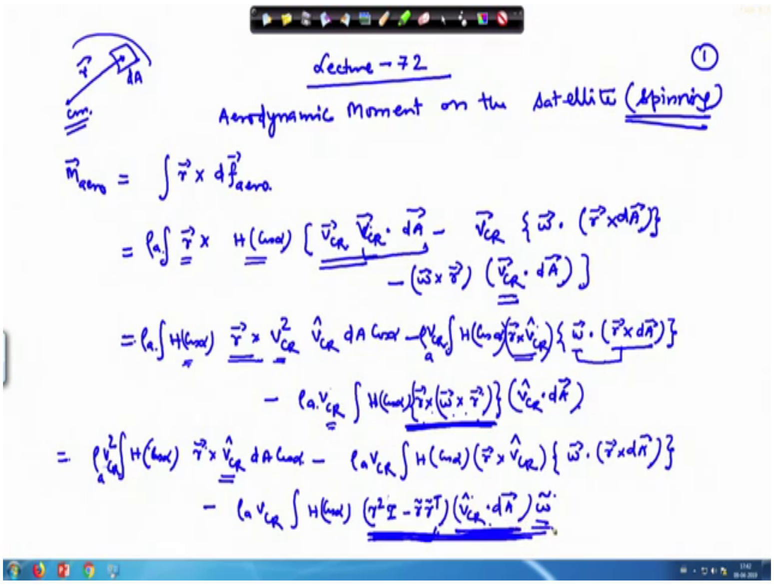
Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture – 72
Atmospheric Force and Moment on the Satellite (Contd.)

Welcome to the lecture number 72.So we have been discussing about the Aerodynamic Force and Moment acting on the Satellite if it is spinning. So, for the last time we worked out the force acting on the satellite. Once it is spinning and in this class, we will finish the torque acting on the satellite once it is spinning due to the atmospheric drag.

(Refer Slide Time: 00:40)



So the torque acting M aero we already I have written here one of the satellite is spinning, so M aero will be written as r cross d f aero. So, this is the basic equation where r is the distance of the point of the elemental area from the centre of mass of the satellite.

So, we need to insert all those values what we have written earlier, so this can be written as H cos alpha and d f aero we have to insert, so if we write those values. So, this is V CR times V CR dot d A minus V CR times omega dot r cross d A ok.

So, we divide into three integrals, so we can write this as cos H cos alpha r cross here this part we can reduce it to V CR square V CR cap and this part we can write as d A cos alpha minus and; obviously, rho is missing. So, rho will put here in this place rho a, this is rho a.

So, H cos alpha as I have told you that we have to be very particular about where the cross and dot product are lined and we have to follow that sequence, any change in that sequence will produce wrong result. Similarly, from here also we can take it out the V CR ok. So, rho a times V CR H cos alpha we are picking up here and here the part r cross; we have missed out.

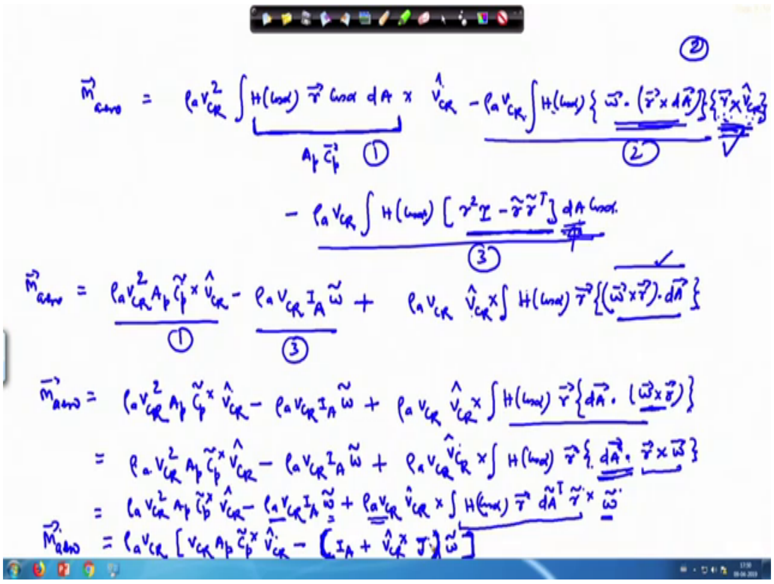
So, this r cross is there r cross we have already taken into account here in this place, so this also must be taken care here in this place. So, that will appear as r cross V CR because this part is a vector; this part is a vector and together it makes a scalar. So, this cross product is going to operate only on a vector therefore r cross V CR.

Similarly, here we have this V CR I have taken it outside and then H cos alpha which comes for this particular one ok, so H cos alpha and then rho a also we have taken outside this is omega r cross omega cross r and then V CR cap dot d A, rho a times V CR square this term can be taken outside H cos alpha times r cross V CR cap times d A cos alpha.

If you remember this quantity is nothing, but r square we have written this as E double bar or in terms of inertia dyadic. If we write in terms of the identity matrix, so this can be written like this r tilde times r tilde transpose V CR cap dot d A, it is little lengthy, but it is not that difficult.

So, if you follow a systematic method of working out, so you can get into any kind of research because the research is a systematic method of doing the mathematics ok; obviously, it’s this area is more mathematical that is why I am telling and together with the physics is also involved a lot of physics it is there ok.

(Refer Slide Time: 08:17)



So, this V CR this is not depending on the integration sign, so therefore, this can be taken out side in the next step we do this, H cos alpha times r time d A cos alpha; r cos alpha times d A this is the first term and then cross is there, so the and cross is operating on V CR ok. So, V CR we already putting here V CR cap and then this cross is present here.

So, you can see that the term which is present here this is nothing, but your A p times C p. So, this way all the terms would emerge out, the second term then we are picking up minus rho a minus rho a times V CR H cos alpha.

And this term is d A cos alpha, so for this we put here d A cos alpha V CR square A p times C p cross or if we put here in the matrix notation, so the same thing can be written as C p tilde times V CR cap ok. So, this is the first term and minus rho a times V CR the second one I am taking.

So, here little bit of rearrangement is required, so I will throw this let us name this, this is the 1st term, this is the 2nd term and this is the 3rd term. So, let me bring the 3rd term first here in this place, so that we sort in the process instead of copying the whole thing again and again, so this is rho a times this is the first term here rho a times V CR and then this quantity which is written here. So, this quantity we write as I A this is the moment of inertia term if we multiplied by mass, so you get a moment of inertia term.

So, we are not integrating with respect to mass, but rather with the area, so we get a equivalent of that. So, the 2nd moment of area basically this quantity become 2nd moment of area, for this we can write as I A times and one part here is missing somewhere, we have omitted while writing this and if you remember that once we write it like this we have to also right here omega tilde for this part, this is r times r cross omega cross r ok. So, this will be these times omega and because this quantity this scalar, so writing it this way is not a problem ok.

So, therefore, this can be written as omega tilde, so this is your 3rd term and now it has got sorted out and the 2nd term we can treat it. So, in the second term we require little bit of treatment like this is a scalar, so we can move it anywhere and we reverse the order of this and write it like this V CR cap. So, and cross sign can be taken outside and this becomes r this is H cos alpha first and then for this the r will appear.

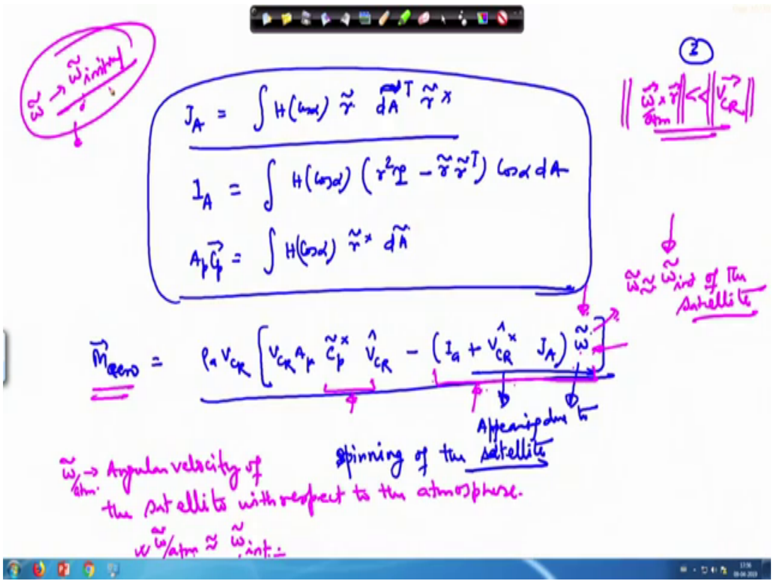
So, what we have done that r cross V CR which we are writing a V CR cross r, so for that a minus sign will appear, so this minus; minus that will make it plus ok, but again we are going to change it for what will see here we are going to put it in a particular order. So, this is r and what appears here this we can write as omega, we need to maintain a particular order for this, so we will write this first as omega cross r and dot d A ok.

So, this has been taken care of, this part is taken care of here in this place H cos alpha is there H cos alpha. We change the order of these we write here as d A dot omega cross r because it is a dot product, so it does not matter. V CR cap rho a times V CR I A times omega tilde cross now this quantity we have to treat properly.

So, if we change the order of this instead of writing it like this, if we write here r cross omega H cos alpha r d A dot r cross omega V CR cap H cos alpha this is r and this quantity, then we can write this is basically d A tilde transpose, this is the area elemental area vector, this is for d A dot we have written here transpose and then r and this will be operating on omega tilde.

So, we finally, wind up this, so if you see here this is omega tilde omega tilde is appearing here, so we take out the related terms and write it in a simplified format V CR A p times C p tilde cross V CR cap minus from this place I is rho a V CR is taken outside, so this becomes I A and plus omega can be taken outside here also this quantity goes. So, we are left with V CR cap cross and the quantity from this place to this place.

(Refer Slide Time: 20:19)

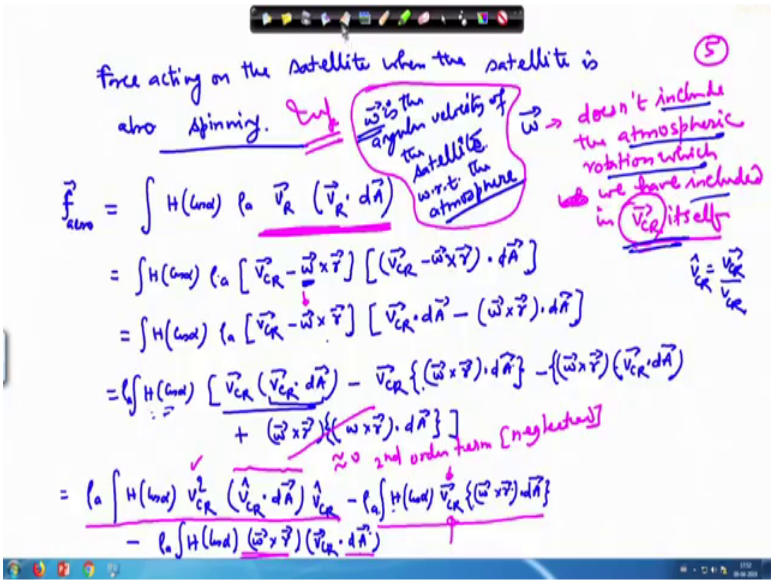


So, this we define as J, so V R cross J and this will be operating on omega tilde, where J we are writing as H cos alpha times r tilde d A tilde transpose times r tilde cross, this is J and I A we have written as H cos alpha cos alpha d A ok.

So, this is M aerodynamic, one bracket will be also here in this place this is the inside bracket. So, if the aerodynamic torque is present; sorry if your satellite is spinning, so I will copy first this expression on the next page and then I will talk about this.

So, what we have got? M aero rho a times V CR, then V CR A p C p tilde cross times V CR cap minus I a plus. So, this term or either side this whole term is appearing due to spinning of the satellite. Now, one thing we must be careful while in the previous lecture also we have discussed while we have written this equation.

(Refer Slide Time: 23:04)



So, omega does not include atmospheric rotation which we have already included in V CR, but omega is the angular velocity of the satellite with respect to the atmosphere this part is important here. Omega is the angular velocity of the satellite with respect to the atmosphere and that will decide at what is speed the particles are coming and a striking the surface of the satellite.

So, this part is your important part to mark. So, here omega tilde this is the angular velocity of the satellite with respect to the atmosphere. However, this omega with respect to the atmosphere means this is with respect to the atmosphere ok. This can be approximated as omega inertial because it does not we if we look into the quantity omega cross r this quantity is much less than V CR magnitude wise ok.

So, therefore, this assumption is safe we can replace this omega atmosphere by omega inertial, so if you do this, so then omega tilde here we can indicate this is equal to omega inertial of the satellite. So, it simplifies your expression then.

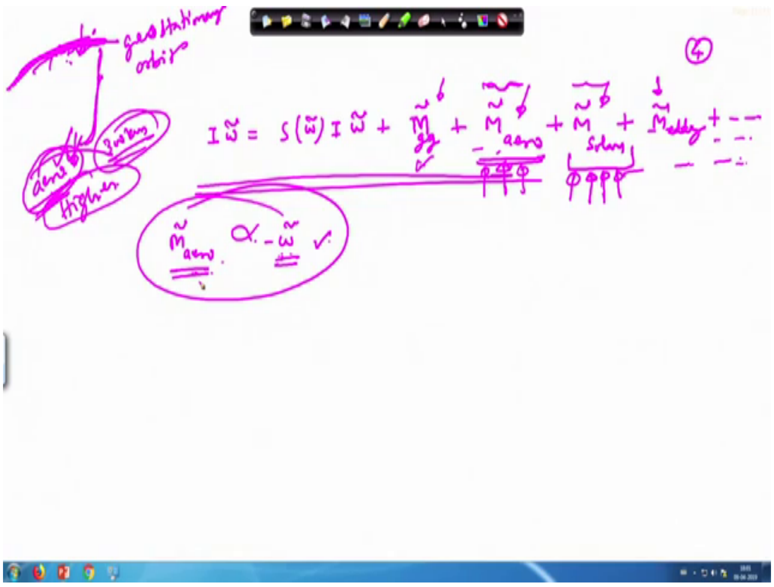
Now, as we are discussing this is a part which is coming because of the angular velocity of the satellite with respect to the atmosphere which we are telling that we can model as the omega inertial. So, if we look here in this expression very carefully. So, it says that the aerodynamic torque, it depends on the angular velocity of the satellite with respect to the atmosphere also it is not only this part ok.

So, this part will, so the rotation of the satellite if the satellite is spinning ok. So, that spin will say the initially your omega here this is omega 0 initial, omega initial this is the write in the beginning. So, because of this it will start decaying it will not stay at this value and therefore, your spinning satellite will stop spinning after some time it may be a long period, but ultimately it will come to a halt ok.

And therefore, in any long range satellite attitude control problem you need to a take into account all these forces ok. If you are looking for the stability problem, so you are looking into the short you are looking for a short period ok. The same thing is in the case of the aircraft also the navigation problem and the control problem.

So, control problem there are the stability problem what we say for the aircraft it is a done over a short period of time, while the navigation it is a done over a long period of time. So, there the curvature of the earth will be taken into account also ok. So, if the satellite is you are putting for a 10 years in the orbit ok, for 10 years you are planning the mission. So, you need to do all these analysis very meticulously, at least you have to do the simulation.

(Refer Slide Time: 28:18)



Now, as far as the satellite attitude control is concerned, so rest of the things are as usual that you have to write the equation in this format and if you have the gravity gradient torque, so write here M gravity gradient torque, then if you have the aerodynamic torque, so you will write here M aerodynamic torque, if you have the solar radiation torque, so you will write here the solar radiation torque and any other like the M due to the eddy one.

So, there are so many things that we can add, so rest of the things briefly, I will write in uploaded material. So, this also I am not going to discuss in details here because this will follow the same procedure as we have done for the M aero. So, I will introduce this and then rest of the things will be uploaded as the written material.

So, all disturbances whatever you can add here in this place and this gives you are the satellite attitude dynamics problem ok. Now whichever is significant like if you take M solar, so M solar radiation pressure if you take at the geostationary height and at this height, so it will be almost constant because as compared to the distance of the earth from the sun this distance is not very large ok so, we will come to this problem later on, so it does not matter much, but if you are looking for the aerodynamic.

So, aerodynamic torque is much higher here in this place as compared to the geostationary orbit, here this geostationary orbit. So, atmosphere is almost nil here; in this place.. So, you will not get the aerodynamic torque as here in this place as you are getting here it is a very strong here in this case say at 300 kilometer altitude, your satellite if you do not correct it is orbit.

So, it will pull down into the earth atmosphere because of the aerodynamic drag. So, similarly for the attitude also this gets affected because of the gravity gradient aerodynamic torque and the solar radiation. So, there after whatever the procedure we have chosen to analyze the system, the same procedure can be chosen and this can be accommodated along with the gravity gradient for the linearized system.

So, we cannot do in so much details all these things here, but the procedure remain same and thereafter; obviously, the theoretical analysis involved, if not you can look into the system behavior over a long period of time just by without the aerodynamic torque and solar radiation torque and with the added aerodynamic torque and with the added solar radiation torque.

So, a multistage simulation you can do and then you can check that what is the effect over a long period of time? So, your whatever conclusion is that the small torque due to the spinning, M aero is proportional to omega tilde here in this place this is proportional to omega tilde; this quantities are constant.

And therefore, this will make this omega tilde and this is coming with a minus sign. So, this will make the system decay this is angular speed will decay over a period of time ok. So, we will stop here and in the next lecture I am going to discuss briefly about the solar radiation pressure and in a very brief manner, so we will continue in the next lecture.

Thank you very much for listening.