Aircraft Structures - 1
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Lecture No -26
Theory of Elasticity - Principal Stress Boundary Condition

So, welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department IIT Kharagpur. We are in the 5th week lectures to the introduction of stresses we have covered different conditions of stresses. We have covered stress transformation and in that sense we have in covered the concept of principal stress. Why we need to find out where and how does it act.

It is on the changing plane in the direction we need to consider and we have also considered the boundary conditions and we will go forward further with some example on boundary condition and a little bit more note on the principal stresses.

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So, as usual the recapitulation slide if we look at already we have covered history of aircraft and aerospace structures as well as solid mechanics. Various types of external loads and conceptual structural details we have covered flight envelope and load factors we have covered how a shear and moment on wing and fuselage of an aircraft truss, aircraft structure comes that we have done truss and space trusses also we have done.

We have covered different unit load methods Castigliano's theorem and Rayleigh-Ritz method under energy methods. we have found deflections different ways of finding deflections of structure.

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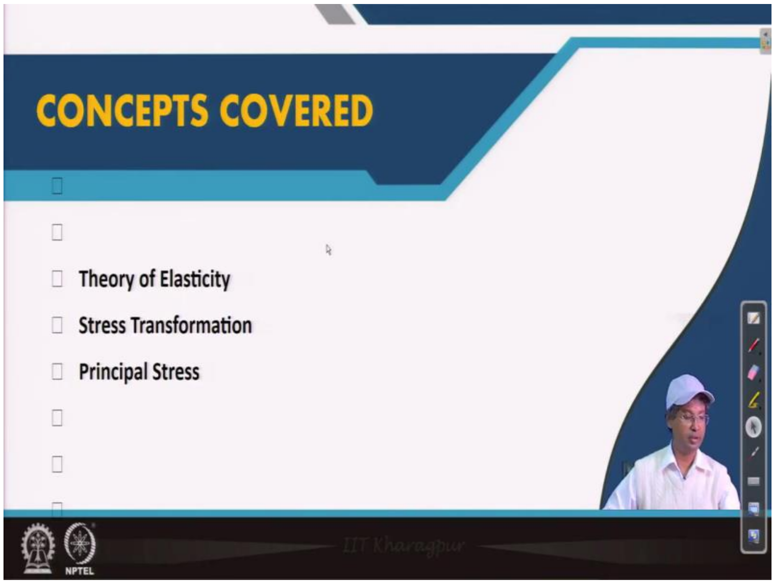

And we will go forward in the theory of elasticity classes we are concerning only about strain and we will see how some example and we will talk more about principle stresses.

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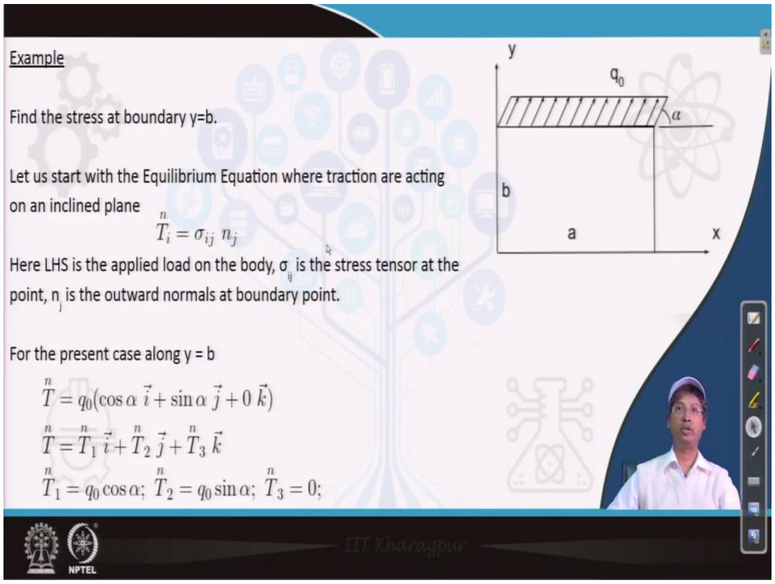

So, the example we have is that it is a an element of length AB so let us start with the equilibrium equation find the stress at boundary y equals to b at this boundary. Let us start with the equilibrium equation where tractions are acting on an inclined plane T. We need to find out T is given by this q 0 we are supposed to it is given this we are supposed to find out this is also given from the element orientation.

So here left hand side is the applied load on the body sigma ij is the stress tensor at the point n j is the outward normal's at boundary points what are the normal's here. So, for the present case along y equals to b here it is angle with alpha, q 0 alpha the two components will be there i and j x and y direction definitely the third direction is 0 there is no such component q 0 cos alpha I, q 0 sin alpha z are the two components.

And accordingly T 1 T 2 we will get T 1 is q 0 cos alpha T 2 is q 0 sin alpha. So, keeping in mind this as we have seen in previous classes we can easily go further and find out the components.

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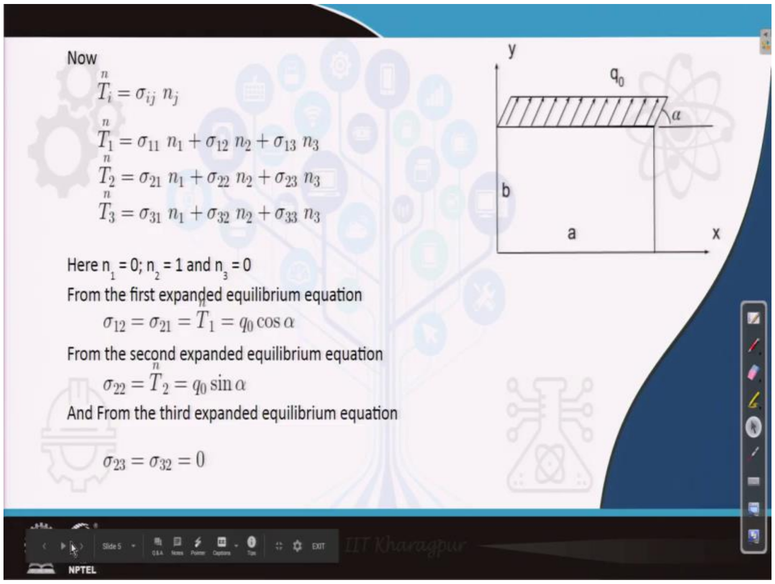

So, what we have is that T i is equals to sigma ij n j n values beta 2 this, this is the expanded form in the previous slide we have found out T 1 T 2 value, n we know we know means we need to denote it that it is in one is equals to 0 because we are talking about this one, n 2 is in this direction equals to 1 and n 3 is equals to 0. If it is inclined some plane then it probably will have n 1 and n 2 value so here it is not having that n 1 value and n 3 is always 0 because we are considering two dimensional problem with help of three dimensional equilibrium equation.

So now what we are putting we are simply putting those values T 1 is equals to this sigma 1 2 or sigma this, this equation we are using this is 0 this is 0 sigma 1 2 is equals to T 1 is equals to k 0 cos alpha and since complementary shear stresses are same that is why 1 2 2 1 is made to 0 made to equal and from the second expanded equilibrium equation what we have is that sigma 2 2 is equals to q 0 sin alpha.

And from the third one what we have is that there is no 2 3 or 3 2 shear component because it is a two dimensional stress condition definitely there would not be any shear component acting from perpendicular to the board so definitely those are 0. So, this is a nice small example this example explains this equilibrium equation use of this equilibrium equation very well. It is aim is to give you that idea why and how we can use this equation so with that concept you can apply this equation for further use.

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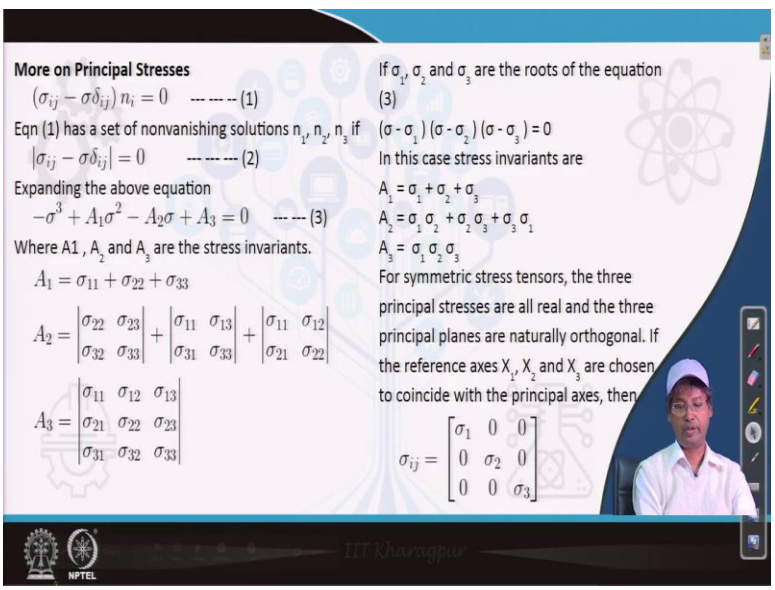

So, we come back to the principal stress principle stress has been derived in two dimension it has been derived in three dimension also. And the equation what we got at the end of the last class is similar to this and we said that we for the non violation vanishing solution of n 1 n 2 and n 3 the determinant of this the previous class if you have seen; previous lecture that portion should be 0 sigma ij minus sigma delta ij must be equals to 0.

And if we make that thing equals to 0 and write it in expanded form this matrix calculus I would suggest you please carry out in some books it is available in what in most of the books it is kept but I would suggest you once do if you have some time. You will find these constants and with respect to these constants we can easily express that equation this equation as minus sigma 3 cube + a 1 sigma square - sigma cube + a 1 sigma square - a 2 sigma + a 3 is equals to 0.

So that this has to obtain while a one is equals to sigma 11 sigma 22 sigma 33 general stresses coming from this sigma i j a 2 also components of sigma i j as we have seen. In this form but this multiplied by this minus this multiplied by this plus plus plus plus this is a Jacobean type of Jacobean matrix anyway we need to call consider that and it is the a 3 is the determinant value of that particular matrix sigma ij.

If sigma 1 sigma 2 and sigma 2 are the roots of the equations then definitely this will have this equations follow this equations and this sigma 1 sigma 2 sigma 3 are the principal stresses and we can easily prove. There are proofs available we are not going to prove here that this a 1 what is expressed here that also follows that if sigma 1 sigma 2 sigma 3 are the roots or the principal stresses in three orthogonal planes.

It also follows that a 1 is equals to sigma 1 + sigma 2 + sigma 3 a 2 is equals to sigma 1 sigma 2 + sigma 2 sigma 3 and sigma 3 sigma 1 and a 3 is equals to sigma 1 sigma 2 sigma 3. For symmetric stress these are known as the stress invariance that means for a; this concept is a kind of something whatever may be the stress condition of a three dimensional stressed element represented by sigma ij.

Better we write sigma ij we can find out a orientation of the axis system or we can say we can find out three planes which are mutually orthogonal to each other those are having 0 cr stresses and acted upon by the three principal stresses sigma 1 sigma 2 and sigma 3. So, again if I look that if sigma ij is with respect to this coordinate system we can find some coordinate system some orientation may be this way this way or the other way.

In those coordinate system we can have three perpendicular or orthogonal planes on those three orthogonal planes there are three normal stresses acting. Those normal stresses are the principal stresses and on those planes there are no shear stresses and from mathematics point of view we can prove that those stresses principle stresses always obeys this rule that means this is equals to this the relation between the first coordinate system to the principal coordinate system is this is equals to this, this is equals to this and this is equals to this, this is known as the three stress invariance.

I hope to some extent I have made it clear some symmetric stress tensors. For some symmetric stress tensors the three principle stresses are all real and the three principle planes are mutually orthogonal just now I have said that it has to be mutually orthogonal following the Cartesian coordinate system if you see that is ensured in the derivation in the with help of the Lagrange multiplier. If the reference axis x 1 x 2 and x 3 are chosen to coincide with the principal axis this is interesting i said this is the first coordinate system x for sigma ij.

Now if this sigma first coordinate system this ij coordinate system is coinciding with the principal coordinate system then it is said that there is no shear stress that is why these components are 0 that is what is chosen the coincides with the principal axis then sigma ij is equals to this. So, as it is said that if the principal axis is the coordinate system of x 1 x 2 x 3 though in that those planes there is no shear stresses. So with that idea of principle stress in three dimensional state.

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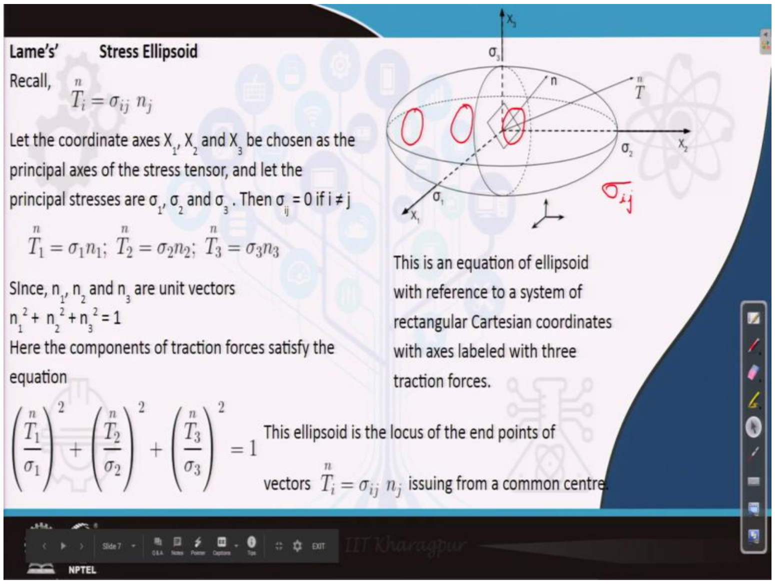

We would like to move forward, this is some interesting observation like the previous one what we have seen this is a some property it holds that is what we will talk its very interesting property. The equilibrium equation condition we have already seen that is the equilibrium equation condition. Let the coordinate axis x 1 x 2 x 3 be chosen as the principal axis of the stress tension and late the principal stresses are sigma 1 sigma 2 sigma 3.

Then sigma ij is equals to 0 while i is not equals to z. Then easily this components are T 1 is equals to sigma 11 n 1 T 2 is equals to sigma 2 n 2 T 3 equals to sigma 3 n 3 this comes very easily this has to follow unit vector. Here the components of the trajectory using this and this we can easily come to this equation which says that T 1 by sigma 1 square + T 2 by sigma 2 square + T 3 by sigma 3 square is equals to 1.

This is nothing but an equation of ellipsoid or three dimensional ellipsoid. Now what is this we have a relation what does that mean it means that if we consider a coordinate system coinciding with the principal axis then for any body which is have showing that system or the stress system follows or represents an ellipsoid by the tip of the vector T i this is I will try to again say better before I will try to let me read this is an equation of ellipsoid with reference to a system of rectangular Cartesian coordinates with access level with the three traction forces.

The ellipsoid is the locus of the endpoints of the vector this issuing the common center. This is this is probably the best way it is expressed. So, it is if both are having the common center the traction and the principal axis then the traction vector tip is if it is plotted we will find different points on this and if we join those points it gives me an ellipsoid that is represented by this equation. This is a interesting mathematical phenomena and it follows that rule.

We sometimes use it for design purpose this becomes some design criteria to check and with that consideration we would like to come to the end of todays lecture.

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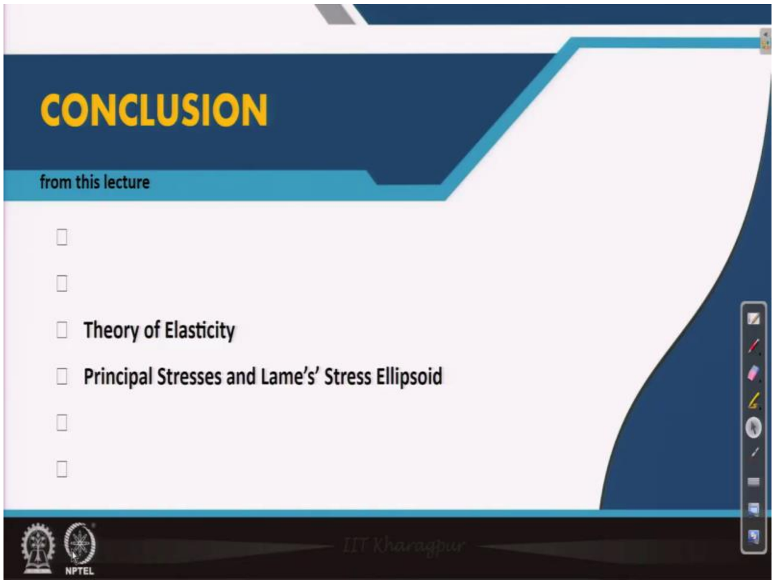

Where we have discussed about principal stresses and Lame's' stress ellipsoid and we will come back with some more things on stresses. And thank you for attending this lecture we will come back with few more idea of stresses and further on strengths and some examples, thank you.