Aircraft Structures - 1
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Lecture No -21
Rayleigh - Ritz Method

So, welcome back to aircraft structures course aircraft structures one this is professor Anup Ghosh from Aerospace Engineering IIT, Kharagpur. We are in the 4th week lecture series and that is why the module 4 and the lecture in sequence is 21. We will get introduced to the method Rayleigh-Ritz method. Rayleigh-Ritz method is really very, very important it is difficult to say how important it is. It is important because probably these lays down the process of approximate analysis and as a whole it lays down the process of computer based numerical methods.

So as such you would not find any link to those methods most popularly known as in structural or solid mechanics as finite element method. But it has correlations it will you will get introduced to it slowly as far as I can I will try to give you those glimpses at this stage it is difficult to discuss those things but I will try my best to give you those introduction. So, with that idea we will we will try to cover the Rayleigh method with one small example.

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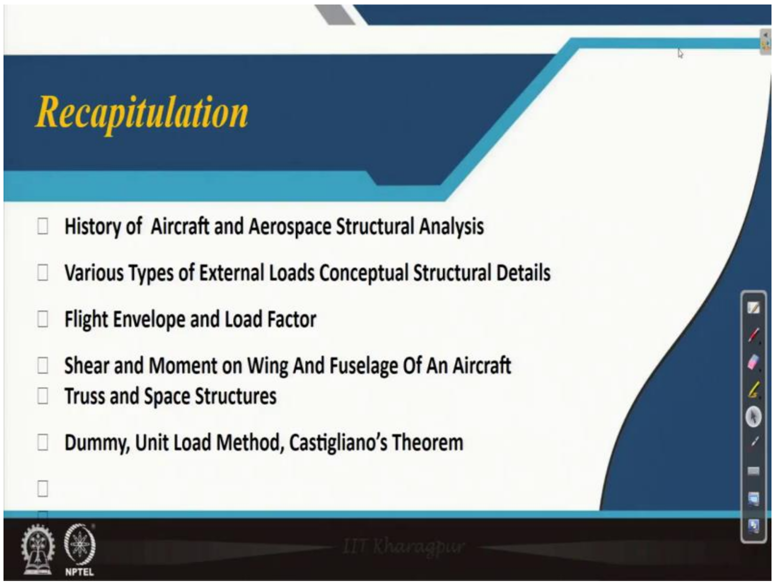

So let us proceed with the next slide which is as usual a recapitulation slide. In this slide what we see is that we have already covered aircraft and aerospace structures analysis history of those we have covered various types of external loads, conceptual structural details. We have covered the flight envelope and load factor, how the flight does the fight envelop looks like and why it is so and how the load factor varies.

We have seen with examples shear and movement coming to wing and fuselage considering a typical example. We have seen truss and truss in the sensor it comes always plane truss but we have seen also space truss or space structures. And we have solved a few examples related to that especially the landing gear problems and then we got introduced to energy methods. And in that sequence we have already done a dummy load or unit load method and unit load method and Castigliano's theorem. Today we will do the Rayleigh-Ritz method.

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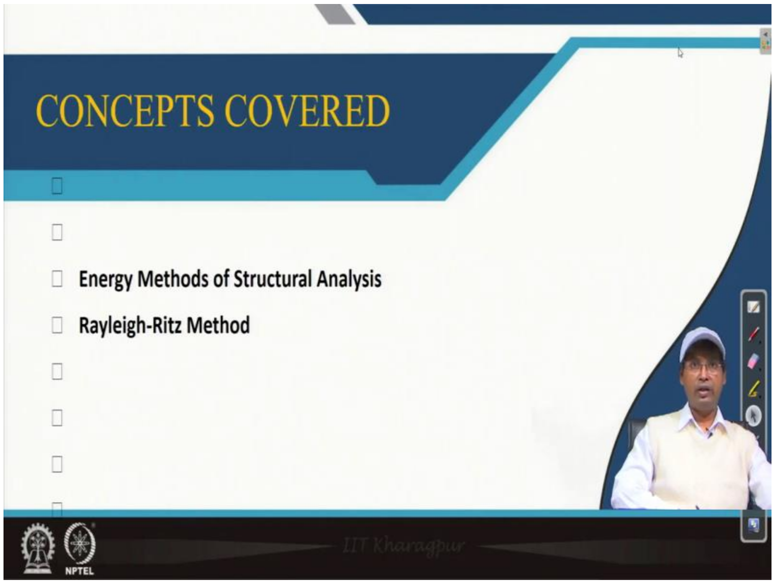

So in the Rayleigh-Ritz method it is better to understand that it is an approximate method.

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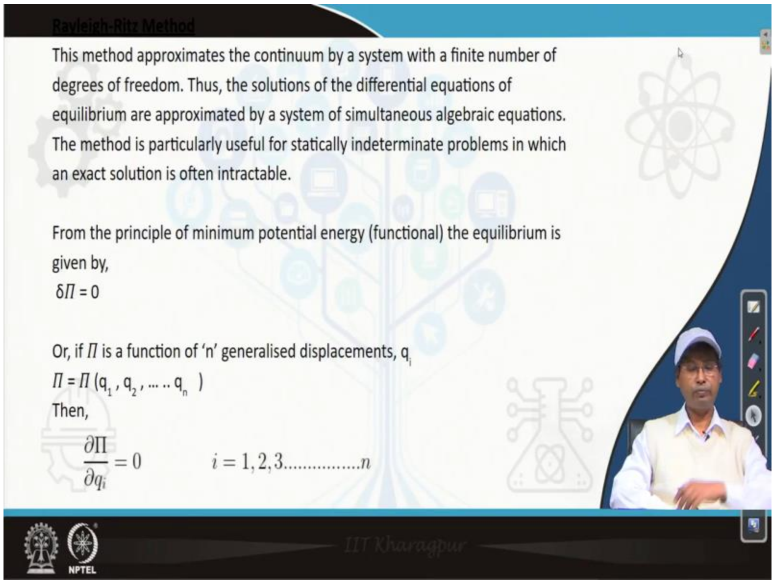

First keeping in mind that idea let us try to understand it is it is a very brief way explained. It is explained or discussed as brief as it can be because it is getting introduced for the first time. There are big books on these methods to do as we have already discussed that these and some other methods like the variational principle along with this method lays down the probably the first step for the numerical analysis procedure in terms of say structural analysis or solid mechanics problems or for fluid mechanics problems or for any other numerical analysis depending on the magnetism or other physics problems.

So let us see try to understand each and every word of what is said here. So, the method approximates the continuum it is a continuous system where continuity of all the variables what we are considering that is persisting by a system with a finite number of degrees of freedom this says a lot as I said so in we will discuss always in purview of solid mechanics structural analysis. So, what we are considering that a structure which is as such as having a continuity or it is a continuum is broken down to finite number of degrees of freedom.

In our solid mechanics we consider the degrees of freedom as displacements and in general we need to define those displacements as functions and we need to carry out those. So, that is the first step it says that it is an approximate method because it is getting divided in finite numbers of degrees of freedom it is not a continuous one. Thus the solutions of the differential equations of equilibrium are approximated by a system of simultaneous algebraic equations.

So that is what as I was discussing these degrees of freedom there may be many or may be repetition for considering small parts but though it is repetition for different boundary conditions it will give different values and definitely it will be easier to get simultaneous algebraic equations and we need to solve those simultaneous algebraic equations to find out the approximate solution.

The method is particularly useful for statically indeterminate problems in which an exact solution is often intractable. So, it is it is especially means it can it has a capability of solving indeterminate problem because the solution approach does not take care of whether it is a determinant or indeterminate problem in bigger senses while you will be using this you will find that the all the problems what in general we solve for practical purpose those are indeterminate problem statically indeterminate problem.

So from the principle of minimum potential energy functional the equilibrium is given by this capital PI shows that the total potential energy and it also says that a variation of it or a small change if I do not talk about in mathematical terms in physical terms if we talk about see by some means if we consider a small change of the functional here the total potential energy. So, that remains that leads to a value of 0 and that that is the fundamental equation and we need as we have already introduced finite number of degrees of freedom and it will lead to simultaneous equation.

This equation will lead to these segmentalian equations considering depending upon the degrees of freedom we choose and how do we choose. The problem remains how the degrees of freedom describes the problem properly and that is the way we get the solution we will consider this. In this lecture and also in the lecture followed by this two problems one in this lecture other one in the followed problem you will find depending upon the assumption of the degrees of freedom of variable description it depends the accuracy though for there are limitations for hand calculations so we will find approximate solutions.

Or if PI is a function of n generalized displacements qi as I have already mentioned here the degrees of freedom or displacements as it is mentioned then PI may be expressed as function of qi to q 2 to q n and then this is nothing but with respect to each and every displacement variable we are considering variation or in mathematical terms we say that it is the partial derivative with respect to that particular displacement and each and every particular partial derivative equation will lead to one equation.

So we will get n equations that is what is said simultaneous algebraic equations we will get and if we solve those n equations we can find out some approximate solution and we get benefited by this. So, let us move forward it is one more definition of the system in a little bit different way let us try to see what is that definition.

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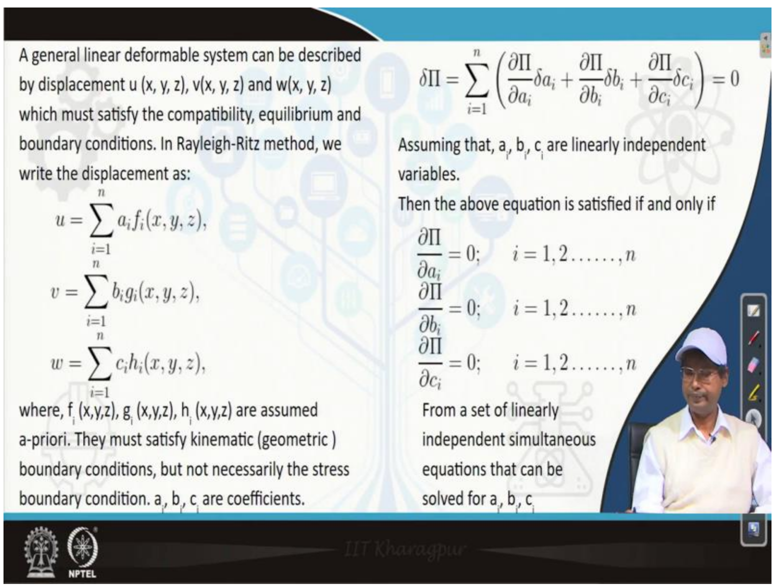

So, a general linear deformable system can be described by displacement u x y z, v x y z and w x y z which must satisfy the compatibility equilibrium and boundary conditions. Compatibility, what is compatibility probably were not introduced that way compatibility is that while we are considering this variable from one segment to the other it must maintain the relation between these.

So the strain has to be compatible or otherwise it will show the property of continuum. Equilibrium, definitely it has to maintain a equilibrium. Equilibrium equations we will do in the fifth week class. So, compatibility probably is covered in the sixth week class and boundary conditions in Rayleigh-Ritz method definitely boundary condition has to be satisfied in Rayleigh-Ritz method we write the displacements as u a i f i summation of i 1 to n summation of i equals to 1 to n a i f i x y z g i x y z where the coefficient is b i here the coefficient is c i and we define w.

So actually w which is anyone say w is function of x y z we also define that that is an equation where we have one more coefficient c i and the function and a function h i which is also a function of x y z where f i, g i and h i functions of x y z are assumed a priory. This plays a very, very important role we need to have an idea what type of function we should assume for f g and h it depends on the problem.

This is the key of the approximate method, infinite element analysis this is conforming I should not say exactly it is conforming or similar probably to the shape functions of an element. So, like that we need to assume this a priory because we need to know what problem we are going to solve is it a displacement problem, it is a stress problem or some other property we are going to solve.

So depending on that we need to consider and not only that what degree of accuracy we want to consider whether it is linear which is non-linear depending on all those things this compatibility equilibrium and boundary conditions will change and accordingly these functions will change. So, this function play a big role in that they must satisfy kinematic or geometric boundary condition but not necessarily the stress boundary condition.

So it has to satisfy the kinematic boundary condition and but it is not always necessary to satisfy the stress boundary condition but it is sometimes desirable to solve to maintain that a i b i c i are coefficients that we need to find out otherwise the functions we are saying assuming. So, the unknowns are a i b i c i only so we need to find out those. So, what we are considering again variation we are considering of the total potential energy and that gives us three partial differential segments in this equations which are del PI del a i del PI del b i del PI del c i then assuming that a i b i c i are linearly independent variables.

It concludes that that then the above equation is satisfied if and only if individually these are zero so that gives us n equations, n equations and n equations. So, we have three n equations we have three n unknowns as we have done and this simultaneous equations algebraic equations we are supposed to solve and we are supposed to find out but see for hand calculation it is not possible to solve that is the reason we will solve small problems probably considering one or two variables but principally it is the same.

So for a set of linearly independent simultaneous equation that can be solved for a i, b i and c i there. So, let us move for the example and try to understand how this method has been applied.

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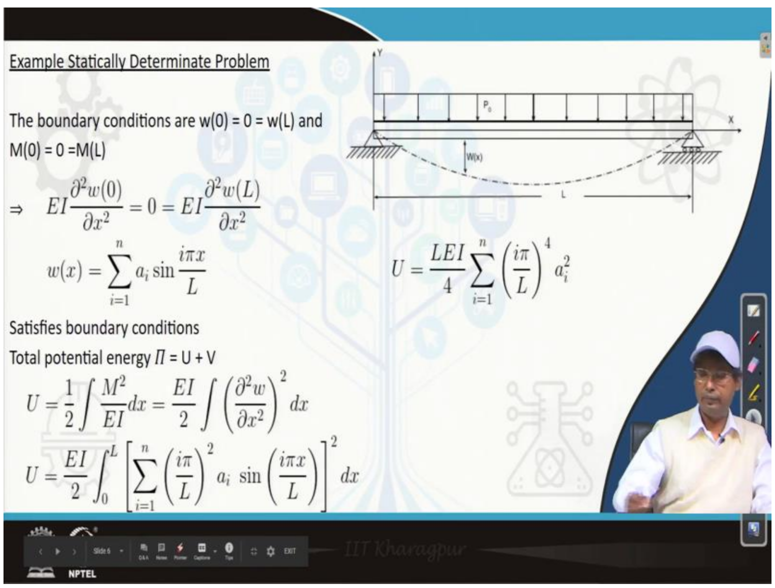

Example statically determinate problem: This problem is quite known problem you have you can easily solve this problem using the methods what we have already described. Those are like Castigliano's principle or unit load method, dummy load method all those things are complementary energy function derivative method all those things you can easily do. So, the boundary conditions are w 0 equals to 0 and w l is also equals to 0 that means we are supposed to assume the function w such that at this point it is equals to 0 as well as at this point it is equals to 0.

The w x is supposed to be like something like this and it also has to satisfy the moment boundary condition here that is the M 0 is equals to 0 and M l is equals to 0 as we know that double derivative of w with respect to the x as it is said from the center line sorry from the center line the EI del 2 w del x square is equals to 0 following this and following this we get that this is that is also equals to 0. So, we need to assume w such that it satisfies all these 4 boundary conditions these two as well as these two.

So now we are assuming it it requires some experience do not think that it may be assumed with two days practice or maybe one years experience of solving problem so the problem is assumed w x sorry the solution or that displacement function is assumed as w x equals to summation of i equals to 1 to n a i sin i PI x by L. So, sin function is quite it matches well let us see how good how good is our approximation to the exact solution it satisfies boundary conditions.

The potential energy total potential energy is PI is equals to U plus V. And U what we have already seen this one, so that has to be its simple derivation is considered here and from 0 to L as usual from 0 to L it is integrated and or square all those things I hope that you will be able to carry out this and finally we get the strain energy as U is equals to L EI by 4 summation i equals to 1 to n i pi by L whole to the power 4 a i square.

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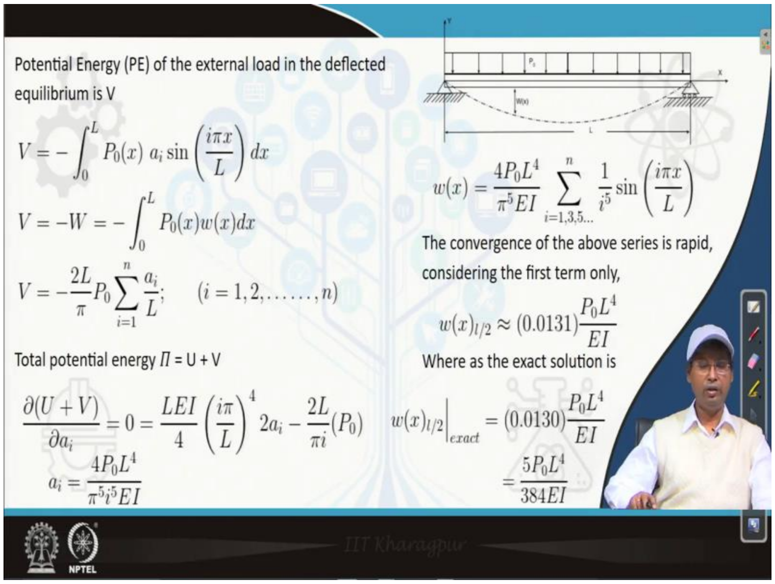

Potential energy PE of the external load in the deflected equilibrium is V. So, we need to find out that portion also the portion V of the total potential energy so that comes definitely minus of load into displacement that is what is given here P 0 x the uniformly distributed load and the displacement function w x what we have considered. So, it is integrated that way it is written the same way it is written twice just to avoid confusion and then integration is carried out that gives us that V is equals to minus of 2L by PI P 0 a i by L sum summed up over 1 to n for i equal value of 1 to n.

And then the total potential energy partial derivative with a i is considered if we consider with respect to a i this is this portion as well as a i is partial derivative is considered so and the previous one if we consider it if you step jumps are there you can easily get that I guess, so summing up of U and V and taking the partial derivative with a i gives us this relation and that is equals to 0 and this gives us that a i is equals to 4 P 0 L to the power 4 divided by PI to the power 5 i to the power 5 EI.

And then once we have the value of a i the a priori assumed displacement function w, w x is equals to 4 P 0 L to the power 4 divided by PI to the power 5 EI summation over i equals to 1 to n and 1 by i to the power 5 sin i PI x by L. The convergence of the above series is rapid you can easily test it probably by this time you have requires some skill of numerical or coding you can easily do a coding to find out how first it get converges so that you can easily do.

But in this work what we have considered only consider the first term and if we consider the first term what we have at L by 2 putting the value of x equals to L by 2 we have the deflection central reflection here as 0.0131 P 0 L to the power 4 by EI and you please note that this is the exact value 0.0130 which is nothing but 5 by 384 P 0 L to the power 4 by EI. So, with the first term only it is quite accurate right.

So with other terms you can easily check how quickly it converges and how accurately we get the solution this point it is good to note that since our a priori assumption of w x y is very, very close to the actual deflection curve this curve is a sine curve that is the reason we are able to able to get almost the exact solution with first term only. So, it depends on the assumption of the basic variable what we are considering or assuming and that way it depends on the solution.

So, with this I hope a very, very important method in solid mechanics is introduced Rayleigh method you may have a look in detail in some from some advanced books definitely will get enlightened and you will learn a lot.

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So we proceed further with to the standard reference page.

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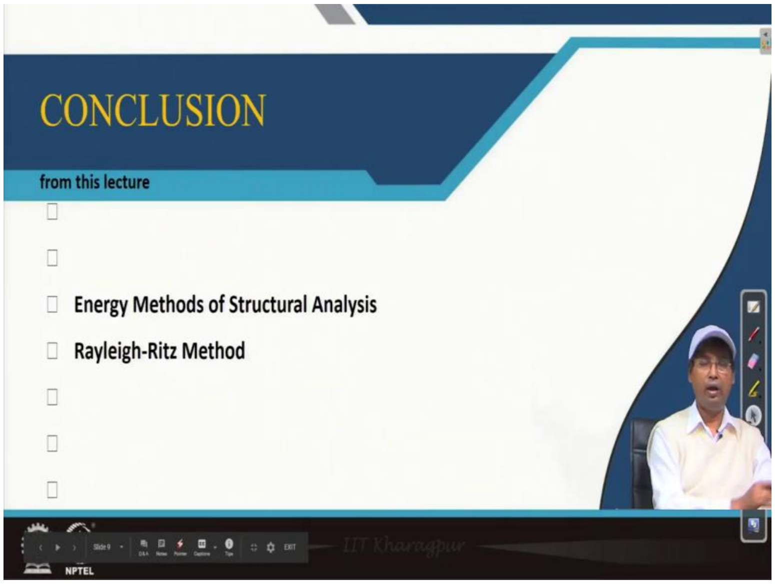

And from there we come to the conclusion page the energy methods of structural analysis and the Rayleigh-Ritz method we have discussed here. Hope you have understood to some extent for further things you please refer to the advanced books basically for those analysis. And thank you for attending this particular lecture we will proceed further for indeterminate problem solving, thank you.