Aircraft Structures - 1
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Lecture No -23
Theory of Elasticity - Stress

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering department IIT, Kharagpur. This is the first lecture of module 5 or week 5 the number in that series is lecture number 23. We will start theory of elasticity the first lecture consists of mainly the stress, definition of stress and components of stresses correlation I would like to put on stress correlations with the previous things we have discussed and accordingly we will go forward this class is with simple thoughts of how to; what is stress and how does it act where what are the components we would not go much beyond.

But slowly in the future lectures we will start go into depth we will go into sometimes two dimensional stress sometimes three dimensional stress considerations will do and accordingly we will proceed for problem solving with that.

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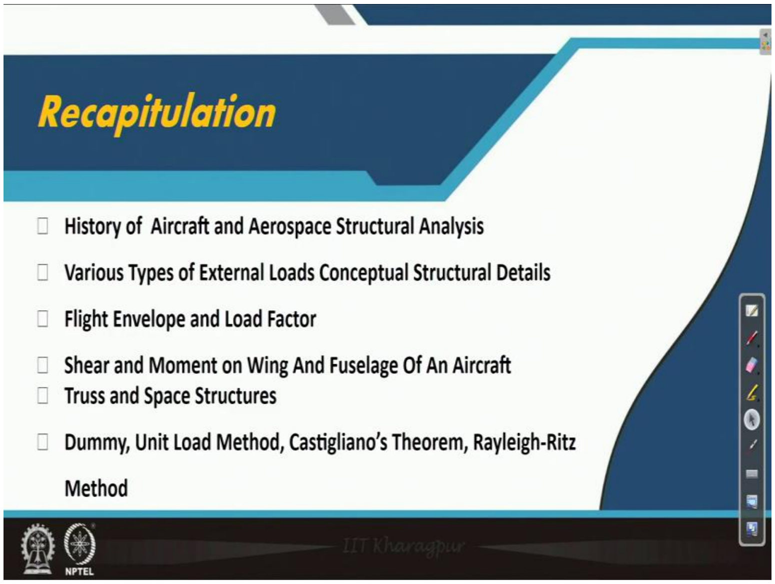

So, the next slide what we see is that usual recapitulation slide. In this slide we have already seen the history of aircraft solid mechanics structural analysis various types, types of external loads conceptual structural details. We have seen what is flight envelopes and load factor is. How do we need to restrict our design within the flight envelope, shear and movement on wing and fuselage of an aircraft we have seen how does it act with a typical example we have solved that those problems.

Unit load methods we have considered then we have gone to the truss and space structures, especially the space structures we have covered there. We have solved problems with landing gear interesting problems with landing gear, preliminary studies for landing gears are still done that way unless you have a facility of advanced computations. But even then many times the way it has been analyzed considering axially loaded members the same way it is still solved for the first iteration.

Then in detail design there are many other ways to do more detail CAD drawings are prepared and accordingly boundary conditions are put to find out things that is also true in case of movement and wing moments on wing and fuselage. But whatever we do you better note down that the thing what we will be learning today that you are already introduced with the stress, components of stress in most of the cases you have solved problems in two dimension.

But our approach mainly is three dimension in some of the cases where three dimensional approach is too complicated it is difficult to give you a proper understanding. On those cases we will go for the two dimensional analysis and we will see how stresses are acting on a body what are the components, how does it act where and how does it act? All that is the main aim of this we will try to see some examples.

We will try to see how the stresses may act and we want to concentrate more about deflection what we have already covered in this the last bullet as you see here that dummy load, unit load energy methods that is the dummy load consisting of dummy load, unit load, Castigliano's theorem, Rayleigh-Ritz method all those comes under energy method. In Rayleigh-Ritz also we have considered energy it is not always the energy sometimes some other functional is also used but predominantly energy is considered.

But those things are not related to stress first of all better too note that in most of the cases we have found out the deflection. Deflection also is a design criteria it is not that only stresses are the predominant thing which we need to monitor. Say one good example I understand is that just try to visualize the wing of an aircraft and say during takeoff it is in most of the cases the civil aircrafts are are made such that the fuel is stored inside the wing.

And say if the fuel is stored inside the wing in that case the wing will definitely bend down. Now if the clearance is not proper the engine may start the ground or the clearance distance may reduce so that may create many other problems. So, deflection plays a big role not only that say when it is airborne the lift is acting upward on the wing and because of that it changes the normal angle of attack.

So in the deflected position what is the changed angle of attack that is also important so in that way deflection is important. We will see how stress is important we have solved some problems in our mechanics related to stakes sorry stress. In your in your previous courses they are probably you have covered the course of mechanics engineering mechanics and there you have covered stresses but will re look into it will try to see what are the stresses how does it act.

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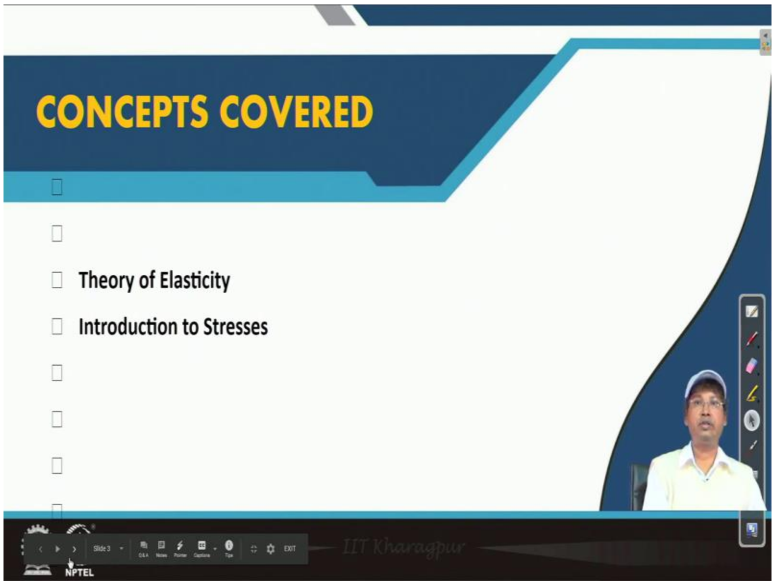

So with those notes initial nodes we will go further to see what is stress how does it act? With those introduction to stress that is the reason we say we are starting the topic introduction to stress and we will go further.

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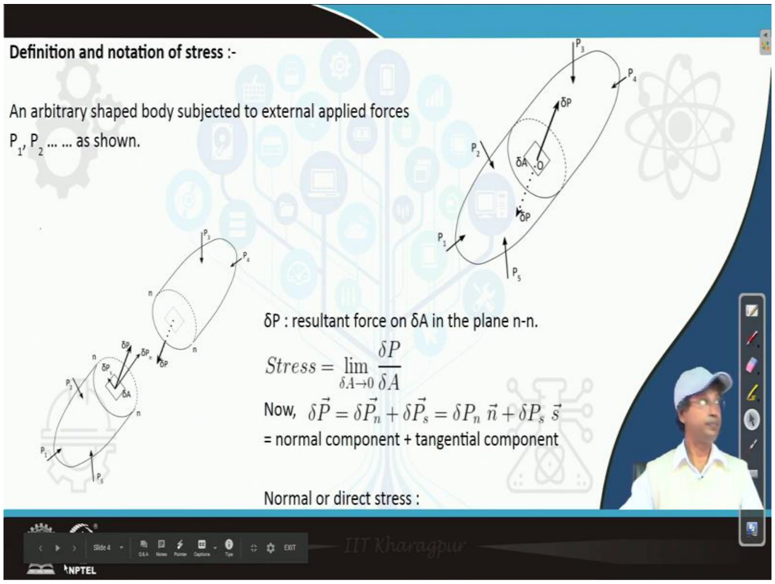

In this if we see definition and notation of stress this is the main topic in one or two slides we will be covering. What we are considering here that an arbitrary separate body subjected to externally external applied forces P 1 to P n here it is up to shown P 5 it may be considered that it is up to P 5. And what we are considering that inside it is not that this surface is perpendicular to this vector this delta the surface what is shown at O consisting of area delta A is perpendicular to delta P.

But what we are considering here that because of these external loads P 1 P 2 P 3 P 4 P 5 there is a resultant force acting which is delta P and that delta P is acting inside the body on the surface delta A which is located at O. So, for balance definitely there is one more force acting there if we separate it out both will act to the two directions that is what is shown here. If we consider the lower body and if we talk show that delta P is acting this way it has two components delta P n and delta P s.

This delta P n whenever this subscript n is there we consider that, that is in the direction to the normal to that surface n goes for normal to that surface and s is tangential to that surface delta s is tangential to that surface that is the reason we have two components of delta P one is delta P n and the other is delta P s. So, these two forces are acting on this area delta A. So, delta P resultant force on delta A in the plane n-n stress this is the definition of stress please note that stress is equals to limit delta A tends to tends to 0 del P by del A.

So from the definition of limit you have already learned in mathematics what we get that that is equals to the stress we denote that by sigma. So, if that is the stress sigma and what we see now as we have already considered that the vector delta P is resolved to two components delta P n and delta P s and it is in the direction normal to the surface it is in the direction tangent to the surface that we have written here.

We have written that normal components plus tangential component normal or direct stress will be there in the next slide. But before that let us consider an example of simply supported beam where we have a central load of P acting downward as P and the length of the beam we consider that is equals to L. So, at this we have reactions P by 2 we have reaction P by 2 and if we talk about the bending moment diagram, bending moment diagram will be P by 2 multiplied by L by 2 is the this amplitude.

And this will become equals to PL by 4 and shear force if we talk about that will be oh sorry like this so what I want to mean with this example is that at any section if we consider this section and if we talk about if it is a rectangular cross section what are the forces we have we have bending moment say this is what is acting. So, on these we have external bending moment acting like this as well as we have shear force acting like this or in may be the other way direction is not important thing what we are considering now.

But what I want to mean that because of this there is a shear force acting in this plane as well as we have normal forces acting on this half as may be tension here and other half it may be compression. If it is pass this way the other half is this way. So, this way it will create a combination of stresses which we need to find out. So, like that this body is considered as a simplest way the stress is what is acting there and accordingly we are trying to define the stress considering a simple force system.

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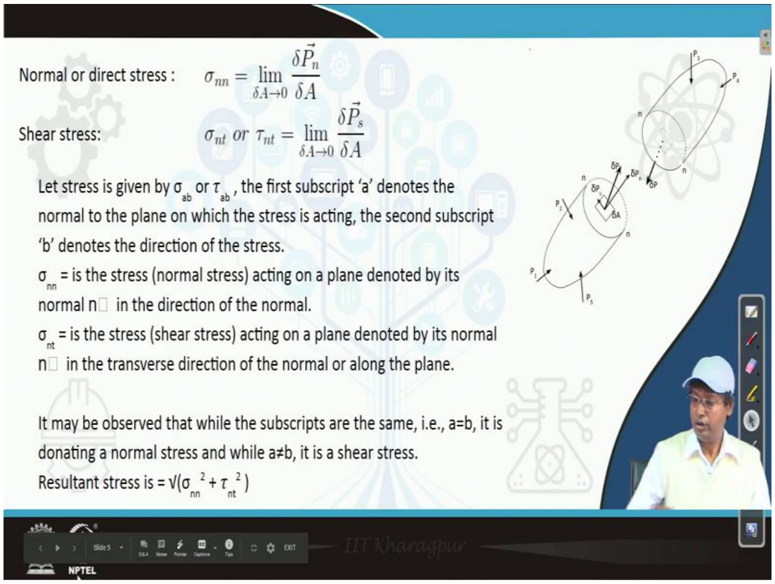

So let us move forward. So, what we have here is that normal or direct stress sigma n n is equals to delta P n by delta L let me delta a tends to 0 and sigma int sometimes tau n t is also used sigma and tau is freely used in this case but sometimes when not mentioned sigma is the normal stress and tau is the shear stress but it is not a very, very necessary rule sometimes both are used for showing normal as well as shear stress.

So let stress in give is given by sigma ab or tau ab the first subscript a denotes the normal to the plane so like that here if we talk about on which the stress is acting, so if we bring back the example of the beam and if we consider this is the axis x it is supported something like this and if we consider a rectangular cross section of the beam in that case the normal stress acting here will be sigma x, x and the shear stress if we consider this is y this will be shear stress tau xy it is on x plane in the y direction.

So that is what is stated here the first subscript a denotes the normal to the plane on which the stress is acting the second subscript b denotes the direction of the stress x is on which plane it is acting y is along which direction it is acting. Sigma n n following that sigma n n is a stress normal stress acting on a plane denoted by its normal n some computer problem in the direction of the normal and sigma nt is the stress or shear stress acting on the plane denoted by the normal’s in the transverse direction of the normal acting on the transverse direction of the normal or along the plane.

So repeatedly we have said probably you do not have any more confusion about it. So, according to the coordinate assumption of coordinate axis if we go for x y z axis considering say this is x this is y and this is z we will be following right hand screw system. So, following that x y and z is this way we can have different such components as it is shown in case of beam considering only the cross section of the beam as rectangular cross section we have two components.

But if the if the beam is loaded also transverse direction say something this way from this direction from the from me towards the board or from the board towards the me that means the j in z direction then definitely there will be components in that z direction and other forces also will come. So, those things we need to keep it in mind. These stresses also will vary depending on the type of material we are considering.

It is not that always the sigma x remains same way it depends on the loading it depends on the type of material whether the material is isotropic or not whether the material is composite or two different materials put together that type of beam or not or beam is the common example you have come across that is the reason we are i am trying to give you example of beam again and again.

So, following that there may be many other components the components total number of components what we may get we will see. So, it may be observed that while the subscripts are the same that is a equals to b it is denoting a normal stress and while a is not equals to b it is a shear stress resultant stress as usual from vector rule what you say is amplitude is that sigma n n square plus tau nt square root of that those two sum of those two squares.

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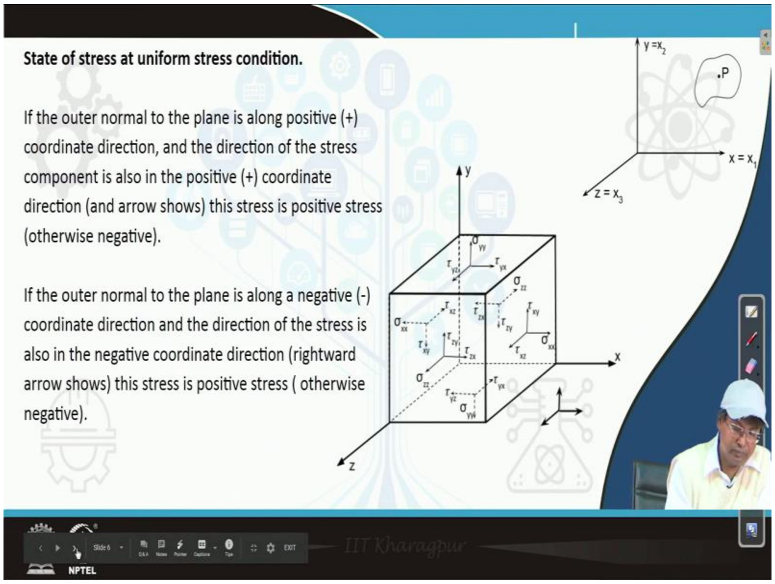

Now we come to the state of stress at uniform stress condition. So, this is something better to note that uniform stress condition. Uniform stretch condition is difficult to prevail in practical cases. So, one example we may think at present some practical example is that say we have a cantilever beam and we have a projection something like this and one load is applied at the tip say P.

So in this particular case what will happen this may be replaced as a beam having whatever the length is say if this is the length l then this is a moment Pl and one axial force P. Now any cross section if we talk about of this is it is a rectangular cross section and as we have already mentioned there is a variation of stress due to the bending but this is there is no change of bending moment Pl throughout the length of the beam and that remains same for the total length of the beam.

So that is a kind of uniform stress condition for the total structure we may talk about and in this particular slide or in the next slide we will consider that there is no change of stress at for that for the shake of definition. So, with that thing keeping in mind that thing this is one example how a material or structure may be stressed where the stresses are uniform is very rare it does not happen in general we need to think a lot to find out this type of case.

Now what we are trying to do we are considering any point save of the beam for this particular case beam and in this beam definitely if we consider this beam a point this point is inside this beam then definitely we would not have all the stress components as it is shown here. So we need to think about a structure which is loaded from all possible sides and inside that we are considering a point and that point is drawn as a cuboid as shown here.

Now let us see about sign conventions and the force stresses acts on that particular point. So, if the outer normal to the plane is along the positive coordinate direction out turn normal say this normal in the positive coordinate direction and the direction of the stress component is also in the positive direction, direction of this also in the positive direction. This stress is positive stress so otherwise it is negative.

So what it is trying to say that if the normal is if I draw a normal on this particular plane say this plane that will definitely in this direction if that direction is matching with this direction as well as if the stress is also acting say stress is also acting but stress is also acting in the same direction then we are cons saying that that is equals to positive stress. So, similar way this tau xy is also in case of shear it is acting in this y direction and acting on the plane which is a shown by a positive normal.

So this is positive this is also positive this is also positive so with that consideration if we look at that sigma x in this particular xx is positive tau xy acting on the plane x in the direction y tau xz acting on the plane x in the direction z as it is shown here is that if the outer normal to the plane is along the negative coordinate direction and the direction of the stress is also in the negative direction coordinate direction this stress is positive stress.

So on the positive we have shown one more way the stress may be positive that way we need to consider this one, this component of stress. What is it says that normal to the plane is along the negative direction this is the negative direction negative x direction and the direction of the stress, stress is also in the negative direction. Direction of the stress that is this one is also on the negative direction.

So in that case we consider that sigma or the stress as positive stress. So, this is also positive stress this is also positive stress otherwise it is negative stress. So, in that way if we talk about we consider there are how many components we may think of we can think of that there are three orthogonal planes that is we can say x plane y plane z plane each are having three components. So, we have 9 components of stresses, so with this 9 component of stresses the possible types of stresses possible variation of stresses and how do they act is to some extent defined.

So to note again let let me briefly repeat what are the things coming. These three are acting on x plane these are also acting on x plane but in the opposite direction. This is and these are acting on y plane and one more we have that is this and these are acting on two z planes. So, in two directions it is acting. So, considering a cuboid in a point what we have shown that there are possible 9 components of stresses and with those 9 components of stresses we let us move forward.

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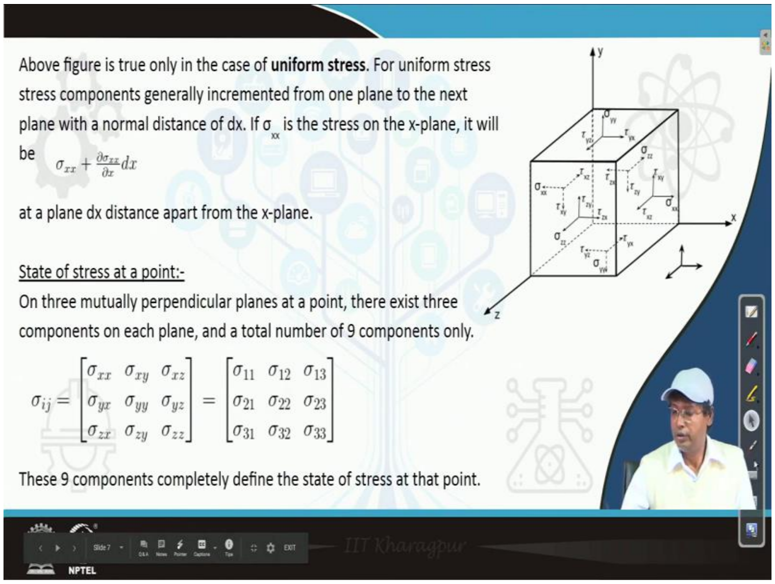

So above figure is true this figure only only in the case of uniform stresses as I have already mentioned it is true for uniform cases one example of a similar type we have seen. For uniform stress components generally incremented from one plane to the next with a normal distance of dx it is for non uniform stress. Please this is for non uniform stress, for non uniform stress, stress components are generally incremented from one plane to the next plane with a normal distance of dx.

If sigma x is the stress on x plane it will be incremented as sigma x plus del sigma s del x which is the gradient and the length is multiplied at the dx distance apart it will be incremented this way. So, in the next lecture we will be considering equilibrium we will see how non uniform stress is considered and how these things are taken care at a plane dx distance apart from the x plane. Now stress at a point if we continue in that sense on three mutually perpendicular planes at a point there exist three components on each plane and a total number of nine components only that we have already seen.

How those things act and generally the total all the 9 components is generally given by sigma ij here we also need to say that where i j is equals to 1 2 3. So, with this definition this sigma ij shows if I say only this and this these two completes the definition of all the 9 components of stresses this is with respect to Cartesian coordinate system it is given. This is a considered sum coordinate system which is also orthogonal and named as 1 2 3 not x y z.

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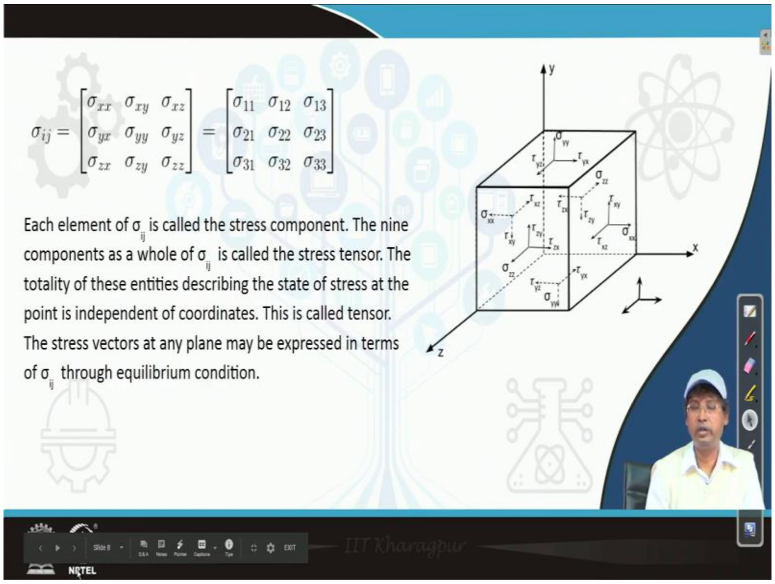

So with this note we would like to complete the definition one more small note is there as it is mentioned in the previous slide that is the each element of sigma i j is called the stress components. The nine components as a whole of sigma i j is called the stress tensor as we have said in the last like last slide. The totality of these entities describing the state of stress at a point is independent of coordinates this is called tensor. The stress vectors at any plane may be expressed in terms of sigma i j through equilibrium condition. So that is the next thing we will learn the equilibrium condition let us proceed.

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In that but before that I would like to thank you for attending this lecture and we will move forward for equilibrium analysis in the next lecture.