

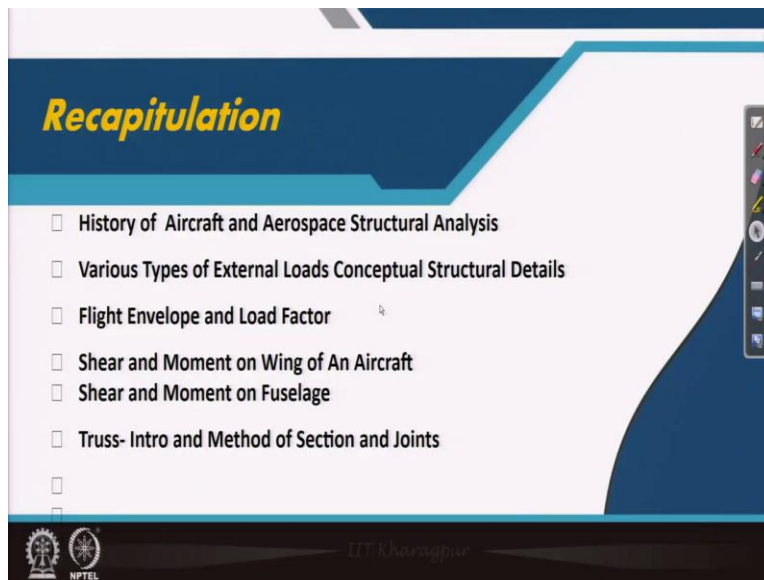
Aircraft Structure - 1
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Lecture-14
Space Structures

Welcome back to aircraft structures one this is Professor Anup Ghosh from aerospace engineering IIT, Kharagpur. We are continuing the lectures for the week 3 or the 3rd week. In the last two lectures we have covered plane truss and this lecture we will start with the space structures then truss is something where all the members and forces are in one plane. It may be any plane XY YZ or ZX any plane or may be in any arbitrary plane.

But in case of space structures it is in more than one plane. Practically if we look at all structures our space structures. So, we need to consider those loads but in general we do not consider the outer plane loads and we take components and try to make the analysis easy and accordingly we do. But there are some cases where we need to analyze space structures. So, let us get introduced to the space structures and I try to follow what the procedure is.

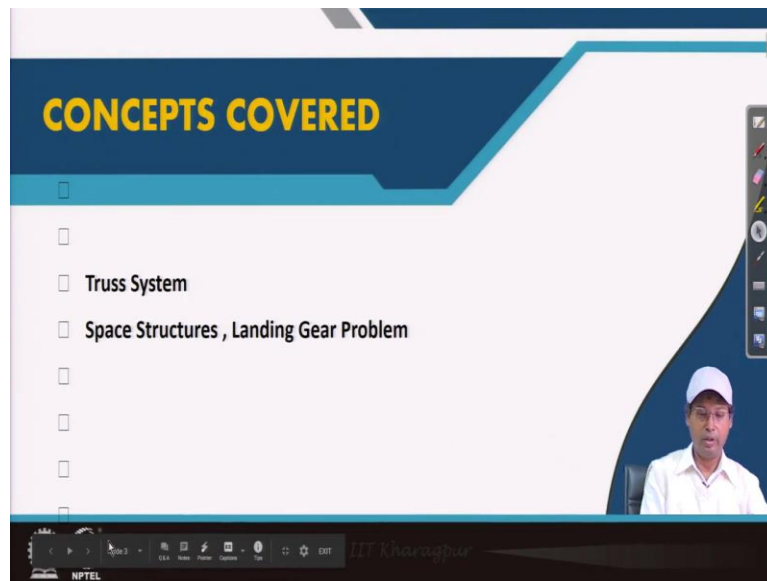
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This is as usual the recapitulation slide already I have covered many times but it is always better to say the salient points. We have learned the history that is very important I feel as you will

grow older you will feel that history is very, very important. So, not only history of solid mechanics our structural analysis but also we have seen the development of fixed-wing aircraft. And then we have come to the loads encountered by aircrafts that is very, very important in comparison to the structural analysis because all the loads has to be he has to be borne by the structures successfully, efficiently and then that will serve the purpose very well. So as, already we have mentioned in the last class we have covered the plane truss.

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And today we will cover space structures and mainly truss will cover though it is written structures but truss in the sense where transverse loads are not there in the structural members structural members are predominantly considered as a member which is two force member. But in one the example what we will solve there it will also experience some torsional moment. So, in that sense we may call this as structures not as truss. But the analysis procedures is similar you can easily go to wherever you want.

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SPACE STRUCTURES

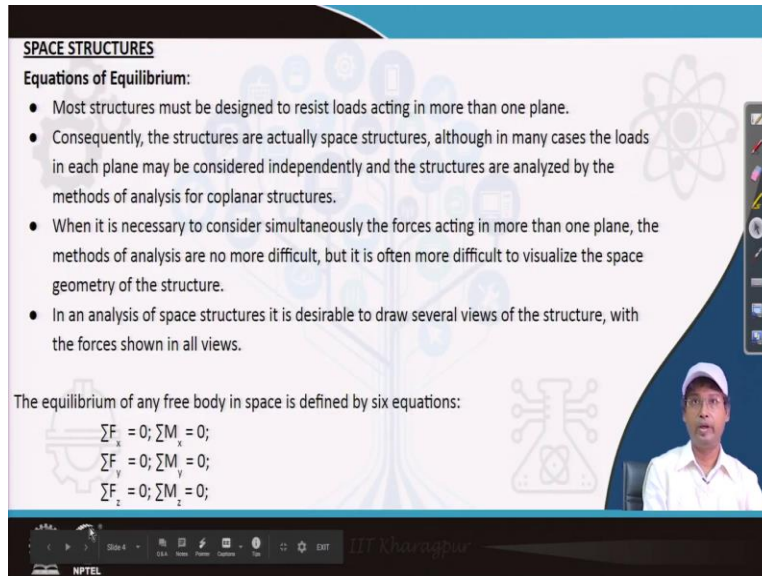
Equations of Equilibrium:

- Most structures must be designed to resist loads acting in more than one plane.
- Consequently, the structures are actually space structures, although in many cases the loads in each plane may be considered independently and the structures are analyzed by the methods of analysis for coplanar structures.
- When it is necessary to consider simultaneously the forces acting in more than one plane, the methods of analysis are no more difficult, but it is often more difficult to visualize the space geometry of the structure.
- In an analysis of space structures it is desirable to draw several views of the structure, with the forces shown in all views.

The equilibrium of any free body in space is defined by six equations:

$$\sum F_x = 0; \sum M_x = 0;$$

$$\sum F_y = 0; \sum M_y = 0;$$

$$\sum F_z = 0; \sum M_z = 0;$$


As a general introduction let us try to get introduced with the problem equation space structures. Equations of equilibrium is most necessary thing we need to find out the equation equations of equilibrium to solve problems. Most structures must be designed to resist loads acting in more than one plane this is what I was telling you a few minutes back. But in general we reduce it to a plane structure so that problems becomes easy.

Consequently the structures are actually space structures. So, all structures are actual space structures that is what is mentioned here although in many cases the loads in each plane may be considered independently and the structures are analyzed by the method analysis of coplanar structures. So, this is important we many times try to reduce the problem length understanding and we solve it in this way.

When it is necessary to consider assume sorry when it is necessary to consider simultaneously the forces acting in more than one plane the method of analysis are no more difficult but it is often more difficult to visualize space geometry of the structure. This visualization creates real problem and this I would like to take the opportunity to say something more in this point and this is the reason it is very very difficult to solve by visual observation the space structures.

And that has initiated the analysis of all space structures with help of finite element analysis where 3 dimensional geometries are created in CAD software's and it is easy to visualize and to

get the force components and moment components. In an analysis of space structures it is desirable to draw several views of the structure with the forces shown in all views. So, unless we draw several views and show the forces it is difficult. The equilibrium of any free body in space is defined by 6 equations 3 equations of forces and 3 equations of moments. This already we have discussed once or twice we have brought in again as a reminder.

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The components of a force R in space along three mutually perpendicular axes x , y , and z , may be obtained from the following equations:

$$F_x = R \cos \alpha$$

$$F_y = R \cos \beta$$

$$F_z = R \cos \gamma$$

where α , β , and γ are the angles between the force and the x , y , and z axes, respectively, as shown in the figure.

When the three components are known, the resultant may be obtained from the following equation:

$$R = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

If the resultant force or one of its components is found, the remaining components may be obtained from the geometric relationships.

$$\frac{R}{L} = \frac{F_x}{X} = \frac{F_y}{Y} = \frac{F_z}{Z}$$

where X , Y , and Z are the components of the length L along the mutually perpendicular reference axes.

The components of force R this force from here to here in space along 3 mutually perpendicular axis XYZ may be obtained from the following equation. So, it is simple resolution of the vector R in X direction $\cos(\alpha)$ component is coming if $F_x = R \cos(\alpha)$, if $F_y = R \cos(\beta)$, and $F_z = R \cos(\gamma)$, where alpha beta and gamma are the angle between the force and the X, Y and Z axis respectively as shown in the figure.

The figure shows it very nicely but it is difficult to visualize these angles in real problem that really creates a big issue we will see how do we solve it. The basic principle is this but we will use in a different way so that it becomes easier to understand and solve. When the three components are known the resultant may be obtained from the following equation this is well known equation I do not think it requires some explanation.

If the resultant force or one of its component is found the remaining components may be obtained from the geometric relation R by L , R divided by the length of the vector and if F_x by

X this component F_y by Y, F_z by Z where X, Y, Z this is Z are the components of length L along the mutually perpendicular reference axis better you we correct this here this is Z.

$$R = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\frac{R}{L} = \frac{F_x}{X} = \frac{F_y}{Y} = \frac{F_z}{Z}$$

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Moments in space structures:

- The moment of a force about a line is obtained by projecting the force to a plane perpendicular to the line and finding the moment of the component of the force in that plane.
- The force P, in the figure has components P_2 parallel to the axis of moments, and P_1 in a plane perpendicular to the axis of moments.
- The moment of the force about the line is $P_1 \times d$, since the component P_2 has no moment about the line.
- It may be noted that a force has no moment about any line that is in the same plane as the force.

How to take moment consider moment that's a really important issue we need to learn moment in space structure. The moment of a force about a line is obtained by projecting the force to a plane perpendicular to the line and finding the moment of the component of the force in that plane. Apparently there are two things I should mention about a line. In two dimensional plane frames structures we were taking moment about a point not about a line.

So this you please note that it is about a line and what do we have need to do we need to consider the component of the force on a plane which is perpendicular to the line about which we are trying to consider the moment. So, let us truss it for the description the force P in the figure has component P_2 parallel to axis of moment this is axis of moment this is axis of moment we may say O Prime and P_1 in a plane perpendicular to the axis of moment.

So this plane what we see this plane is perpendicular to this axis of moment. The moment of the force about the line is P_1 cross d. So, d is the perpendicular distance from this P_1 force

component to the axis of about which we are trying to find out the moment. Since the component P_2 has no moment about the line so this will not have any moment because it is in the same direction another point you should do monitor here that see \cos with this component we can easily have a plane with this line.

And this component of the force can exist on a plane so that is the reason it cannot have a moment about that plane about that line whereas this is on a plane which is perpendicular this to this O prime. So, it is having a moment so looking at this simple principle if you keep it in mind it helps to solve the problem. It may be noted that may be noted that a force has no moment about any line that is in the same plane as the force that is what I said it in a different way.

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Landing Gear Problem
 Find the forces at points A, B, and C for the landing gear shown in the figure below. Members OB and OC are two-force members. Member OA resists bending and torsion, but point A is hinged by a universal joint so that the member can carry torsion but not bending in any direction at this point.

A problem is of a landing gear, landing gear is having three components this is the main component with which the wheel is attached. These are two supporting members. So, dimensions are given here as well as it is given here. This view is if we look at this view is if you look at from this side this side I want to mean it is on yes it is here VD plane and this view is on that SD plane this way. So, this view is this one and this view is this one, so please keep it in mind that these are the two views landing gear problem find the forces at points A, B and C for the landing gear shown in the figure.

Members OB and OC are two force members this OB and OC are two force members. Member OA resist bending and torsion but point A is hinged by a universal joint so that the member can carry torsion but no bending in any direction at this point. So, this member cannot carry bending it can only carry torsion. So, what are the six we have six equations of equilibrium what are the six unknowns? It is mentioned here one unknown the first unknown is force here second unknown is force on this the two force member.

Third one we may say that this is the torsion acting on this member and they maybe three more forces acting at this we may consider those as A_D , A_S and A_V please do not keep in your mind the directions because for problem-solving direction maybe the other way taken in the drawing will follow that drawing. But for principle to understand what are the unknowns these are the unknowns and let us proceed to solve it.

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First consider the components of the torsional couple at point A. The resultant couple vector T , shown in figure below, must be along the member, and it has components T_V about a vertical axis and T_S about a side axis.

By proportion or considering component of vector T

$$\frac{T_V}{40} = \frac{T_S}{30} = \frac{T}{50}$$

$$\Rightarrow T_V = 0.8 T \text{ and } T_S = 0.6 T$$

First about the torsion components first consider the components of the torsional couple at Point A here. The resultant couple vector T shown in figure below must be along the member. So, it is along the member as I showed you in the last slide and it is it has components T_V about vertical axis and T_S about the side axis. This is please you may note that this is vertical axis this is dragged access this is side axis.

So that way T_V , T_S and T is acting on this member by proportion of concentration of component of vector $T_V / 40$, T_V by this member 40, $T_S / 30$ and $T / 50$, T_V this is the length we get a relation which is $T_V = 0.8 T$, $T_S = 0.6 T$. If you have probably understanding this relation you can easily think about resolving the T along two components you can easily find out the cos components from these three known values.

So it is nothing but those cos components are considered and a ratio in that form it has been stated here.

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The forces at B and C act along the member (two force member); therefore there is only one unknown at each point. The force at A is unknown in direction and must be considered as three unknown force components, or as an unknown force and two unknown direction angles. Usually it is more convenient to find the components, after which the resultant force may be obtained from $R = \sqrt{F_x^2 + F_y^2 + F_z^2}$

The couple T is also resolved into components about the S and V axes.

Taking moments about an axis through points A and B,

$$\sum M_{AB} = 4,000 \times 36 - C \times (40/50) \times 30 = 0$$

$$\Rightarrow C = 6,000 \text{ lb}$$

Note: 1000 lb is in the same direction of AB; OB is passing through AB.

In the free-body diagram for the entire structure, shown above, there are six unknown forces.

So, here we have a relation between components of torsion those components are shown in this figure. Here all the unknowns are also shown as I mentioned there are three forces acting on a and we need to find out those forces and let us see slowly how do we get as I mentioned previously that in the free body diagram for the inter structures shown over there are six unknown forces. So, six unknown forces are these are components do not take it that way the torsion is the unknown.

And this actually the torsion acting on along this is the unknown and this is say A_D , A_S , A_V three and this and this; these are the six unknowns. The forces at B and C act along the member two force members therefore there is only one unknown at each point the force at A is unknown in

direction unknown in direction and must be considered as three unknown force components or as an unknown force and two unknown direction angles.

Usually it is more convenient to find the components after which the resultant force may be obtained from the relation R is equals to over root F_x square + F_y square + F_z square ($R = \sqrt{F_x^2 + F_y^2 + F_z^2}$) The couple T is also resolved in two components about S and V that is what is shown here. Taking moment about an axis through the point A and B so this is what we are doing that M_{AB} is written in C we are considering moment about this line that is why M_{AB} is written here.

So sum of moments; moments about AB what are the components will come since 4000 is written here this is coming 36 where from 36 is coming this distance is 6 in the previous diagram it is noted that there is in 36 this is $30 + 6 = 36$ and it is acting upward is 4,000 is acting upward that is there is not it, it is 36 then multiplied by C , 40 by this is cos component only C is on this plane the front plane this is on this plane the C is acting.

So if it is acting on that plane about what is the perpendicular distance that perpendicular distance is 30 this is the perpendicular distance as it is mentioned where the component? Component is 40 by 50 from the previous diagram or if we consider the angle it comes 40 by 50, 50 is this length because this is 30 and this is this is also 30 that is, there is 30 by 30 square top portion so this is 50 and accordingly 40 by 50 comes as the component.

$\sum M_{AB} = 4,000 \times 36 - C \times (40/50) \times 30 = 0$, $C = 6,000$ lb Now it may arise in your mind why 1000 is not considered that gives us the value but why 1000 is not considered? 1000 is pound is in the same direction of AV , so it is acting in the same direction so it does not have any component OB is passing through a B this is the two Force member this is passing through the axis about which we are considering movement so it is not having any moment component that is there is in 1000 and this AB .

B force is not having any component and we get the value of C at the end of this exercise okay let us move to the next slide.

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All the unknown forces and the 4,000 lb load act through member OA. The torsional couple T may be found by taking moments about line OA. The 1,000 lb drag load has a moment arm of 4.8 in., as shown in the figure.

$$\sum M_{AO} = 1,000 \times 4.8 - T = 0$$

$$\Rightarrow T = 4,800 \text{ in-lb}$$

$$\Rightarrow T_V = 0.8 T = 3,840 \text{ in-lb and}$$

$$\Rightarrow T_S = 0.6 T = 2,880 \text{ in-lb}$$

Note: 4000 lb vertical load is coplanar to AO

The other forces are obtained from the following equations, which are chosen so that only one unknown appears in each equation.

$$\sum M_{OS} = 2,880 - 40 A_D = 0$$

$$\Rightarrow A_D = 72 \text{ lb}$$

The subscripts OS designate an axis through point O in the side direction

In the next slide what we have is all the unknown forces and the 4000 pound load act through member OA the torsional couple T may be found by taking moment about line OA if we consider a moment about this line then component of these the forces of A are getting cancelled because it is on that line. So, we do not need to consider that B is also acting or crossing that line so there will be any component of from B.

C also there would not be any component only the component will remain from this 4000 pound also will not have any component Y that 4000-pound is on the same plane as the OA is that is the reason you do not have this 4000 pound is actually acting on this plane and OA is also in that plains of this 4000 pound will not have any component in that plane. So, the torsional couple T may be found by taking moments about line OA the 1000 pound drag load has a moment arm of 4.8 inch that 4.8 inch is shown in the previous figure.

If you do simple geometric calculation you can here it is shown here it is shown four point eight but where from do we get this is 6 inch from the 6-inch component in this direction is having 4.8 so that is what is used here and so what we have is the as shown in the figure and we are considering moment about AO that means about this axis no component of A is coming there no component of 4000 pound is coming there no component of B no component of C.

So easily we can find out 1000 cross 4.8 this way it is actually is the contribution to the torsional moment and that is equals to 4800 inch pound. And as we have already found out the components of T_V and T_S , those are $T_V = 0.8 T$ which is equal to 3840 inch pound and $T_S = 0.6 T$, again there is a typographical mistake please excuse me this is T may be written more precisely if I can okay so that gives us the force 2880 inch pound.

So the reasons why 4000 pound is not coming that I have already explained because it is on the same plane vertical load is coplanar to AO I have mentioned again for more clarification. The other forces are obtained from the following equations which are chosen so that only one unknown appears in each equation. So, we are considering moment about OS, OS is this exists and what we have A_D in 40 A_D which one is A_D ? This is A_D other are not having because this is on the same plane.

This is also on the same plane only this is out of plane and acting perpendicular to this plane that is the reason it is AD is coming there and these are definitely these two are definitely crossing that line this axis so there that would not be there would not be any component. So, considering that equation what we have A_D into 40, 40 is this length or this length and we have A_D is equal to 72 pound.

The subscripts OS originated designates and axis through the point O in the side direction we have mentioned it here and we need to continue what the other forces.

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$\sum F_D = 1,000 + 72 - 6,000 \times (30/50) + 20/53.9 B = 0$, where $OB^2 = 40^2 + 30^2 + 20^2 \Rightarrow OB = 53.9$
 $\Rightarrow B = 6,820 \text{ lb}$
 $\sum F_S = A_S - 6,820 \times 30/53.9 = 0$
 $\Rightarrow A_S = 3,800 \text{ lb}$
 $\sum F_V = 4,000 + 6,820 \times 40/53.9 - 6,000 \times 40/50 - A_V = 0$
 $\Rightarrow A_V = 4,270 \text{ lb}$
 Check: $\sum M_A = -1,000 \times 36 + 6,000 \times 0.6 \times 30 - 6,820 \times 0.557 \times 20 + 3,840 = 0$

So we are considering equilibrium in the drag direction solution of F_D equals to 0. If we consider summation of F_D equals to 0 ($\sum F_D = 0$) it gives us the force B because in this direction this is known these are orthogonal to that direction whereas this will have a component in that direction this also will have a component in that direction but we we have already found out this force this is known force that is 6000.

So that is the reason the component of B we can find out but this 53.9 where from it comes it comes from this simple calculation the lid this is 53.9 where OB square is equals it is a continuation. So, OB square is equals to this and that helps us to make a component in this direction horizontal direction so that is the reason this is the 20 and this is the length and that gives us the component in drag direction.

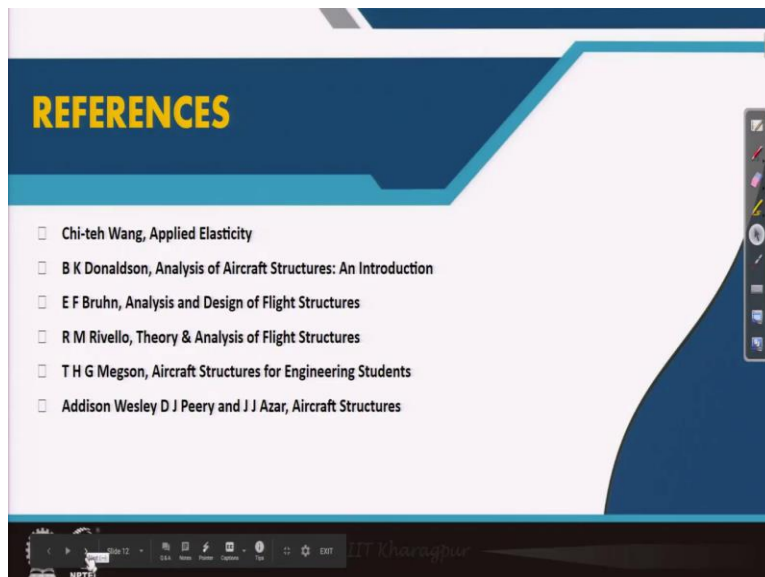
The other component is this is 30 and this is sorry 50 this is this is the 50 length this is 50 that gives us the component here and 1000 and 72 this is 72 already known and 1000 is acting here. So, as a result we get the force B and similar way if we consider in the S direction summation of forces in the S direction we find out the force A is. As I had described in very well the previous equation please try to understand these equations why this component is coming.

And definitely you need to spend some time to understand this equation also I would suggest you find out this component F_V and find out the vertical force A_V . So, considering that A_S and A_V are

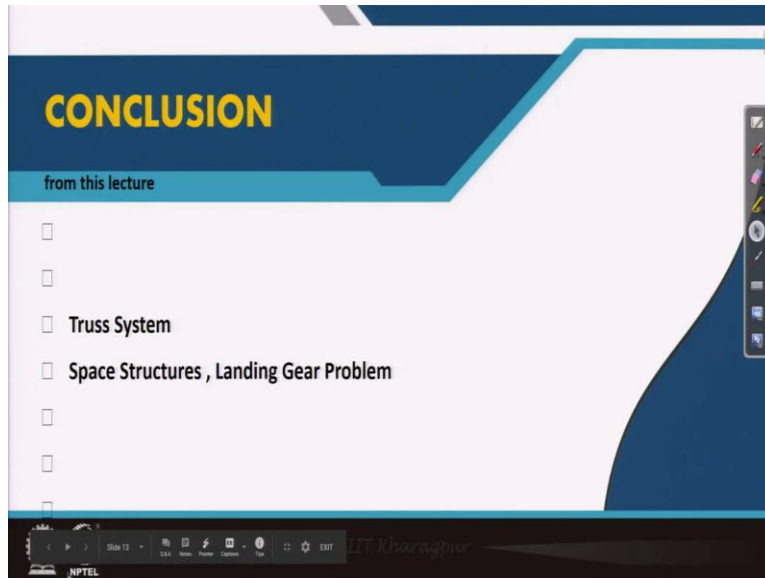
known following the similar logic you can easily understand the components of forces in this direction and you can find out the force A_V . And as a check since it is a 3 dimensional problem and it is better to have a check summation of A_V we are we have checked and we are considering the moment components about A_V this axis and we are trying to see whether it gives us that moment 0 or not.

So you may consider that as a homework if it is not clear we may discuss it all. So, with that note let us conclude today's lecture in the next lecture again will solve one more truss problem and interesting landing gear problem with landing gear components, concepts of Oleo strut and all those who will discuss and will continue further.

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So references are all same references and what we have learnt from these slides are these space structures introduced to space structures and we have solved a learning gear problem we have how to consider three-dimensional structures to for the solution and that way we conclude today's lecture and hope you will join back again in the next lecture that is another space structure problem will solve, thank you for attending this lecture.