Aircraft Structures - 1

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Lecture No -18
Dummy and Unit Load Method

Welcome back to aircraft structures one course this is Professor Anup Ghosh from Aerospace Engineering Department IIT Kharagpur. We are in the mid of fourth week lectures where we will consider mainly the energy methods. Energy methods preliminary did definitions and derivations are covered in the last lecture. This week we will cover the dummy load method and unit load method.

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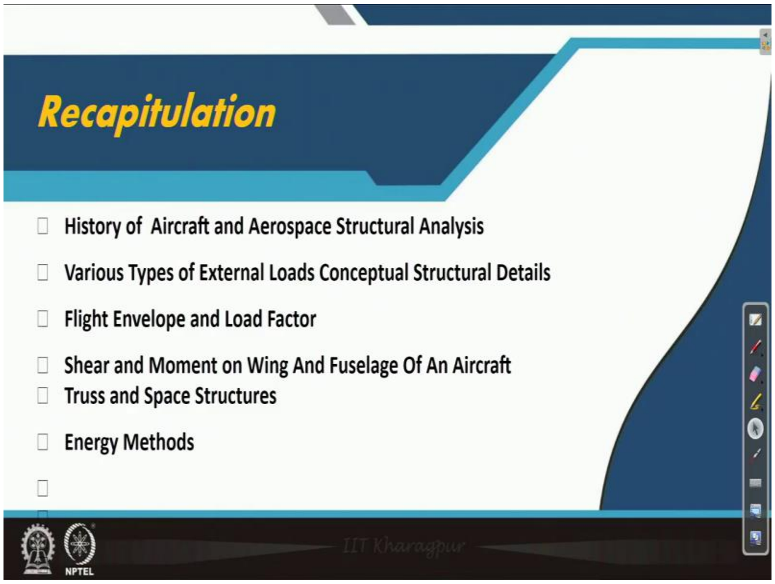

Before we go into details in the dummy load method and unit load method better we recapitulate what we have learned. We have learned history that's very important I always say always fill of solid mechanics or structural analysis or aerospace structures. Various types of external loads and conceptual structural details, flight envelope and load factor C r and moment on wing and fuselage of an aircraft, truss and space structures we have done and then we have come to the energy methods in our last lecture so we proceed.

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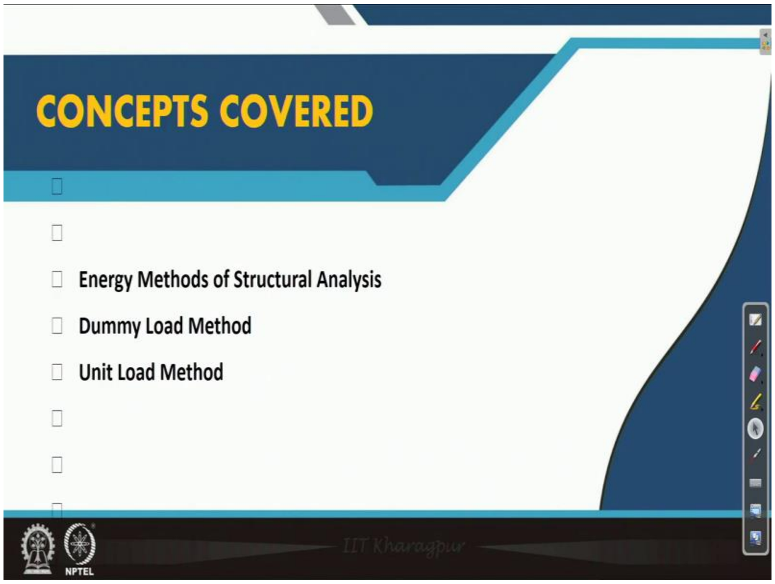

Today we will continue with that to dummy load method and unit load method.

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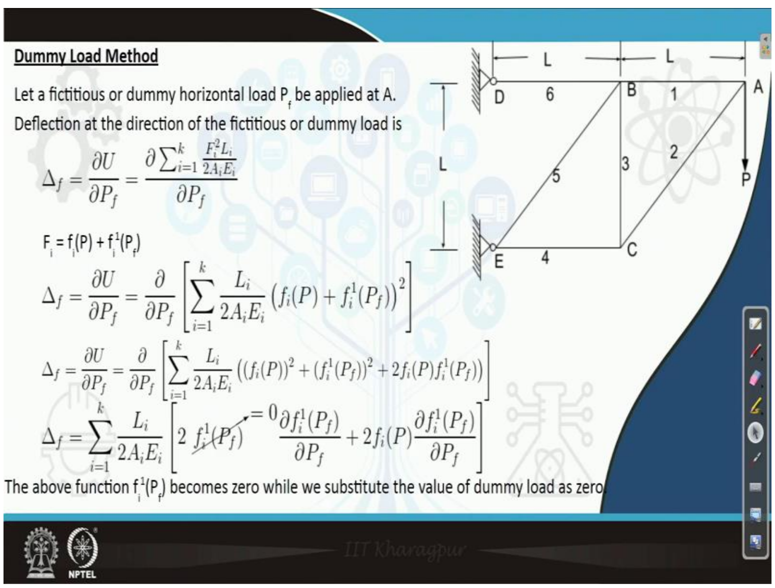

The dummy load method what we will discuss today is we will take the help of the example what we have solved for vertical deflection as it is shown on the right hand side of the slide, here. This if it is asked to find out the horizontal deflection of this point A. So, how can we do? So, to do that we need to take help of the dummy load method, say let A fictitious or dummy horizontal load fictitious or dummy horizontal load P f be applied at A.

Deflection at the direction of the fictitious or dummy load is following the previous derivation we can write that it is if it is capital Delta F is a partial derivative with respect to that P f or the fictitious load and then the total energy gone for that in all the members that is F i square a l i divided by 2 A I E i that has to be taken a partial derivative with respect to the P i and it is summed up for the members 1 to k. Now let us try to see how it changes if we talk about in a general way.

So we can say that that force whatever F i is or the member force generated in each and every member is F i and there is a contribution of two forces one is the F i here where it the value is P because of P whatever force is coming that is a F i P and because of the fictitious load whatever value is coming that is F 1 P f. So, if it is a sum of those two functions we are considering that linear superposition is possible and then we can continue for the derivation this is square L i is considered that square is expanded here.

If we consider the partial derivation with respect to this term definitely this is equals to 0 and this term is here written as 2 fi PF this is constant and this is the derivative we are supposed to take and in this term we have 2 f i 1 P f this is written equals to 0 because since the load is fictitious at the end we are supposed to put a value r of P f equals to 0 and definitely this total value will become the function value come zero for any case and that will make the total this portion of the equation zero. The above function f i 1 P f becomes zero while we substitute the value of dummy load as zeros.

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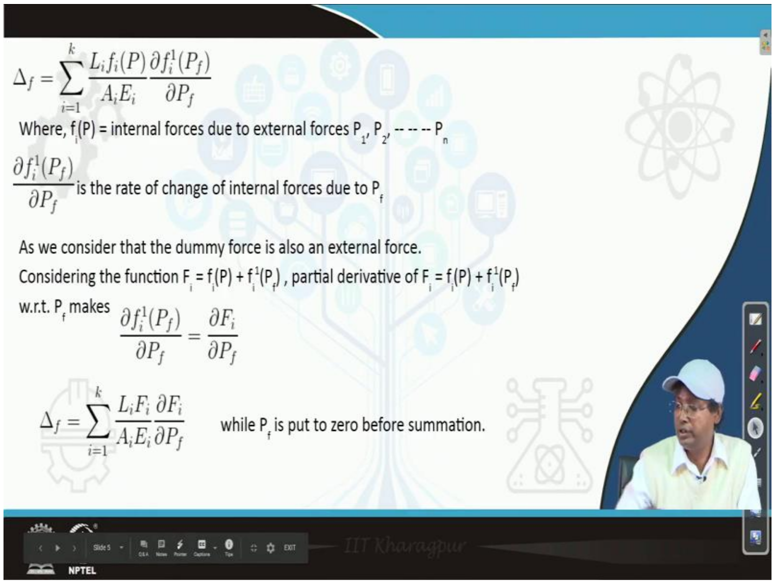

So, the finally what we have it boils down to the equation as it is shown at the top of the slide it is it is - that's this slide where f i P because to the internal forces due to the external loads P 1 P 2 P n it could be as many as possible considering general case and f i 1 P f is equals to is the rate of change of internal forces due to the P f or the application of the fictitious load. We will see look at this equation in a different way that will help us to solve the problem.

So let us bring back the equation what we have put before we go for the partial derivation. As we consider that the dummy force is also an external force. Considering the function F i equals to f i P f i 1 P f + f i 1 P f partial derivative of this gives us that this is is equals to this, so what we have we can easily replace this term with or this term with this term now remains F i what to do for F i? For to use this formula we need to be precautious and we have to put that while P f is put to zero before summation.

This while we are calculating the f i we need to put that those values for the P f equals to 0 and we need to calculate that so that actually because this but this is more popular way to remember and that is the way we generally carry on with keeping in mind that we need to put P f equals to 0 before we go for the summation.

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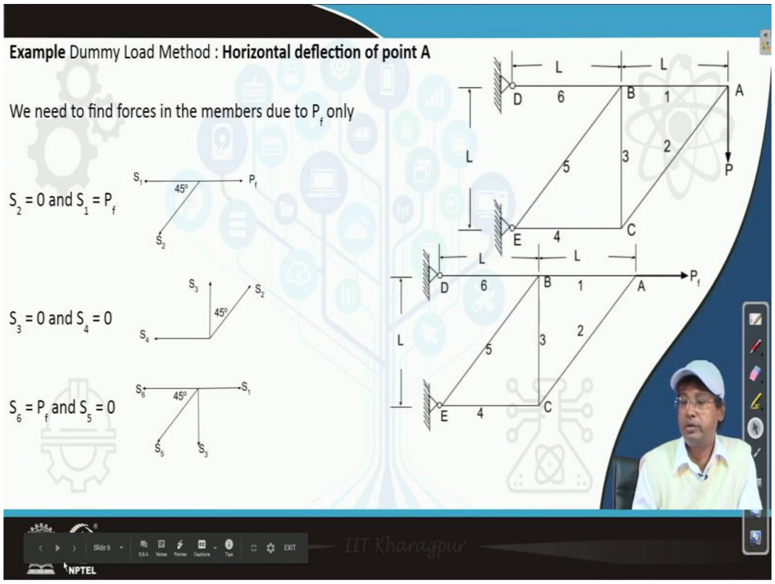

So, let us solve that same problem what we have solved in our last lecture, problem is easy procedure is, you better concentrate on the procedure. How the procedure is? How do we do?

This is the original problem and in this case what we are doing is that we have removed all the forces and we have put a force P f because of the P f we are supposed to find out the member forces which will become that F i give us that f i 1P f, so this case will give us that value. Okay so let us see the solution is very easy we need not to do much for this. For this joint we have considered here and we have considered that.

Considering that this equilibrium of this what we can see that this S 1 is equals to P f and S 2 is equals to zero and similar fashion if we go for the joint, in this joint what we can see that from the summation of horizontal and vertical equilibrium as the process may be if we go for since S 2 is equals to 0 definitely S 3 and S 4 is equals to 0. Now if we come to this joint we have already S 1 is having a value of P f, S 3 is already 0 this is equals to 0 and then S 6 + S 5 definitely S 6 will have a value of P f but S 5 will not have any value if we consider the vertical equilibrium. So that way what do we have? We have values for S 6, S 1.

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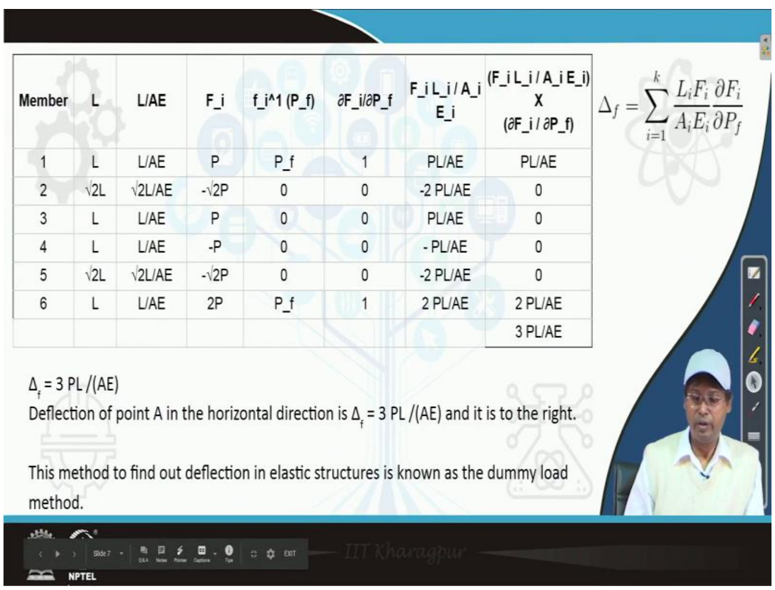

And with that we move to the table that helps us to carry out the calculation. In the table what we have put we have put those members and one after another this is the length of the member this is L by AE assuming that all the members are having same A and E where always A and E is divided there may be problems where A and E are not constant. Practically it is not constant but for problem solving purpose in most of the cases this AE is generally becomes a constant value but A vary E sometimes in most of the cases are constant.

Anyway these are the load for the external loads that here in this case the vertical load P. This already we have solved in our previous class. This portion we have solved today. So, please here in this please you please note that whatever is written underscore that is the subscript what is with cap is the superscript. So, in that fashion with that we have the member force this way and then we have the derivative of those forces partial derivative of those forces as 1 and 1.

And then we calculate the first this portion value here it is that portion value it is nothing but multiplication of this and this, this and this and that way we have those values. And this is total multiplication of with this, this is multiplied and we get those values. So, definitely since these are 0 we have only summation, summation gives that 3 PL by AE. So, Delta f the deflection of point A in the horizontal direction is 3 capital Delta f equals to 3 P l divided by AE and it is to the right, why?

Because the direction of the P f we assumed on the right hand side Delta f has become a positive value so our assumption of displacement on the right was correct and it is deflecting on the right hand side. This method to find out deflection in elastic structures is known as dummy load method. Why dummy load method? Because we are applying a dummy load P f we are applying a dummy load P f and using that we are finding out the deflection. Let us move to the next slide.

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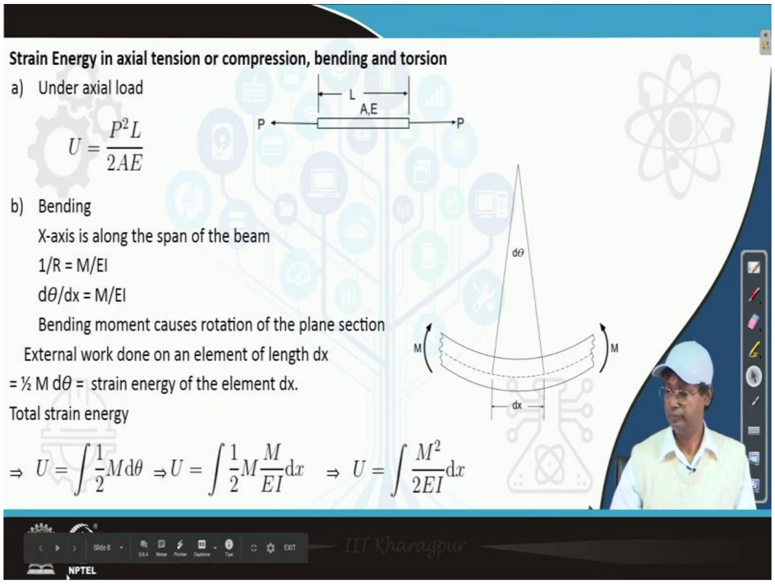

Before we move further for the unit load method we have already observed that energy is required to be found out for various cases. And in the next example what we will be solving using dummy load method we need these expressions so that is the reason in this, in a very brief way energies for tension compression bending and torsion is derived here. This is not derived definitely I would suggest you to please find out how do you get that curve U for a certain member for a tension P Square L by 2 AE you get having a cross section A modulus of the A i E and length L.

I think a little effort if you put you can easily do we will see the other portions. If we look at the next one, if a bar is a beam is bending like this the radius of curvature is R and x axis is along the span of the beam, so we are considering x axis in this direction and then we have a relation for the curved beam under pure bending is that 1 by R is equals to M by E i. Sometimes we get minus also that depends on the way we consider x and the way we consider R.

So since R is 1 by R is easily can be written as d theta dx this equals to the M by EI bending moment causes rotation of the plane section external work done on an element of length dx as it is shown here is equals to half M d theta the strain energy of the element dx and that this strain energy we are going to integrate for the length. So, U is equals to half M d theta we are not putting limit because depending on the case we need to put the limit and we need to find out the total strain energy.

So that way d theta is substituted from here and it becomes M by E i dx and then we get the most popular form of U for bending or strain energy for bending as equals to integration of M square dx divided by twice E i. So, we have a similar expression for torsion let us see.

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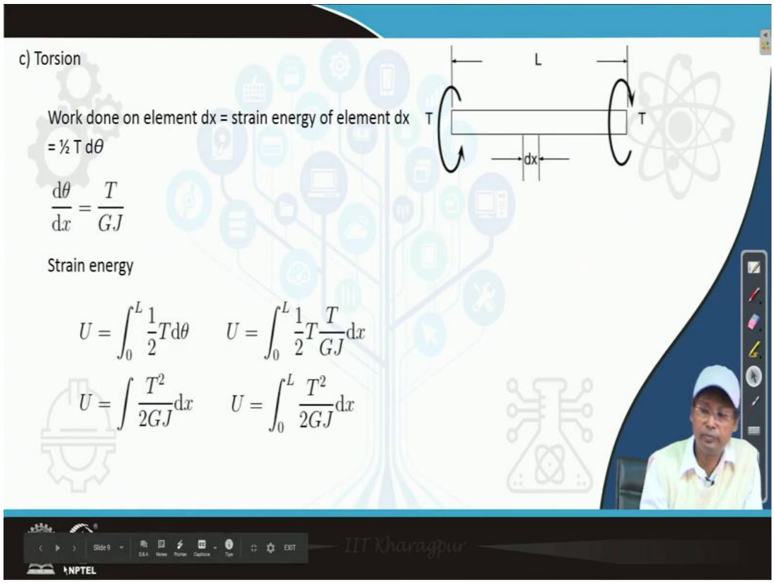

In case of torsion which is a bar of length L loaded at two ends by T the torsion a dx length if we consider the work done on element dx is equal to strain energy of the element dx which is equal to half T D theta and same way we continue d theta dx we as we know from the torsion formula is equals to T by G J and we integrate that substitute that d theta this becomes half T Square 2 GJ dx and we get the formula it is my mistake repeated.

So for the torsion the formula is this. So, with this consideration or introduction of a calculation of total strain energy let us move forward for further learning on the unit load method that is a very beautiful method.

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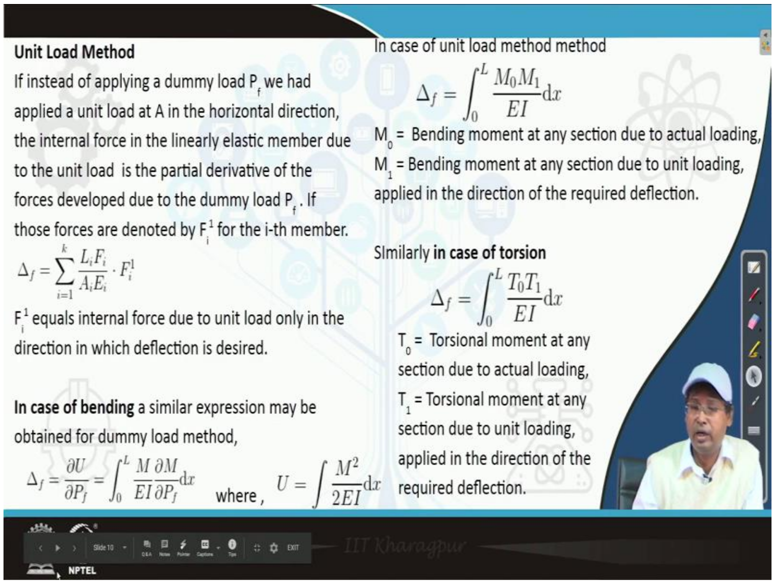

Let us see how do we go for the unit load method it is very, very similar to the development load method and let us have a reference with that method to understand this. Unit load method if instead of applying a dummy load P f we had applied a unit load A in the horizontal direction. It is we are talking about the same problem what we have solved and the internal forces in the linear elastic member due to the unit load is the partial derivative of the forces developed due to the dummy load P f.

So from observations from mathematics it is quite clear instead of applying P f dummy load of any value if we have like unit load the derivation we need not to take it becomes the member forces becomes automatically the partial derivative with respect to the P f. If those forces are denoted by f i 1 for the ith member then easily we can put this value here and the Delta f becomes summation of i 1 to k L I F i by A i E i multiplied by F i 1 where F i 1 is the forces developed in the members due to application of unit load in the desired direction.

So if one F i 1 equals internal force due to unit load only in the direction in which deflection is desired. In case of bending a similar expression may be obtained for dummy load method that is what is written here this del M del P f becomes this value this is nothing but how do we get that's why it has been as a reminder written. So, this value becomes M 1, M 1 is the is the moment developed because of the application of unit load.

It sometimes get confusing please keep it in mind this statement in your mind while you are confused and we get the value. M 0 or M not bending moment at any section due to the actual loading M 1 bending moment at any section due to the unit loading applied in the direction of the required deflection. Similarly in case of torsion we can have similar equation T 0 T 1 T 0 or T 0 or T 0 is the torsional moment at any section due to actual loading T 1 is the torsional moment at any section due to unit loading applied in the direction of the required deflection.

So in a summation if we look at dummy load method and unit load method is that in dummy load method we are supposed to carry out the partial derivation and in unit load method we are statically finding out the moment torsion or the member forces in case of truss or the part of the partial derivative because you are applying unit amount of load.

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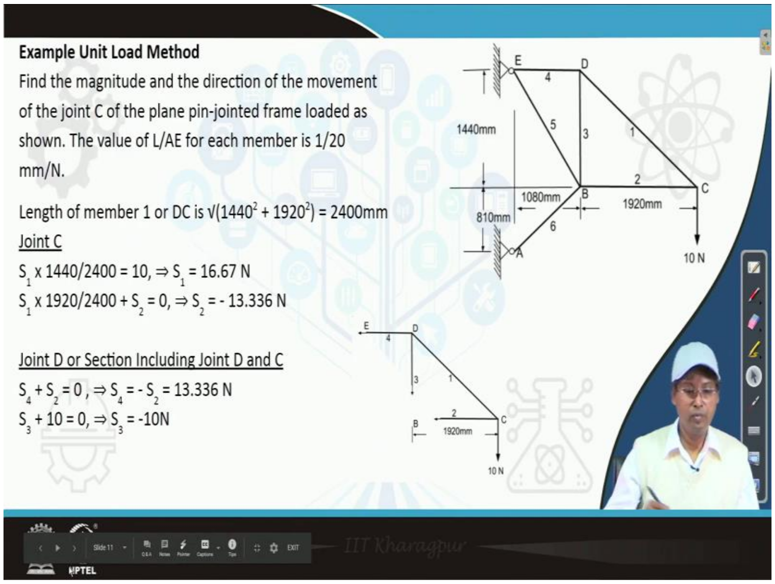

So, let us try to solve a problem example unit load method, find the magnitude and the direction of the movement of the joint C of the plane pin jointed frame loaded as shown. The value of the value of L by AE for each member is 1 by 20 millimeter per Newton because we are trying to find out who in dimension earlier things were not in dimension so we did not look at it. But here L by AE value is given so we can find out in millimeter.

Length of member 1 or DC from the other dimension this and this easily we can find out square root of those and it is 2400 millimeter. Now if we consider the joint C is 1 this is simply considered as the summation of vertical forces equals to 0 and from there we get that S 1 is equals to; since the members are all known 2400 is also known so we can easily find out that cost component this is the cost component is equals to 10 and what we get that figure is not given here it may be said something like this.

This is 10 Newton this is S 1 this is S 2 so we are considering equilibrium summation anyway we get this and from the other component considering the horizontal direction we get the summation we get that S 2 is equals to minus 13.336 Newton. Now if we cannot talk about joint D or section including joint D and C so to do that what we have done we have considered a section this way and that section is shown here.

So either we can consider to find out a consider moment to find out member forces like say for S 4 we can consider moment about this point or we may have a horizontal and vertical equilibrium because S 2 is already known. So, any way you may go for go and find out the values of S 4 here in this particular case S 4 is equals to 13.336 Newton and S 3 is equals to minus 10 Newton. So, let us proceed further.

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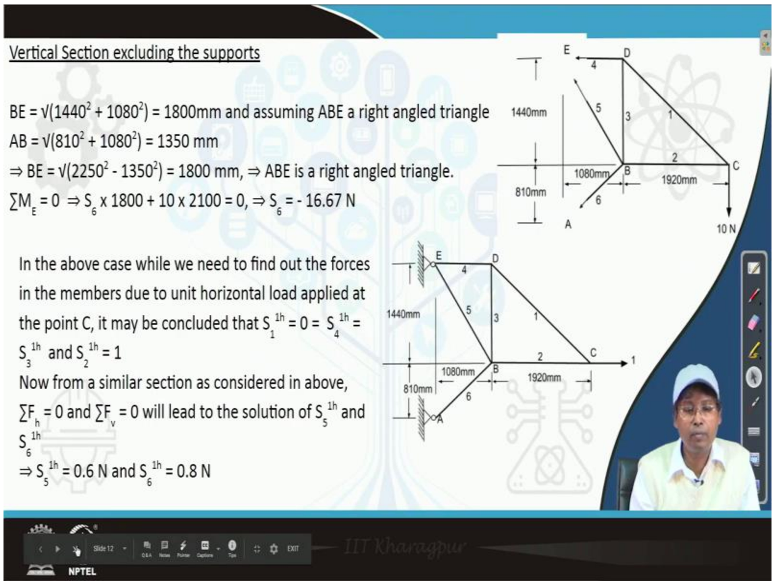

If we consider a vertical section this is nothing but a section considered from this axis from here a section is considered. Now from the dimensions we may easily find out whether the triangles formed particularly this triangle this triangle, this is right angle or not this is right angle or not may be checked from the dimension. So, that is what is done the BE this length BE is found out as he calls to 1,800 mm and then this is definitely a right angle so we can do that.

AB if we talk about this AB this is also a right angle so from there easily we can find out that this length is equal to 1350 now where why we have this length and this length if this matches with square of this square root of that 2 matches with this we can easily control that this is also right angle that will help us to consider a moment and know the forces. So, that check is considered here 1800, so the other way it has been done I think you can easily check it so considering moment about this point gives us the force S 6.

S 6 this is then the perpendicular distance this distance is equal to this distance and we get that S 6 is equals to -16.67 Newton this is done for the load given in the vertically downward direction but the question is that to find out the movement of C it is not that ask that whether the C is moving how much it is moving downward or how much it is moving horizontally rightward or leftward anyway we need to find out this.

This much is probably sufficient to carry out a vertically downward movement but let us see a horizontal movement we need also to find out that is the reason we apply one unit load here this direction. In the above case while we need to find out the forces in the members due to unit horizontal load applied at the Point C it may be calculated that S 1, S 4 and S 3 are equals to 0 and S 2 is having value 1. And similar way if we proceed now the way we have done in the previous portion now from a similar session as considered in the above summation of horizontal and vertical forces are equal to 0 individually will lead to the solution S 5 and S 6.

S 5 for 1 horizontal load is equal to 0.6 Newton and for S 6 1 horizontal load is equals to 0.8 Newton. So, with that we move forward for the next slide.

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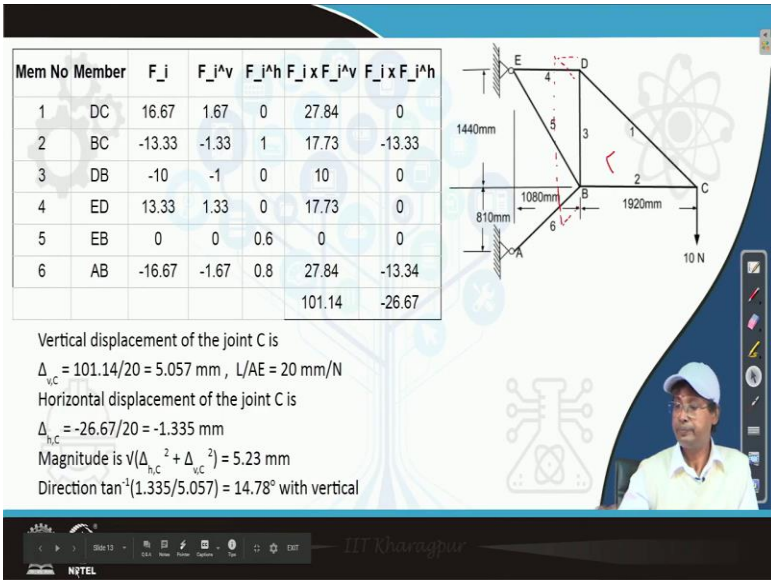

We have got all the values we need to find out the deflections as we have already described following that procedure we can easily find out the deflection. So, please keep it in mind that these are not the values of deflection because in the earlier tables L by AE was also included in the table in this particular case it is not included. So, that is the reason the vertical deflection at C becomes this divided by the 20 what is given in the question and that gives us that this joint moves downward as 5.057 mm.

And if we follow similar way the horizontal is minus of 1.335 here comes why minus how do we handle that minus. So, since it is minus we assume the load acting on the right hand side so the value has become minus, so it is coming something 1.335 it is a small value and this value is 1.335 and the net resultant will be this. So, the magnitude is this that is 5.23 mm and the direction is tan inverse 14.78 with vertical.

So this is the theta indicated that is 14.78 degree so with that the solution ends and we let us move forward for one more example.

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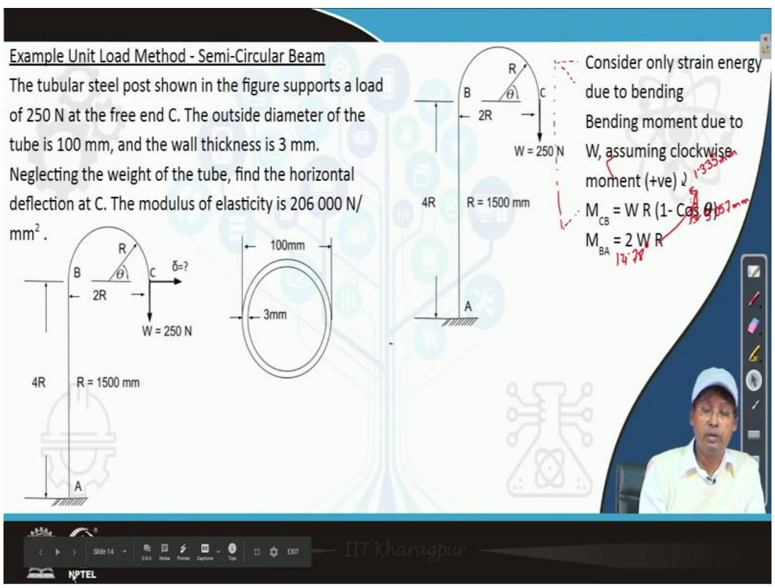

Quickly we will try to cover this example in this example need to be cleaned. This example is associated with bending calculations the structure is shown here. Let us read it carefully example unit load method semicircular beam. The tubular steel post tubular means the it is annular section shown in the figure supports a load 250 Newton this is the load 250 Newton at the end see here it is supporting 250 Newton.

The outside diameter of the tube is 100 mm as shown here and the wall thickness is 3mm as shown here neglecting the weight of the tube that means we are supposed to neglect the self weight find the horizontal deflection of C. The modulus of elasticity is 206 000 Newton per millimeter square. So, what we are supposed to do? We are supposed to find out the horizontal deflection as it is indicated here.

We first calculate the strain energy because of moment. So, to do that with the original load existing on the structure as it is shown here we are supposed to find out the moment at for moment in between from these two this C to B if we consider this, this distance is nothing but this one, multiplied by R definitely, so that multiplied by the W gives us the bending moment considering that is acting this way as it is shown here.

And similarly it will it that the bending moment here from here to here it is since it is acting in this direction that will remain constant for the value what it at it achieves here that is equals to W into twice R this is twice R and that will remain constant for the length AB that is what is written that in BA will remain constant. So, let us proceed further we need on in the horizontal direction so what we need to do for that let us see.

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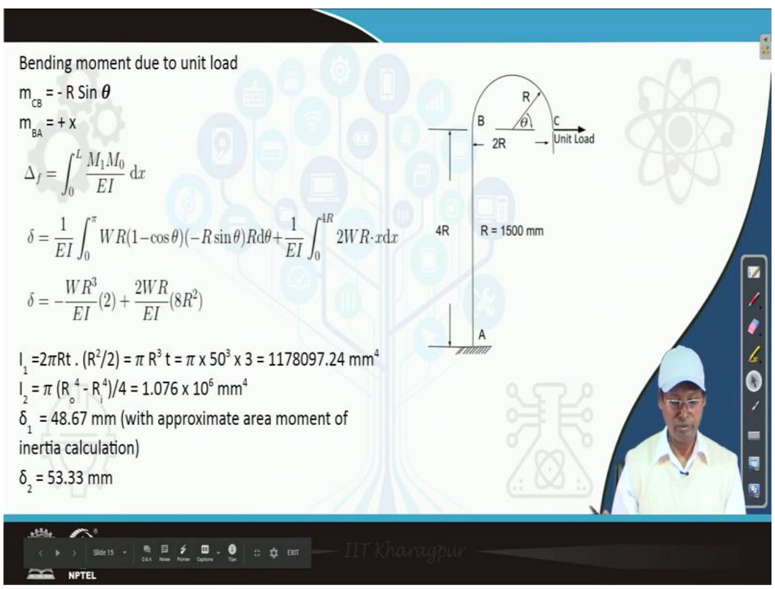

To do that what we have done is we have applied one unit load here in the horizontal direction and because of the application of the unit load we are supposed to find out the bending moment.

That bending moment because we have assumed the bending moment acting this way as positive this will act in the opposite direction and that is the reason if we again consider this, this, this is nothing but the sine theta, R sine theta and that is what the MCB is because load is one that is why not nothing else is there and here it is we are considering X from the here which is equals to X because it is acting this way it is acting in a positive direction so that is the reason we are considering that this is equals to X.

Now we are supposed to find out evaluate this equation that that equation is evaluated M 1 is written here W R 1 - cos theta this is M 0 and dx is equals to R d theta and then what we have done that has that is integrated from zero to PI 0 to 2PI and the remaining portion W to WR is the M1 and this X is M2, so with this we integrate and integration is not shown here in detail you can solve it as a homework and check whether you are getting this value or not.

So finally this del Delta or maybe written as capital Delta F any way you can write this and these are same please keep a note up about it and you see we can find out the deflection either 48.67 mm or 53.33 mm that depends on how do we assume the moment, area moment it may be assumed as a thin wall structure equals to PI R cube t or following the exact derivation it can be found out using this formula that makes the difference between this and this and this.

Anyway the value whatever we get is reported here and with that we conclude our discussion with energy method related to dummy load method and unit load method and we will also solve some more examples to have a clear idea about the process.

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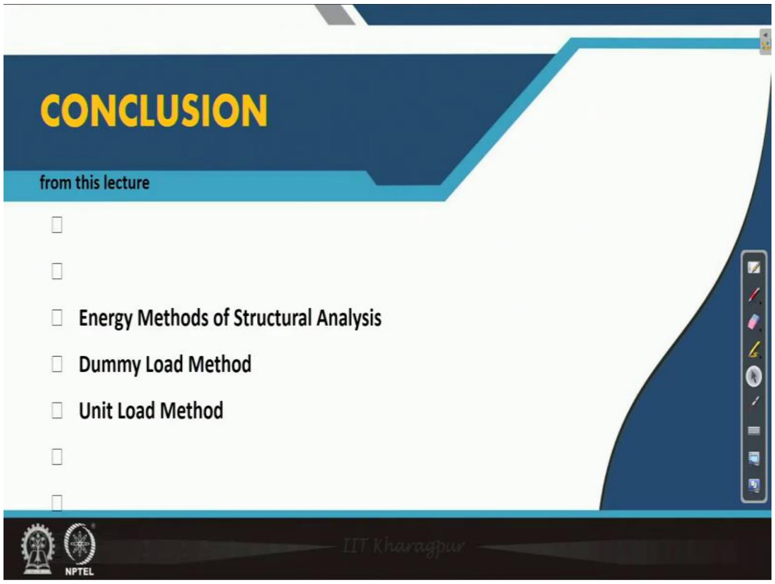

So, as usual the reference slides remain same and what we see is that what we have learned is that dummy load method and unit load method with examples and also with derivation. And with that I would like to thank you for attending this lecture and I would like to see you back in the next lecture with some more example, thank you.