate with respect to sigma 1 and sigma 2 that is the reason I said it may be something any position and it is be difficult to determine or indeterminate.

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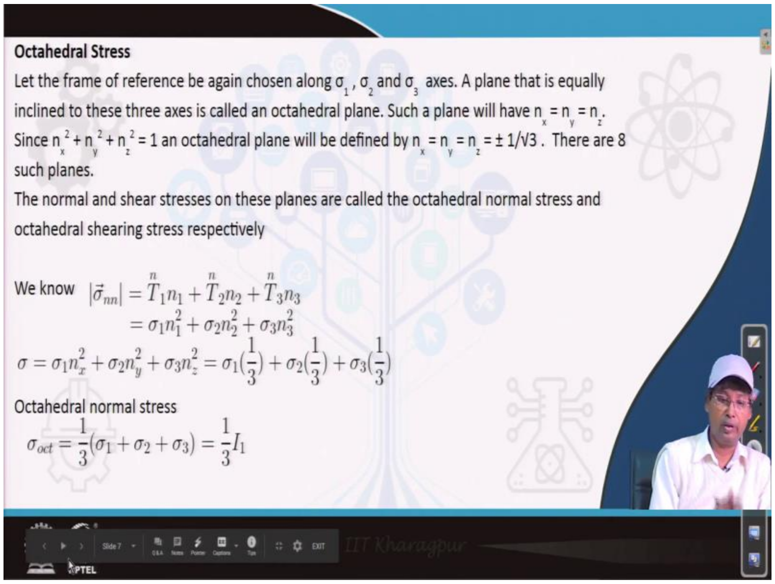

So we move forward to our next topic next topic shear stress is to some extent introduced if you are interested more about this we can easily have a look. This topic is to some extent also related to shear stresses. So, it is a nice visualization like the stress ellipsoid lame is a stress ellipsoid we have seen in the last class or the last lecture. So, here what we see let us see let the frame of reference be again chosen along sigma 1 sigma 2 and sigma 3 axis.

We are choosing those as the axis and a plane that is equally inclined to that to these 3 axis is called an octahedral plane. This is a also used in some book for equilibrium equations for surface. This plane looks like this plane; so, if we look at the description what is said that a plane that is equally inclined to these 3 axis is called an octahedral plane. We are talking about this plane because this is equally inclined to all the 3 axis I have not given any axis number it can be easily given following the system what we are doing.

This is x this is y and this is z. So, following this we can say that this is the octahedral plane and such a plane will have n x n y n z equal definitely it has to be equal because it is inclined to same way in the to the axis and definitely to for a Cartesian system that this has to follow an octahedral plane will be defined as plus minus 1 by root 3. There are eight such planes now it becomes difficult to imagine.

Why eight such planes? Let us try to observe, say this is the plane which is xz and we also have one more plane better to change the color ok one more plane is bisecting it perpendicular way and that plane we can say that this is zy plane. Now we can imagine one more plane in the Cartesian coordinate system which is something like this. Now with this notation with this drawing we can easily imagine that and this may be this is then what which plane this is then xy plane.

So which quadrant we have drawn here, this quadrant this quadrant is; this quadrant we have drawn we have talked about this octahedral plane yellow is not visible at all anyway it is there. So, similar to that we can have so there are 8 quadrants in 3 dimension and we can find out that is the reason here it is said that there are 8 such planes. So, this is in one quadrant and similar way we can have 8 quadrants because that is created by 3 mutually perpendicular planes and that may have 3 quadrants.

So let us go to the to the visualization or octahedral states what we are we are discussing. The normal and shear stresses on these planes are called the octahedral normal stress and octahedral shear stress respectively. Now from the previous equations what we have already derived the sigma we can easily find out putting those values for the sigma n n and sigma n or sigma n n that becomes sigma 1 one third of sigma 1 + sigma 2 + sigma 3 and that is what is written here as i 1 that is the stress first stress invariant.

This first stress invariant is also introduced in the last week and it is said why it is said invariant because irrespective of coordinate system it remains same. So, whatever way the it is stressed whatever value the stress system it means external load it remains same then in any condition the sigma 1 sigma 2 sigma 3 also remains same and the sum of those 3 is also a constant known as the first stress invariant or i 1 sometimes this is also given as a 1 in some books. So but both are same.

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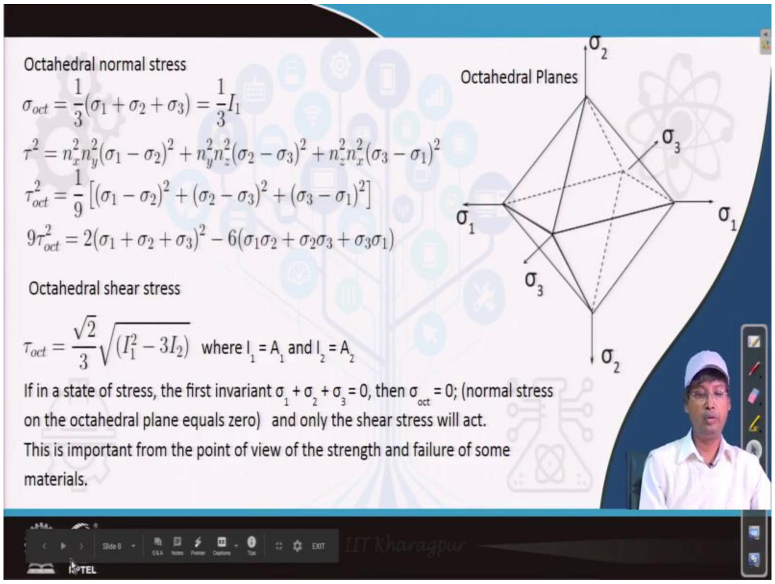

So the octahedral stresses we have this is what we have and the tau the shear stresses the normal stress is this and the shear stress we can find out substituting the values of direction cosines in this equation. And what we can find out that the tau is equals to this and in do simple maths we have 9 tau octahedral shear stress square is equals to 2 into i square or the first invariant square plus 6 into this is the second invariant.

If you look at the equation in the previous class this is the second invariant i 2 or a 2 there it is given as a 2 and that simplifies to that tau octahedral is equals to root 2 by 3 root over i 1 square - 3 i 2 so that 2 is taken outside 9 is coming so it is there is no such problem. And those 8 planes are drawn here to give you the understanding in those octahedral planes the normal and shear stresses are having some invariant value because it is a combination of invariant so that those values always remains constant.

On those planes normal and shear stresses and that makes an interesting observation sometimes for some material property determination we need to evaluate this. So, for a better visualization what is what we can do we can mark the access system here. So, if we mark the axis system here this is the first octahedral plane what we have already seen and the remaining are quite visible from there. So, with this if in a state of stress the first invariant is equals to 0 that means the octahedral normal stress is equals to 0.

And only the shear stresses will act normal stress on the octahedral plane equals 0 it may happen that in some cases the first invariant if it is 0 there only the shear stresses will act. Some example I have drawn a typical case in 2 dimension I have drawn for pure shear. So, there was no normal stress acting so that type of pure shear may act this is an important from the point of view of strength and failure of some material.

So that pure shear sometimes govern the material property and that is the reason we need to find out this and we need to check whether the pure shear condition in 3 dimensional stress concept is with sustained by the material or not and or how much it can sustain.

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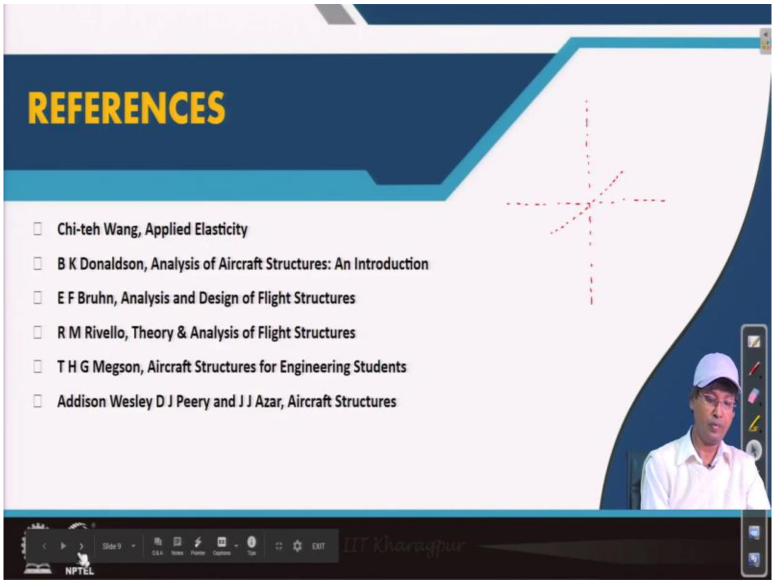

So with that same note we have come to the end of the lecture today the books are remain same that is why it has been written there.

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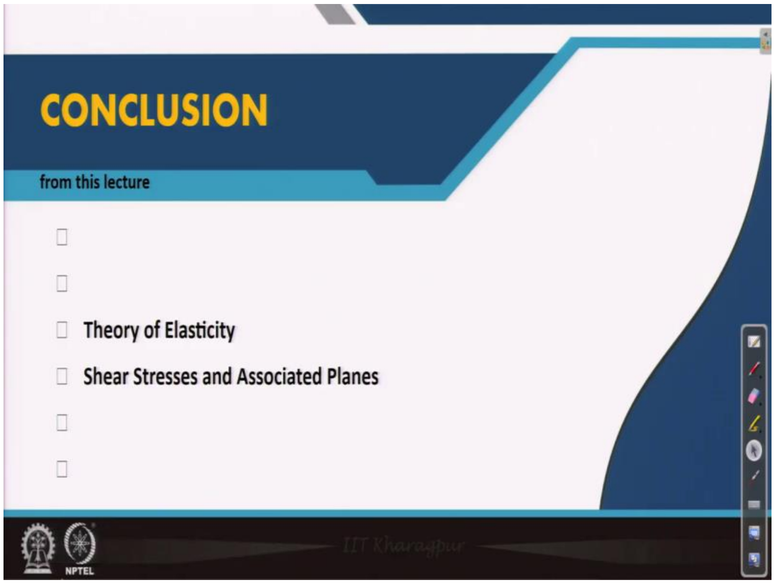

All the text whatever I have covered so far or may be covering in future slides all are taken from some books either this book or that book so I would suggest if you do not find. And it is difficult to pinpoint what is taken from what which book it is sometimes a mix of different books that is the reason I would suggest you follow those books if required. And with that I thank you for attending today's lecture and we will meet again with our next lecture soon, thank you.