Aircraft Structures - 1
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Lecture No -19
Dummy and Unit Load Method - Examples

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering department IIT Kharagpur. We are in continuation of the fourth week lecture this is lecture number 19. We are already introduced with dummy load and unit load method but unless we solve energy method problems it is difficult to understand properly that is the reason we will try to solve few more problems in this lecture.

And you should keep it in mind that the examples are solved in one method must be tried with other methods which will be covered in the forthcoming lectures.

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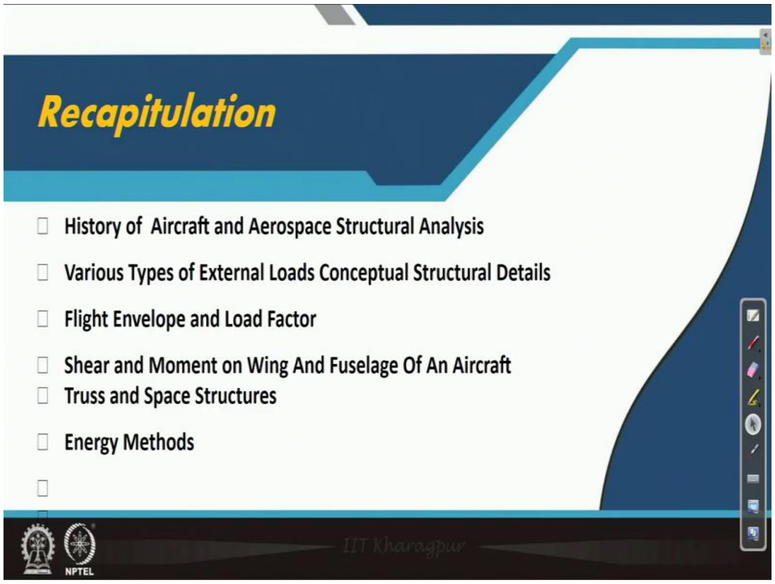

So before we go further it is better to have a recapitulation history of aircraft now already we have done history of aircraft and aerospace structural analysis, various types of external loads conceptual structural details that also we have done, how does it look like? Rib, spar frames long runs all those things we are introduced and which portion carries what type of load. And then we are introduced to overall source of load on any aircraft how the loads come and how what is flight envelop.

Why do we need to maintain it and in design how does it help, limit load ultimate load all those things we have already discussed. Shear and moment on wing and fuselage of an aircraft has also been discussed. Those have been discussed in with examples and then we have discussed problems related to truss space structures especially landing the gear problem. To cover the space structures and then we got introduced with energy methods and we are continuing with that.

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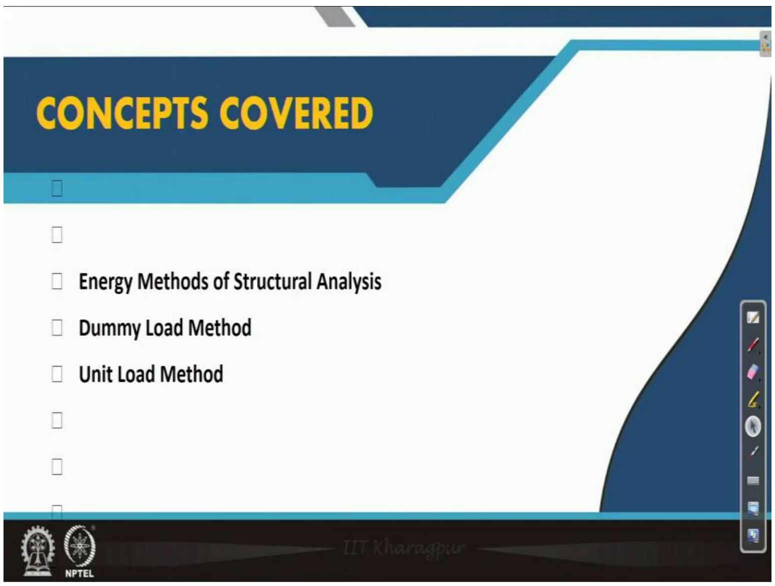

So, today as it is the mentioned already that we will solve if you example problem.

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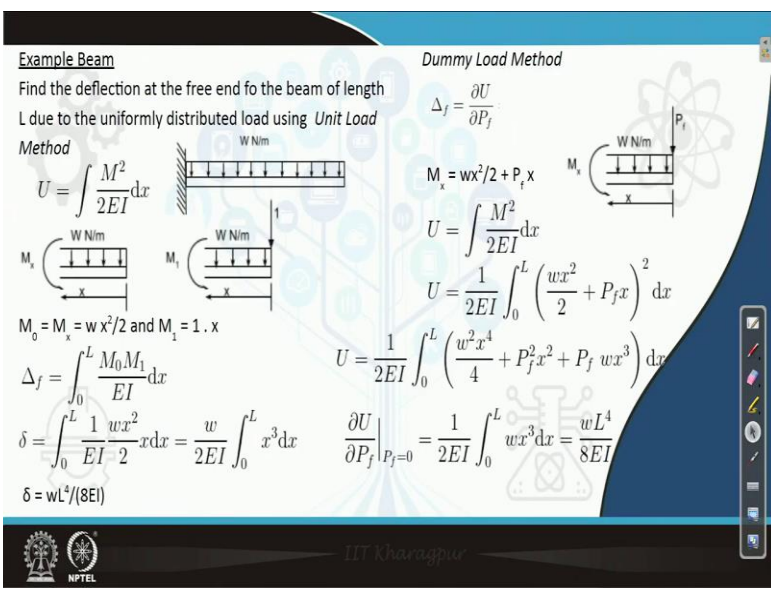

So the first example if you look at this example is considering the a beam, a cantilever beam cantilever beam is loaded uniformly distributed load W Newton per meter as it is shown in the figure. And you see these problems who will be solving both first by unit load method and then by dummy load method. But whatever we do we need to need to consider that this energy strain energy for bending this is predominantly a bending one we need to consider the bending energy and using the bending energy we can easily find out U either using unit load method or dummy load method, what is the deflection?

So to find out the tip deflection in this particular case free end or the tip, tip deflection in this particular case for to follow a unit load method what are we doing, we are applying one unit load here. Once we apply the unit load here and accordingly we may find out M 0 and M1 here MX whatever is there without the unit load that is actually the M 0 and for this particular section and considering the direction of moment shown as shown in the figure we can easily say that the M 0 is W equals to x square by 2 W multiplied by x square by 2 and M 1 is when this uniformly distributed load is not present and then the M 1 that is equals to 1 cross X.

So this case is better to neglect this portion of the uniformly distributed load and then what do, we have already the formula what we usually use to find out and that formula we are using and putting the values like M 0, M 0 is WX square by 2 1 by E I is there and X is there this M 1 value that gets multiplied W by 2 EI comes out. We do integrate from 0 to L here at the length of the B means L not mentioned only mentioned in the state text.

So visually we can mention again this way, this is L. so, with this if we that's the reason the integration is considered from 0 to L and X is considered from this into the left ward that x to the power cube by dx and it is a simple way x4 and y4 will come and WL to the power 4 by 8 EI is the deflection this is following unit load method. Now if we try to follow same example try to solve same example using dummy load method what can be done is that we can we can solve the similar way.

Only we need to consider del U del P f and instead of applying unit load will have need to apply the load P f now it makes is equals to W x square by 2 + P f x because of this P f load so we need to that the total strain energy as we substitute this value here this is Wx square by 2 + P f x whole square and the constant term is taken out integration is definitely again carried out for 0 to L dx and then if we carry out further we have this formula is expanded this way W Square extremes to the power 4 by 4.

So this is not there is no nothing of P f so this will Delphi derivative of this will go to 0 there is 1 P f there is 1 P f, so what will happen this P f will make that P f twice P f x square and while we will put the value of P f equals to 0 this term also will lead to 0 only remaining term is this one because P f will become 1. So, in that case this becomes integration Wx cube dx. Similar way W will come out and it will lead to the same result.

So whether we follow the unit load method or the dummy load method in both the case the answer has to be correct that is what we have learnt problem we thought made differ but answer should be same. So, the first problem for today's discussion is complete we will go forward for the next one.

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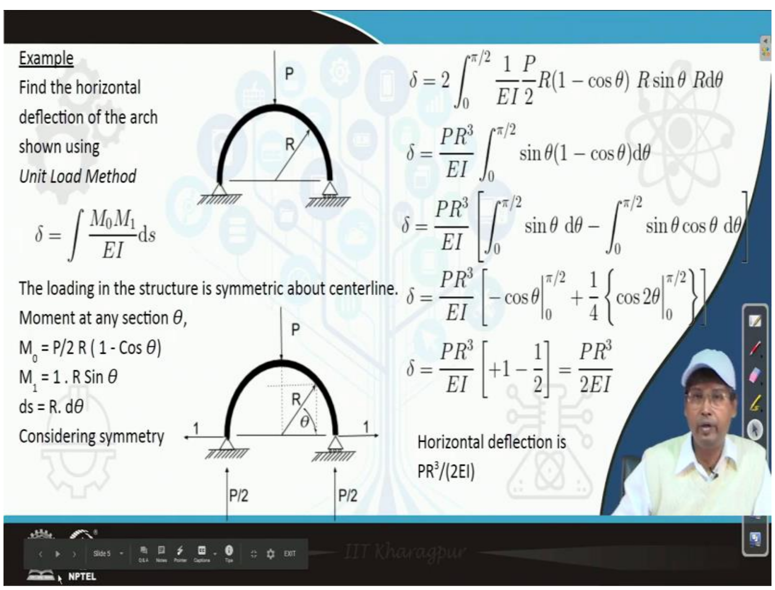

This example is a different type of example find the horizontal deflection of the arch shown using unit load method. So, first we will try unit load method how the unit load method works in this case, find the; to do that the equations remain same as for the unit load method. It is the loading is symmetric if you look at it is centrally loaded. So, without any doubt we can easily find out that there the reactions are P by 2 at the two ends.

And for unit load method we need to apply one unit load P 1 because this end is supposed to displace whereas this end is supporting so there definitely will be one more reaction and that reaction is shown here it is not that two unit loads are applied if the concept is not that like that but the concept is to apply one unit load. The loading in the structure is symmetrical about center line so that is what just now I mentioned.

Moment at any section theta moment at any section theta if we look at how can you find out that is it will have two components one is this will give us the M 0 and this will give us the M 1. So, M 0 if we talk about this is the momentum, so this distance is nothing but 1 minus R into 1 minus cos theta and that is what is given shown here you see and P by 2 is also given. So, accordingly we get the M 0 calm part for M 1 the on is this much and this is nothing but R sine theta.

And since it is R sine-theta without doubt who put that one in to R sine theta and ds is equal to definitely R d theta. So, that is the thing is put in into the in this form this is the value is put and we get this value. So, one thing you must notice that there is a two in front and the limit is from 0 to PI by 2 we are not considering 0 to PI. The our structure is symmetric that's the reason if we find out the energy up to this and then make a double of that that gives us the total energy that is what is done here.

So 1 by E IP by 2 R into this portion is because of M 0 and the other portion R sine-theta this is M 1 so while we put this to value and we need to carry out the integration that is the remaining part need to do. So, in unit load method we need not to differentiate, so if we follow steps sine theta multiplied by 1 minus cos theta is the only variable part and PR cube PR R these are also is there. So, makes PR cube by E I outside and then we need to carry out the integration this is sine 2 theta will become.

And then that is there is in one fourth and one more half will come because of the 2 theta while we do integration minus plus is because of the integration sin to cos and here also sin to cos plus has become minus you put the boundary values 0 to PI integration limits 0 to PI by 2, 0 to PI by 2 and accordingly this gives us 1 and this gives us minus 1 by 2 and it finally leads that the end deflection this end deflection if it deflates like this then this deflection is PR cube by twice EI so that is what we have solved and let us move to next slide for another method the other method what we can use to solve this.

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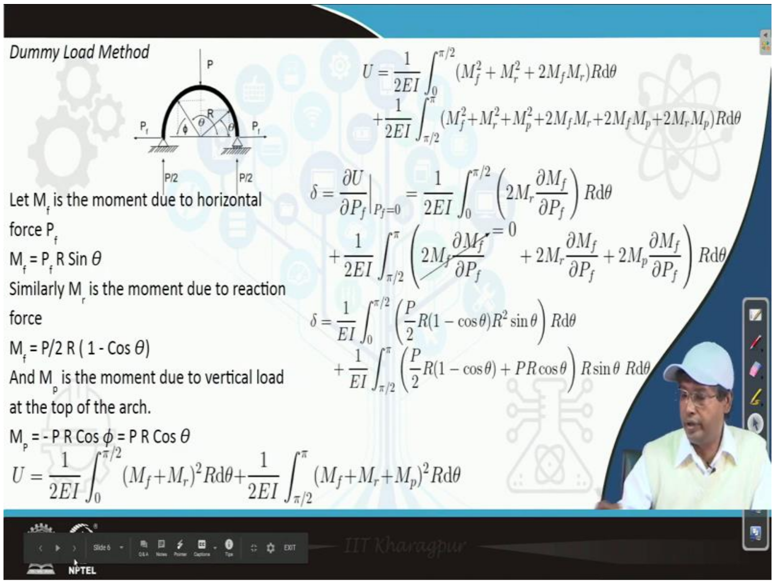

Same problem we will try using dummy load method in this development not only that we will solve using dummy load method will also solve it without considering the symmetry. So, it becomes the equation becomes little bit lengthy and we need to solve it. So, let us try once using that method also how what the result comes and we will see will solve in that process. So, let us follow. It is same only since P f is applied one more P f reaction P f is there.

Since we are not considering symmetry we need to consider moment at this part also that is the reason Phi is defined to help us to understand and after Phi is defined we can easily solve the equation. Let M f is moment due to horizontal force P f if this is the thing as we have discussed earlier this is the arm for P f and that is R sine-theta and similarly if M f is the moment due to the reaction that becomes P by 2R sin into 1 minus cos theta.

So MP is the moment due to the vertical load due to this load due to this load and if we talk about that load it is becoming minus of P R cos Phi minus of P R cos Phi it is acting this way all moments those are acting this way on this section, this way or this section in just this way whereas this is acting in this way that is in minus F has come and P is the load R cos Phi is this angle and that is transfer transform to theta as cos theta.

So that is the I think you can easily get why it is like that from 180 degree minus Phi and that way we get it. So the total energy total strain energy is here 1 by 2 EI M f plus M r whole square Rd theta plus this is for 0 to PI since as we said we are not going to consider the symmetry that is the reason in two parts the integration is take considered from here to here one part where P f that MP part is not there.

And from here to here one more PI by 2 2PI where MP part is there so that square is considered. Similar way it is expanded both part this is the first part and this is the up to PI by 2 this is up to PI, PI by 2 to PI and then again the derivation is considered and if we go for the derivation this part is better we can easily concentrate. So, we are supposed to do derive it with respect to what I say that I am sorry P f. So, this is noted here, this is going to be 0 why this is becoming 0 because as we will put that P f is equals to 0.

Here P f will become equal we put 0 that there is in this part will become 0 this is automatically 0 this MP is also automatically 0 that is because MP does not contain any P f this is the part will remain that a M r this part M f + P f will remain. And here also MP part will remain this is also this also will go to 0. So, similar way if we do this also we can get as twice M r twice M r del M f this part only remains but these two will not be there this will not be something like this and will be 0.

Now what we have done we have put the values of the moment what we have found out here those moments values are put here P by 2 R 1 minus cos theta this is this part and R square sine theta is this one M f del M f by Del P f and it continues that way. So, in this case also what we have we substitute these values and we get this equation.

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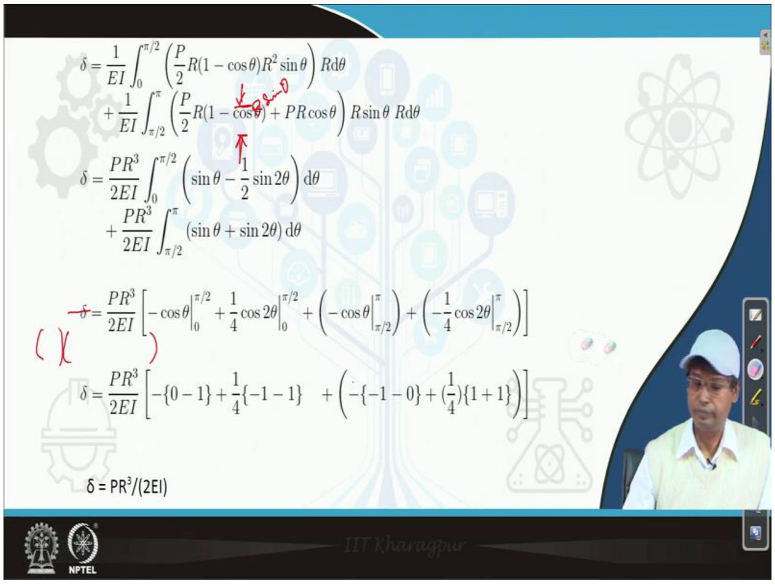

So, this equation is repeated from the previous page and if we simplify that it becomes I guess there is a mistake of this R square this because this R, R square and R makes it R to the power 4 probably this is a mistake you please ignore that otherwise it is not going to be P R cube. In the previous page also that mistake is there please ignore consider that this is not there. So, PR cube by 2 EI comes out then sine theta minus 1/2 sine 2 theta it becomes.

And similarly from the other one also PR cube by 2 EI is coming out and then we carry out the integration sine theta cos it becomes minus cos theta. Similar way plus 1 by 4 cos 2 theta 0 to PI by 2 this is PI by 2 to PI cos becomes minus sine a sorry sine becomes minus cos and say here also sine becomes minus cos 1 by 2 concern comes out and similarly we get it this way. So, I think they are also some typographical mistakes are there you may consider this two part if you carry out this integration you will definitely get this value.

So I would suggest you carry out the part and check whether these two equations are written correctly or not but finally the deflection what we see is correct.

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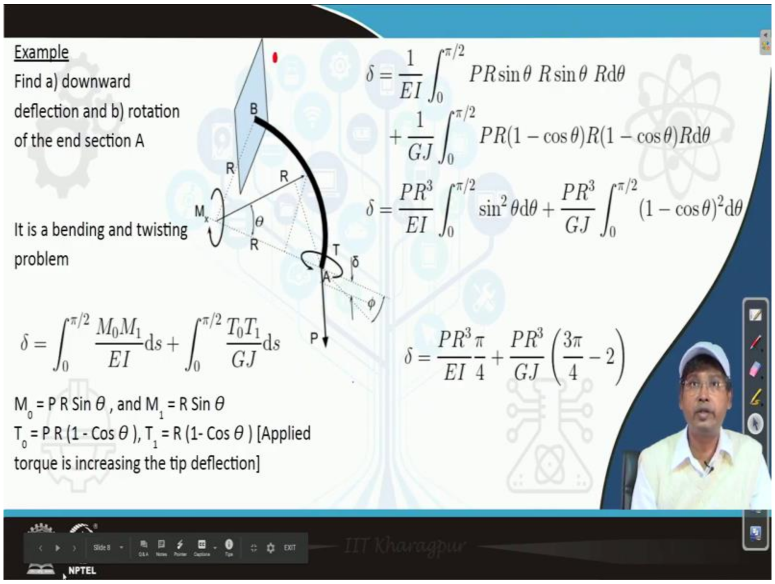

So, if we move forward for the the last example this example we have a curved beam. In this curved beam what we do we need to find out the deflection, the downward deflection of this free end and the rotation of the free end rotation is considered in this way the rotation how it is taking place. So, let us see first here to how the deflection is coming, it is a bending and twisting problem unless both are there, there would not be any translation downward as well as rotation that is the reason following in unit load method we have both the energy due to moment as well as energy due to torsion.

And once we do that once we carry out that the M 0 this is the important part here is to find out M 0, M 1 and T 0, T 1. So, let us try to see how which arm is taking and which part is being considered for M 0 and M 1 that is the most interesting part. Remaining part it is simple calculus I think you can easily do probably better than me. So, that you can easily check and find out the values finally what you are getting.

So to do that let us do come we are to concentrate to find out the M 0 first we have a load P acting downward in this particular case we are not considering this T or there is no T present at this tip it is because of only the applied load P. So, do not get confused with that T is shown here just to give you the idea which way and how the torsion may act and how the Phi is acting. In this problem the there is no T is applied so what how the moment is acting in this particular section.

If we look at it at this section this P is acting from here downward. So, the arm for moment is actually this portion and that is nothing but the R sine theta this component. So, that is the reason P R sine theta we get for M 0 and for M 1 that is nothing but while unit load is applied here removing the all other loads we get the R sine theta. Now about T 0, T 0 because of P what is the torsion at this particular section?

So in this case particular case actually this is the point where P is acting downward and what is the arm? Arm is actually this much and this is nothing but one minus cos theta multiplied by R. so, that is what that T 1 we get P R 1 minus cos theta T is acting downward R 1 into 1 minus cos theta, theta and P equals to 1 gives us the applied torque is increasing the tip deflection this is interesting point.

See P is being producing a torsion which is which is moving the tip downward that is the reason we say in this particular case the torque what is produced by this P at any section which is theta apart is actually increasing the deflection vertical deflection downward. So, with this concept while we have all this value while we have this formula we are supposed to put that those values where and those values are put here P R sine theta, R sine theta and then 1 by G J therefore the torsion part is also it is put.

And then the simple integration is carried out and that gives us that Delta is equal to P R cube by a PI by 4 plus PR cube by GJ 3 PI by 4 minus 2 so with this consideration let us move to the other part of the problem that is rotation of the end section A.

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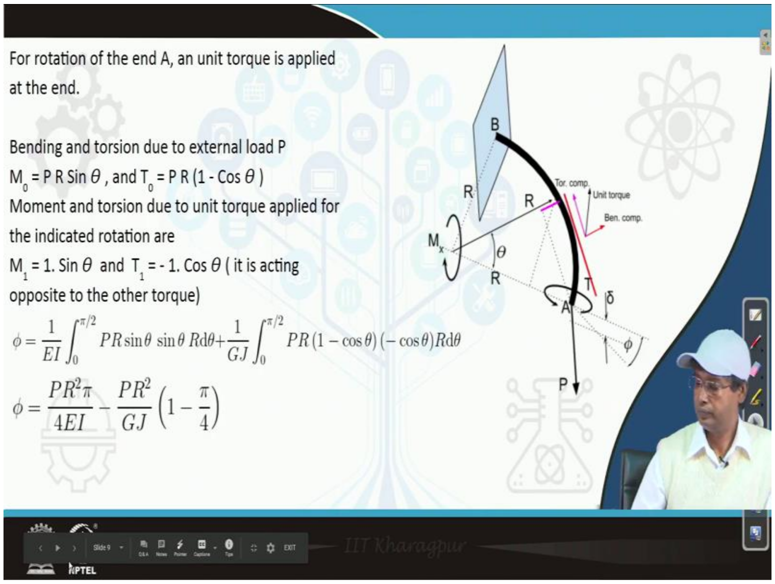

So here we have considered some additional drawing we have prepared for better understanding. What we need to see is we need to apply one torsional load, torsional unit load in the direction of in the direction where the theta is increasing and accordingly we can find out whether it is what is the value of Phi or the end rotation. So, for the rotation of end A and unit torque is applied at the end bending and torsion due to external load T.

This we have already found out for the external load P the other load is the most interesting in this part of example that is moment and torsion due to unit torque applied for the indicated rotation R. Now you see we are applying this unit torque why it is in this direction because this is tangentially tangential here and this is coming from bottom to the up from it comes from bottom to the and that is the reason following right hand screw system this is the vector.

Now this vector is applied here as it will go further these two components will change following this arc. If we one component will create a bending and the other component will create the torsion or induce the torsion. So, if this is in two separate colors are used here if we make a component this way this is this is the bending component. And this bending component is a increasing the deflection downward and if it is increasing the deflection downward that component will be this is one this component is one sine theta because see as we as we increase this is increasing.

So this angle is the theta, this is theta so if that is increasing so this component bending component is a sine theta and the torsion component is cos theta. So, the bending component one unit torque and sine theta that is M 1 is acting there and cos theta is the component fine one is then magnitude of the force is fine but wise minus that minus is because you see this is the way it is applied this is actually in the opposite direction it is acting opposite to the other torque that means the torque.

The way we have considered previously in the previous example it is acting opposite to that and that is the reason it has been considered as minus, so in this sense we can see that this point will go up because of this torsion this point will go up because as I said as I repeat this is coming going down and then coming up. So, that is the reason minus has come here and accordingly we have put those values P R 1 minus cos theta multiplied by minus cos theta Rd theta and this is the bending moment part and then if we integrate we get the value of Phi.

Phi of is in this direction considering this is positive while it is rotating this way if this total value is positive we will be considering that this is rotating this way. So, PR square PI by 4 EI - PR square by GJ 1 minus PI by 4 is the rotation value. So, with this solving 3 examples we conclude today's lecture the examples are very good.

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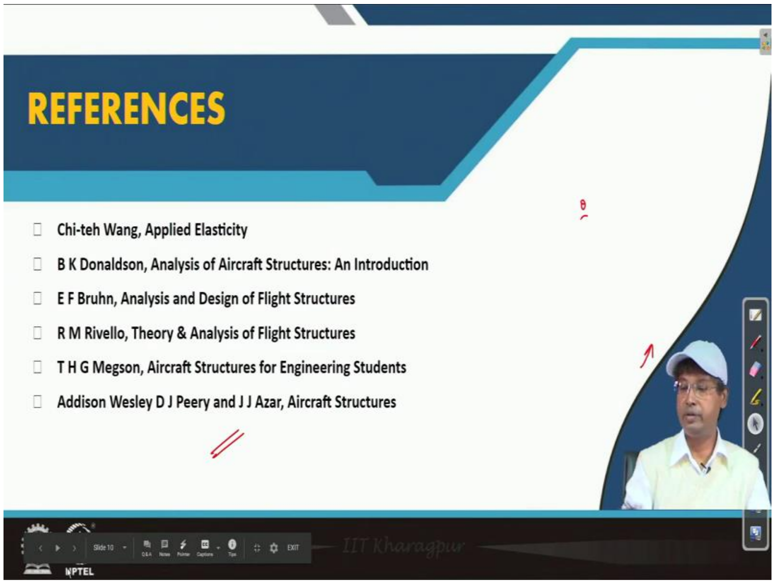

You may solve the same examples references for this is similar to or same as we have done previously and with this we come to the end of the lecture slide today and thank you for attending this lecture. We will move forward for the next lecture, thank you.