Space Flight Mechanics
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Lecture No - 68
Orbit Determination

Welcome to lecture 68 we are just finished the perturbation analysis for general Orbit perturbation theory and now we going in to the Orbit determination problems, but before we take the actual for the determination process. We have to go through the frames different reference frames are available for modelling the motion of the earth and without that; because we are interested in finding the orbit of the satellite.

So there for reference frame it becomes mandatory to define. We have the reference from like inertial reference frame, Terrestrial topocentric and various other sort of reference frame are there as for the need. And there are algorithms are the equations available for transferring from 1 reference frame to other reference frame and in that process let us start with what the reference frame is there and the reference system.

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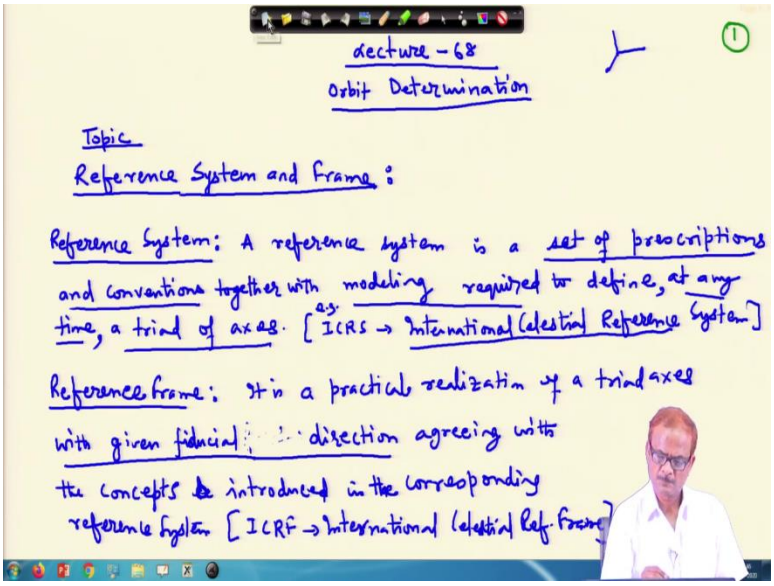

So first we define the reference system and then go for the reference frame. A reference system is a set of prescription and conventions together with modelling required to define at any instant of time, a triad of axes. So, how the reference frame will be formed so that defines the reference

system. And currently see here in this case we have the ICRS this is example International Celestial Reference System this is an example of this and so we will discuss further about this.

So reference frame it is a practical realization of a triad axis, triad axes means you have three orthogonal axes, so triad axes with given fiducial points. Fiducial points are nothing but reference points, fiducial direction and points fiducial directions which triad of axes which given fiducial direction some reference direction along which direction your triad and axes point agreeing with the concepts introduced in the corresponding reference system.

For example I can have the ICRF where ICRF as usual written it is international celestial reference f stands for reference frame.

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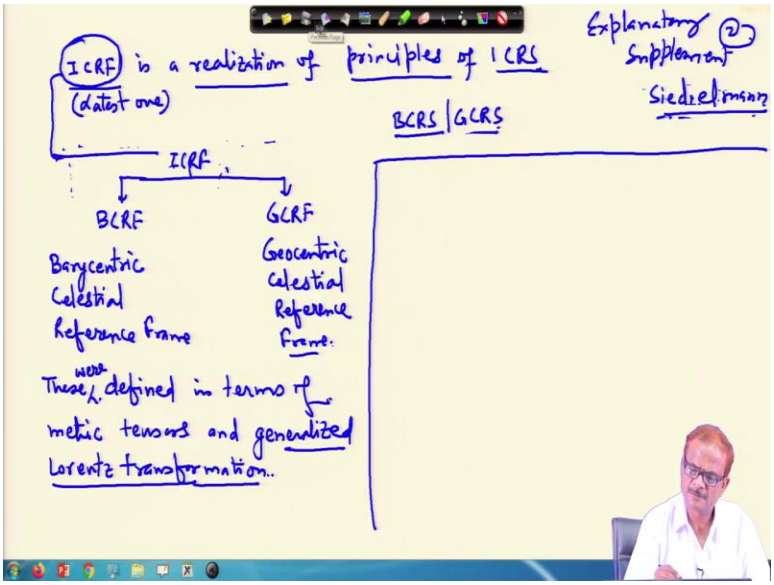

So ICRF International celestial reference frame is a realization of principles of the ICRF what were the principal has been laid down in ICRF according to this, this frame is realised and this is the latest one. So this is latest one, but before this we have other reference frame. So, ICRF it can be divided into two parts is called the BCRF and the GC one is GCRF and another is GCRF, this is called the; we can write it as.

Here ICRF this is called the Barycentric Celestial Reference frame and this is called the Geocentric and accordingly you have the barycentric celestial reference system and geocentric celestial

reference system. So these are defined in terms of; if the general relativity becomes important here in the case where the largest distances are involved. So, BCRF this is the barycentric as you know, the barycentric is the centre of mass of the solar system and GCRF the geocentric is at the centre of mass of the earth.

These are defined; these were defined in terms of metric tensors and generalized Lorentz transformation of course this is beyond the scope of our course. We are not going to discuss all those things but conversion from one frame to another frame, general relativity plays its role. Special relativity deals with the case where the absence of mass. But once the mass is present for the space time take it affected and therefore you have to take into account the general relativity in the case we are dealing with the BCRF and the GCRF conversion between them.

And the book by Siedelmann, explanatory supplement to astronomical. You can just look into the explanatory supplement full book name I do not remember. But this is my Siedelmann s i e d e l m a double n, this may be a Siedel, I will write the name of this book by Siedelmann, spelling exactly all do not remember but it is a Siedel something like that Siedi is there or not I do not remember that and then ok.

So, that gives you; so if you look into that book. So, this will become conversant with this topic of reference frame how the time is measured, organized and various other things. So, this book may be of more than 800 pages or may around thousand pages and it gives all the great details all the things available. So, this ICRF the International Celestial Reference Frame. So this is kinematic realization I will come to this point.

What is the kinematics realization and what is the dynamic realization? Can we can go to the next page.

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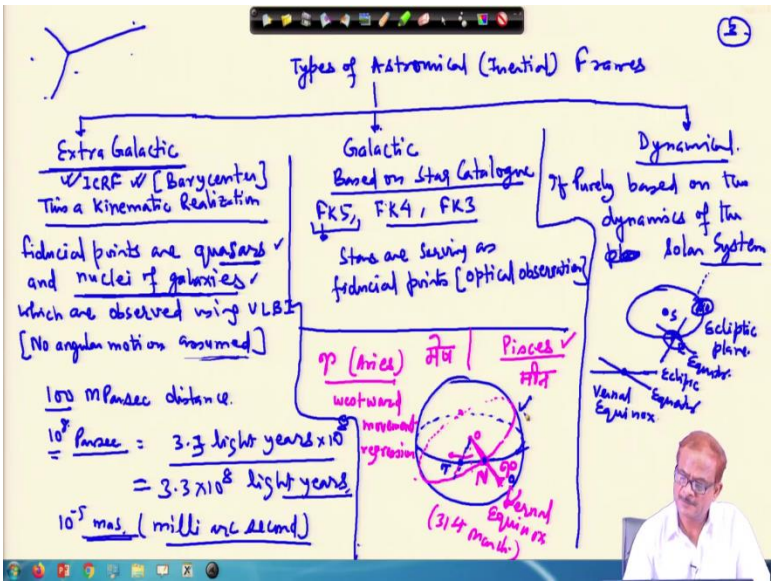

We can divide this inertial reference for formulation in broadly in three categories this Extra Galactic in this category your ICRF lies and moreover this is a kinematic realization. So this is defined kinematically. In this the fiducial points the reference points are quasars and nuclei of galaxies which are observed using very long baseline interferometry. So kinematic realization no angular motion involved or no assumed angular motion.

Actually what happens the quasars and the nuclei of galaxies. They are located at some 100 mega Parsec distance. so that makes it . 100 mega Parsec is 10^8 Parsec. Mega is 10^6 and then 100 so 10^8 parsec that makes it . 1 parsec is around 3.3 light years, so, 3.3 $light\ years \times 10^8$ so many years. So at such distance those fiducial points which are the quasars.

These are the source of X rays and the nuclei of galaxy. So they show almost no motion. Over 1 year there maybe; so it is a line at a distance of 3.3 $light\ years \times 10^8$ means 3.3×10^8 light years distance. If something is lying at this much of distance and it is almost is not showing any angular motion. So, angular motion may be in the range of 10^{-5} milli arc second.

So this is milli arc second. So almost they are not showing any variation; so if you have many points in the space and if you look from the earth and they are not; where not showing any motion over the years. So you can easily fix reference frame using those points and this extra; and this ICRF is based on that and so its centre is lying on the barycentre what we called as the barycentric.

So at the barycentre is centralized, barycenter. And shortly, I will show you how it is different from the other one.

Galactic this is based on star catalogues such as Fk5, Fk4, Fk3 these are old and this is still being used, but this is the latest one. So Fk5 this is called the fundamental catalogue. Fk stands for fundamental catalogue and this ‘k’ instead of a small ‘c’ this ‘c’ there you replace it with ‘k’. So here numbers of stars are observed, so stars are serving as fiducial point and these are obtained through optical observation.

And dynamical reference frame it is a purely based on, we shall call dynamical itself if purely based on the dynamics of the Solar System. So as we know that in the case of the solar system we have the; already we gone through the perturbation theory .So in the perturbation theory we have sun is here around that earth is moving. This is your sun and earth is somewhere here.So this plane we have defined as ecliptic plane.

But because the earth is bulged earth is moving here so this is the equator. This is the pole of the earth so the intersection of the ecliptic and this is your ecliptic and the equator this defines the point we call as the Vernal equinox which will; so I will show on the next page or maybe here also I can have a look at; this is the earth and this is the Earth equator. Then the Sun apparent motion it will appear like this.

If you look from the Earth so this is the way the sun will appear to me. For the point where it is cutting the equator that becomes your the line of vernal equinox or this is your vernal equinox which we define at the 31st March. During the time of Christ, vernal Equinox was lying in the constellation of theories, we call this Aries with the symbol RAM. Aries means RAM. In Hindi we call this as **‘MESH’**.

During the time of Christ vernal Equinox it was lying along the Constellation of Aries, but currently it is lying in the constellation of Pisces. This is called the **‘MEEN’**. So currently it is lying in the constellation of Pisces. What happened that over the period, because of the perturbation

this moves regressors. Ok. So this point will put a 0 also. This point shifts in the backward direction here in the westward direction.

So this is the westward movement and is called the regression. And you can simply say that this O and this is the nodal line. . O and let us say this point we write as N. So this becomes your 'ON' is your nodal line here in this case.. You will further explore this; what is the precision nutrition or other things involved. But first we were discussing about the dynamical reference frame.

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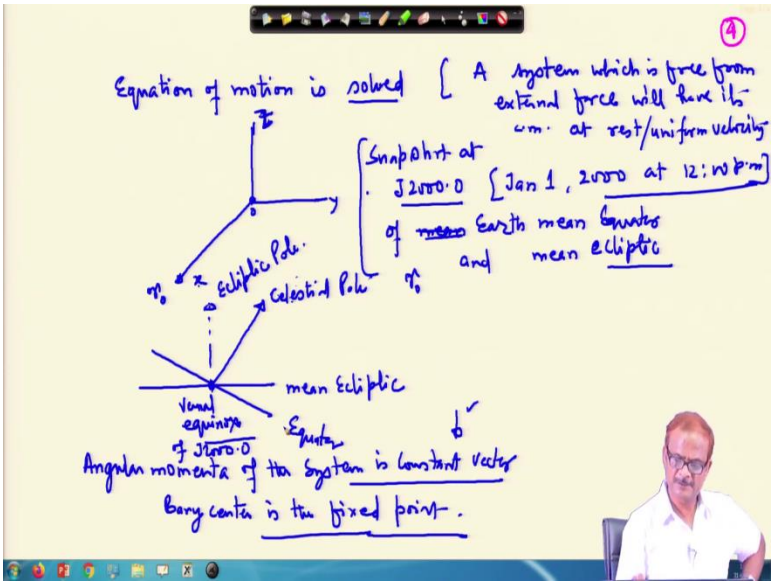

So, dynamical reference frame; in this case the equation of motion are solved. Already we have done the general perturbation orbit perturbation theory. equation of motion it is all and you know the multi body system what we have discovered that a system which is free from system which is free from external force will have its centre of mass either at rest or moving with uniform velocity. So center of mass can be taken as a reference point because then it forms a non inertial sorry the inertial point if it is fixed triad in the case of the solar system and if this is the barycenter if I take the origin here and take x in the gamma direction. And write these as X Y and Z and this as O. So these forms an inertial reference frame, but under what condition? Now if you look here in this point, as I was telling that this point will regress it will come here to this point.

So this is gamma 0 this point will be written as say Gamma 0 it is basically a sign of Ram so it is written in terms of Gamma. So therefore this point itself is not fixed it is a regressing on the equator of earth. So if that is happening then if we take the barycentre as the centre of the triad and its

direction along the gamma direction then of course this will be rotating as you see and therefore instead of doing this.

This is fixed at this; this is taken at a snapshot ok snapshot at J 2000.0. What does this mean? This mean implies, this is January 1 2000 at 12 pm at this time whatever the snapshot of the equator and the mean equator the mean ecliptic and whatever the snapshot of mean equator; so this is the snapshot at 2000 1st of January of mean earth equator or mean equator or earth mean equator and mean ecliptic so this Gamma 0 is defined by this.

This defines your Gamma 0. This is the mean ecliptic. What does this mean by mean? Totally this will also be clear and this is your equator and this is North Pole. This we will not call is as North Pole we will rather call as the celestial pole and perpendicular to the mean ecliptic is called the ecliptic pole. So while solving the dynamical system what we find that we get; if you remember, so the angular momenta is fixed.

Angular momenta of the system is a constant vector this gives you fixed direction either we can take this as a fixed direction or we can take the barycentre and barycentre is the fixed point. Let me right that also barycenter is the fixed point because its centre of mass so it is either at rest or moving with uniform velocity, so therefore that makes it inertial. At particular instant of time and therefore this is not called inertial because in fact this frame, if you go by this, we will see that this frame will recede.

Because you are always defining to what the gamma and therefore, we need to fix at a particular instant of time, which is done by writing gamma 0 at J 2000. At J 2000 we are fixing it. So once we have taken the snapshot and then fixing the frame, we will call it as the pseudo inertial and this is not perfectly inertial this is snapshot in that respect your ICRF the International Celestial Reference System the extra Galactic which is which shows hardly any motion.

And it is kinematically realized because there is no equation of motion needs to be solved here. Only the observation of some quasars and Galaxies are required using the VLBI. In this in the galactic one, we require the position of the various stars and which are done using the optical

observation and using that then you fix the triad and again the Central, at the barycentre but the direction you have to fix. The dynamical one here the direction is fixed using the vernal Equinox.

So this is your vernal Equinox J 2000. Any other date that will be also a vernal Equinox but not of J 2000. That will be of another date. . So this gives rise to various terms like the mean true of date or what we call is the mean of date or mean of J 2000 so on many terms that are involved. So, I will continue this is in the next lecture we stop here and we will again get back to this. Thank you very much.