Satellite Attitude Dynamics and Control
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Lecture – 70
Atmospheric Drag on the Satellite

Welcome to the lecture number 70. So, we will start today with the drag due to the aerodynamics, means aerodynamic drag arising because of the molecular impact on the satellite ok.

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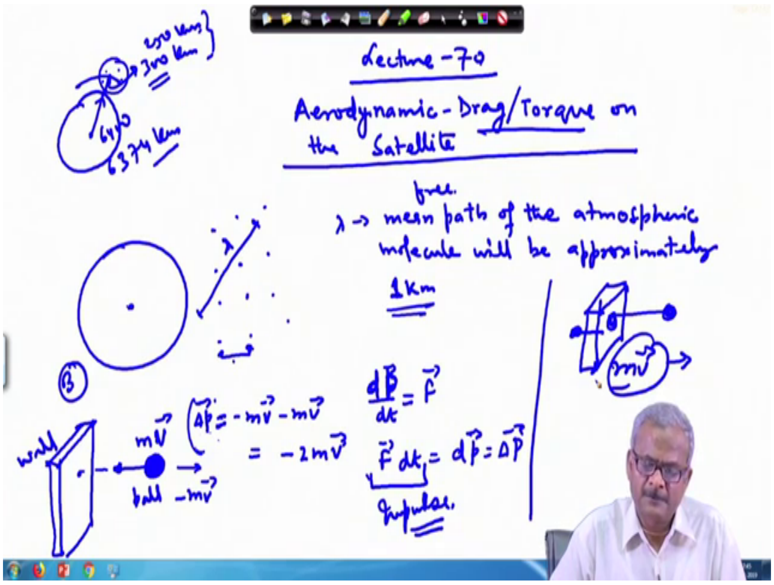

So, molecular impact means in the atmosphere. So, this is the earth and earth radius is around 6400 or approximately 6374 kilometers and say you are satellite is in the orbit of 300 kilometer altitude. 300 or 250 kilometer altitude which is the normal altitude, 250 kilometer is the normal altitude at which the satellite is launched. So, at this attitude drag is considerable ok. So, if you leave the satellite in that position so, slowly slowly it’s orbit will decay and it will the satellite will enter the completely into the earth atmosphere and it will burn out or either it will, if the whole thing does not burn out so, it will hit the earth.

So, this molecular impact on the satellite, this generates drag well an earlier I have told you that the aerodynamic drag can be and the solar radiation drag can be model as the disturbance to the system. So, aerodynamic drag can be modeled as a constant term and solar radiation as a periodic term ok. However we are going to work it out here, how the aerodynamic drag and the molecular this solar radiation torque will drag and the torque in which form they appear will try to work it out.

So, if this is your body, the satellite body and it is a moving in the atmosphere at in the orbit. This is say 250 kilometer or the 300 kilometer orbit where the atmosphere is considerably rarefied and molecular distribution may look like this. So, we can have the mean path of the molecule here, lambda mean path which we called the mean free path actually. Mean free path of the atmospheric molecule 1 kilometer. So, as you go to higher altitude this distance will keep on increasing ok.

Distance between these molecules may be a small. It is a order of magnitude is small as compare to this 1 kilometer. But this mean free path before 1 molecule collides with another molecule it will be quite large ok. So, if these drag basically it arises because of the two factors. As you have learnt in your physics that say if this is the ball, is it is s wall here and another hard ball it is moving toward this one. And says V is the velocity. So, it goes and collides with this ok. And then returns back without loss of any energy we considered that it is a perfectly elastic collision. So, it is a linear momentum is m v and while it returns the linear momentum becomes minus m v.

So, the change in linear momentum becomes minus m v minus m v. So, this is delta p change in the linear momentum. So, this is minus 2 m v and how much time this impact has taken place? So, accordingly you can calculate the corresponding force imparted to the wall or imparted to the wall by this ball; this is your ball. So, this will be given by. So, basically you are this is the change in momentum so let us write it another way. This is delta p this is the change in momentum. So, if F is the force applied or say d F is the force applied; so, d F by d t sorry the d p by the linear momentum is p. So, change in momentum d p by d t that gives you the force applied so, F time d t that gives you d p or delta P. So, this is the impulse.

So, the total in this case what we are going to do that there are two types of things. Here one is the returning another type can be like this ball goes here and then gets stuck to this. So, it is a losing its total momentum which was m v. So, this lost momentum will appear as a force applied on this wall. The same kind of phenomenon it occurs with the satellite.

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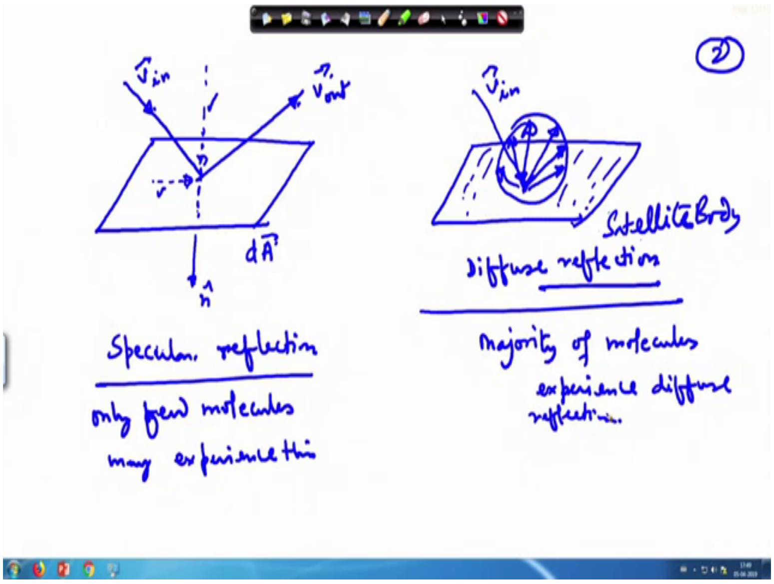

So, in the case of the satellite say this is a small area of the satellite d A whose normal we define like this n cap which is going inside here in this case. So, the n I have shown here in this place remember and this velocity this is incoming velocity and this is the V out going velocity. So, what we see that if we break the components of this velocity along this direction and this direction. So, this component will remain unchanged only this component vertical component will change ok. And this we call as the spectacular sorry the specular reflection. On the other hand we can also have the case where the particle is coming here in and thereafter it is absorbed in the top layer of the surface of the solar cell or it may be the, it may be your satellite body. So if we say that this is let us take this to be satellite body right now.

So, this can be the satellite body ok so, then once it is coming inside and then it loses all its velocity and later on it is ejected. So, this ejection it is a probabilistic phenomenon and it can be ejected in any direction ok. And this type of ejection it is a call the diffuse reflection ok. So, this a specular reflection it happens only with only few molecules may experience this; while majority of molecules experience diffuse reflection.

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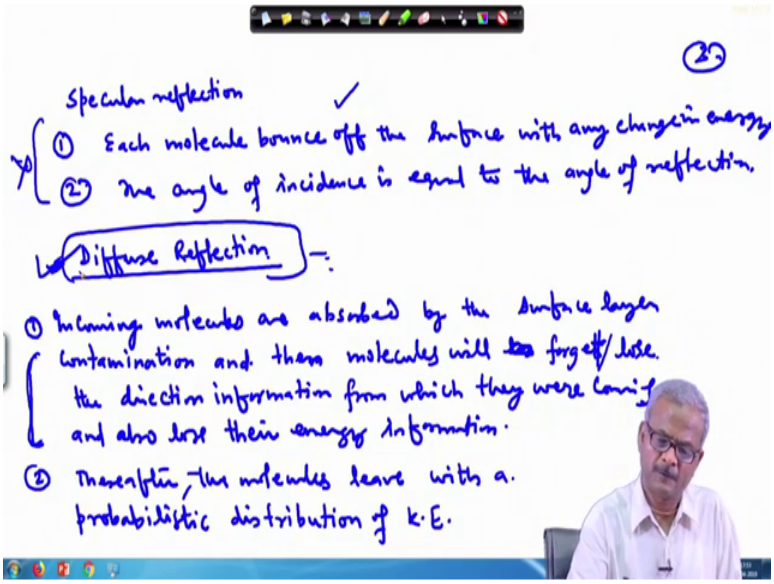

So, in specular reflection two things happen, bounce off without any change in energy. And the angle of incidence is equal to the angle of reflection. While in the diffuse case, incoming molecules are absorbed by the surface layer contamination and then the molecule will, molecules will forget slash lose the memory of its direction for molecules forget slash the direction information in from which they were coming they were coming and also lose their energy information. Means how much energy they were containing that is also lost. So, thereafter the molecules leave with a probabilistic distribution of kinetic energy ok.

So, this is a these two are the different mechanism totally and this mechanisms it happens only with few molecules, but majority of them go diffuse reflection. So, we will take up this and all the calculations are done based on this only not on this. So, this is not used only this will be used.

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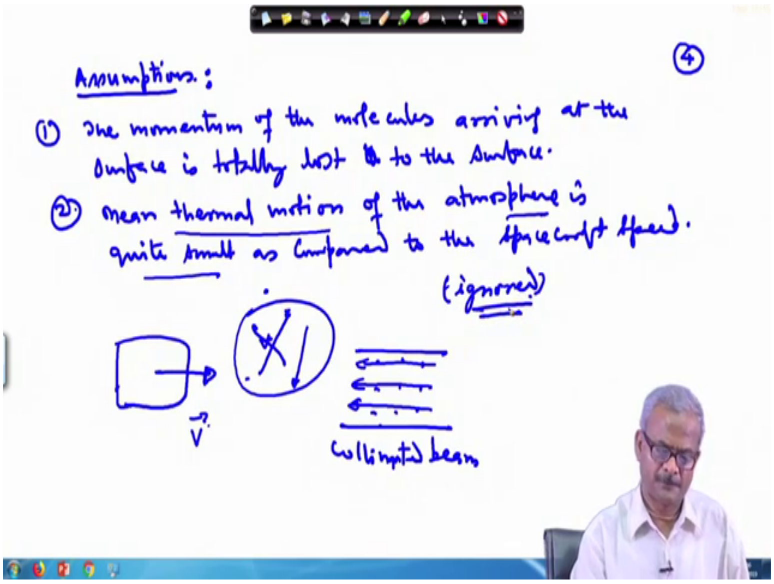

So, the following assumptions will be made; totally lost to the mean thermal motion of the atmosphere, as compared to the spacecraft speed. Why this is so? Because say this is your spacecraft and if your molecules are arbitrarily moving, so here like someone is going there, someone is going there this way it is a moment is there. So, then analytical analyses in the pro analyses we will get the problem. While if we assume that let us say that, I assume them to be static ok.

So, whatever the velocity of the satellite is there so the relative velocity of these two will be nothing but the velocity of the satellite which I can show like this. So, they generally assume to be a collimate this say for a number of beams are there and going like this ok. So, then they form as the collimated beam. And then the analysis becomes easy here in this case. So, therefore mean thermal motion of the atmosphere is quite a small or it will be ignored. So, this will be ignored.

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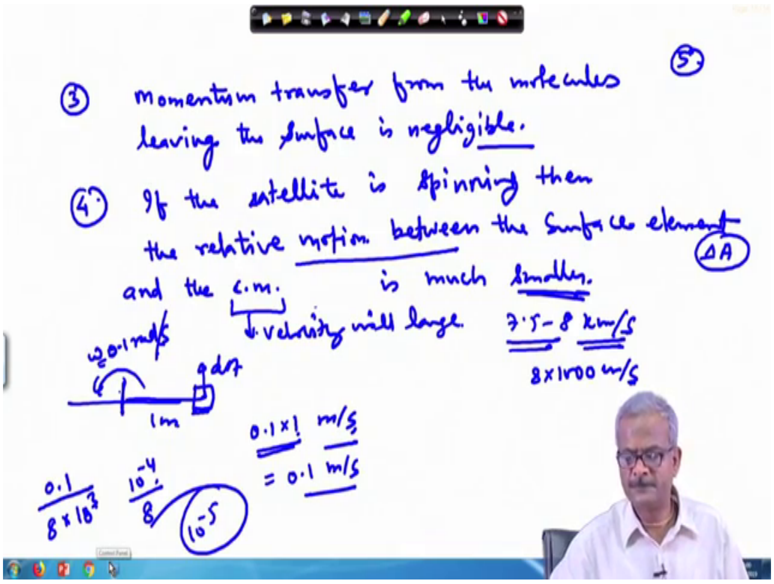

And momentum transfer from the molecules leaving the surface is negligible ok. So, this simplify the situation a lot ok. If the satellite is spinning, then the relative motion between the surface element means a small element of the surface say delta A or d A and the relative motioned of the or if it ok. The relative motion between the surface element and the centre of mass surface element and the centre of mass speed is and the centre of mass is much smaller.

This is centre of mass for the satellite as you know this will be this velocity, will be large ok; 7.5 to 8 kilometers per second ok. So, this is very large and your satellite may be rotating say at most at 0.1 radian per second. So, you can calculate here say if it is 0.1 radian per second and some distance here d A element is located at 1 meter and omega is 0.1 rad per second 0.1 radian per second. So, what will be the velocity here? Omega cross r. So, this is 0.1 times 1 this meter per second ok.

So, that means, this is just 0.1 meter per second the speed of this element on the other hand this is 7.5 to 8 kilometer per second. So, let us take this to be 8 kilometer. So, 8000 meter per second. So, what the ratio? Ratio is so this is 10 to the power minus 4 divided by 8. So, approximately that turns out to be 10 to the power minus 5 ok. So, this is very small.

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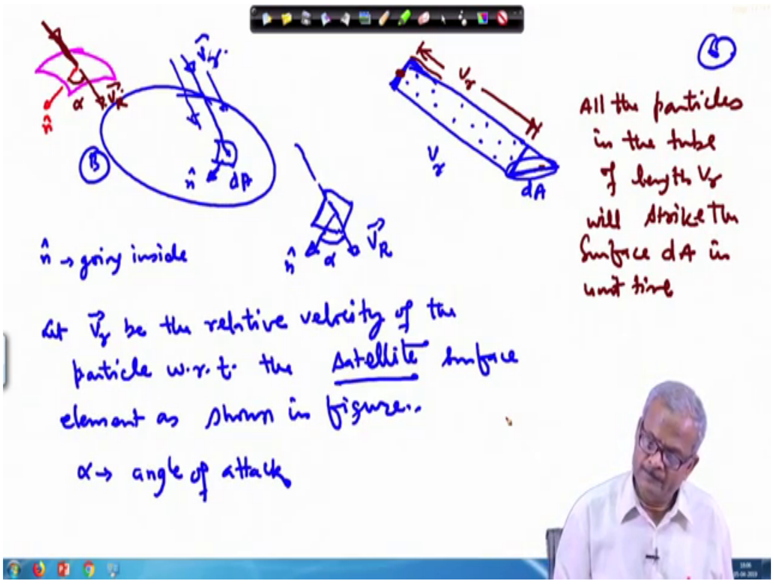

So, under this assumption then we can calculate what will be the drag and the torque on the satellite. So, we have this body and on the body there is a small element which willso as d A and n cap this is the unit vector which is going inside, so n cap going inside. So, a small surface small element on the surface of this satellite V r the relative velocity of the particle with respect to the satellite. So, if you take the atmosphere to be at rest so, the V R is nothing but the velocity of the satellite.

So, your particle is coming here in this direction. So, what will be the force exerted on this small element? So, let us write this let V r or this is your d A as shown in figure. So, as I have told you that if that is not random motion in this random thermal motion of the atmosphere so, all the particles will be coming here in this direction so, as a collimated beam. Then the analysis then becomes easy ok.

So, let us say this is a tube of length V r. So, all the particles contained in this tube and here is my surface ok. This is d A. So, this is d A. So, all the particles contained in and what will assume that this is your vector here, let us say I will make a separate figure, n is the vector going here n cap and V R is the vector which is going here in this direction. This is the V R. This angle we write as alpha. So, alpha we write as the angle of attack. We can do little better say for we have this surface and normal to this surface is going inside this is n cap and the velocity vector is from these direction ok. So, it is a going like this angle is alpha; so this is V R.

So, all the particles contained in this tube of length V r. So, from here to here this length is V r. So, they will strike the surface in 1 second ok. So, all the particles in the tube of length V r will strike the surface d A in unit time. Because all of them are having the same velocity so any particle which is here so in 1 second it will reach here in this place and whichever is near so it will strike. So, in 1 second total all of them all the particle in this tube will go and hit this ok. So, the momentum transfer in 1 second will give you the force ok.

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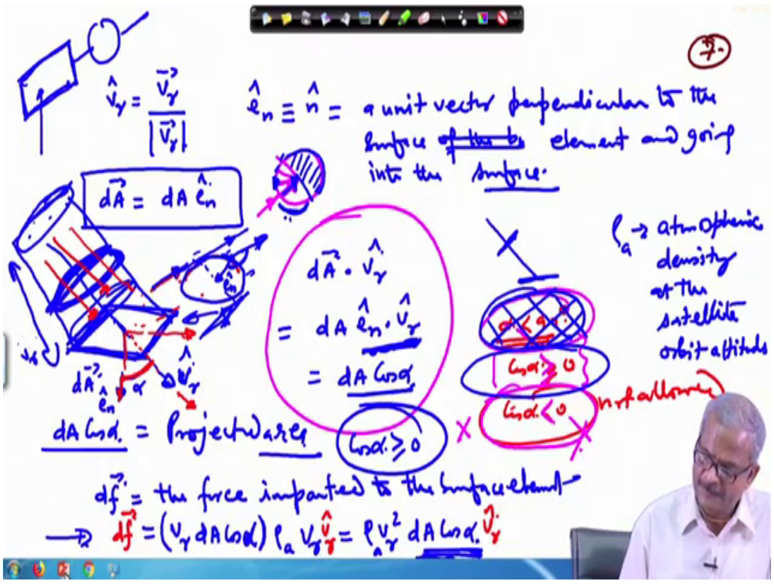

So, we write V r cap equal to V r magnitude. And n cap or e n cap this is a unit vector perpendicular to the surface of the or the surface element perpendicular to the surface element, and going into the surface. So, the surface element d A we can write as d A times e n cap. So, if you have the surface element here and beam is coming like this in an inclined way ok and perpendicular to this is going down this is the angle alpha. So, beam is contained in a tube of cross section which is given like this, so that will be contained in a tube of cross section.

If this total area is d A ok. So, d A cos alpha is the projected area, projected area ok. So, in this direction your d A vector is there ok. So, in this direction is component how much this will be along the V r direction. So, along the V r direction you will have d A dot V r cap and d A is nothing but your d A times e n cap and then V r cap. So, this two are the unit vectors. So, V r cap is along this direction, v r cap and e n cap is along this direction ok. So, what will be this dot product this is nothing but cos alpha. So, this equal to d A cos alpha and d A cos alpha is your projected area means this area, this cross section not this one. So, this is the actual area and this is perpendicular to the beam ok. So, beam is coming like this. So, perpendicular to this while this area is like this.

So, what will be the impact force applied to the? This area can be calculated as d f equal to the force imparted to the surface element and this will be given by all the particles contained in a tube of length V r. So, this tube is of length V r. So, V r times the cross section area projected cross section area d A cos alpha ok. So, this becomes the volume and in this volume from this place to this place ok. All the particles contained in this and this will be a very small length ok. So, all the particles contained in this it will be hitting the surface

So, let us say rho a is the density of the particle per unit volume. So, rho a times how many what is the number of particles? So, mass of particle per unit volume so, this is we are taking as rho a ok. So; that means, this is the rho a is the atmospheric density there. Atmospheric density at the satellite orbit altitude so, you multiply by this. So, this becomes mass ok. And then you multiply it by V r. So, that becomes momentum ok. And this total momentum will be imparted to this surface, this surface in 1 second. So, therefore, this is the force. So, this gives you V r square rho a times V r square times d A cos alpha. And in which direction it is a taking place? That also we have to consider.

So, this force will be applied along because the all the particles are working acting like a collimated beam. So, they will be coming and hit in this direction. So, we also have to multiply here by V r cap. So, this will be in the v r cap directions. this is your d f the force imported to this surface and if you integrate it over the whole area then you get the total force applied to the surface which constitutes the drag ok. another important part is what we can see from this figure that, this angle alpha should be less than 90 degree ok or the cos alpha here, cos alpha should be positive. It should be greater than 0.

Up to alpha equal to 0 this is you can put it as alpha equal to greater than equal to 0. If it is becoming negative. So, cos alpha less than 0 this is not allowed. Why? As you can see here in this figure if this alpha this is the angle between this line and this line. So, if as the angle increases, that means it will become tangential to the surface ones alpha this is 90 degree and thereafter it will become here in this direction. That means, beam is not hitting the surface at all ok. If beam is not hitting the surface some beam is coming from this direction. Let us assume that beam is going here in this direction and this is your surface ok. So, here your surface element is this ok. And this is your e n vector ok. So, you can see that the angle between this vector if you put here this vector here in this place parallelly ok. So, this angle is obtuse angle, or it is a greater than basically 90 degree ok.

So, until unless it is a less than 90 degree it is not going to hit. So, if you know the surface equation the how the surface is oriented with respect to the beam ok. So, you can calculate the total force acting on the system. So, here in this case the beam is coming from this direction. So, this is the beam here this is the beam. So, the projected area will be only form in this so, only on this area this force will act and this area it will not act ok. So, we do not have to calculate for the whole area. Similarly in the case of a solar panel if we have a solar panel here the satellite is there and beam is coming from this side ok. So, we have to take this area, not the area on the back this is the front area.

So, this part must be kept in mind that you do not have to take the case of cos alpha which is less than 0 ok. So, this case is not considered. Only the case we are that cos alpha, alpha is less than equal to 90 degree or nearly 90 degree so, this will be considered. So, you can consider this to be mathematically, apply it mathematically make it alpha equal to less than equal to 90 degree. So, that cos alpha is greater than equal to 0. So, this part is, now let us consider this part here say you measure theta from alpha from this place. So, beam is coming here, surface element is also it is a going inside.

This is the normal to this surface the alpha is 0 here. As you take this surface so alpha increases as you take this take this increases. So, it will keep on going keep on going and then in this domain alpha will be negative once you come to this domain again this alpha will be cos alpha will be positive. So, you have to take this domain also. So, for that reason we will also eliminate this part. So, we will not use this, what will be use only this condition will use. So, cos alpha should be, so what we conclude that cos alpha should be greater than equal to 0 this is the condition to be used.

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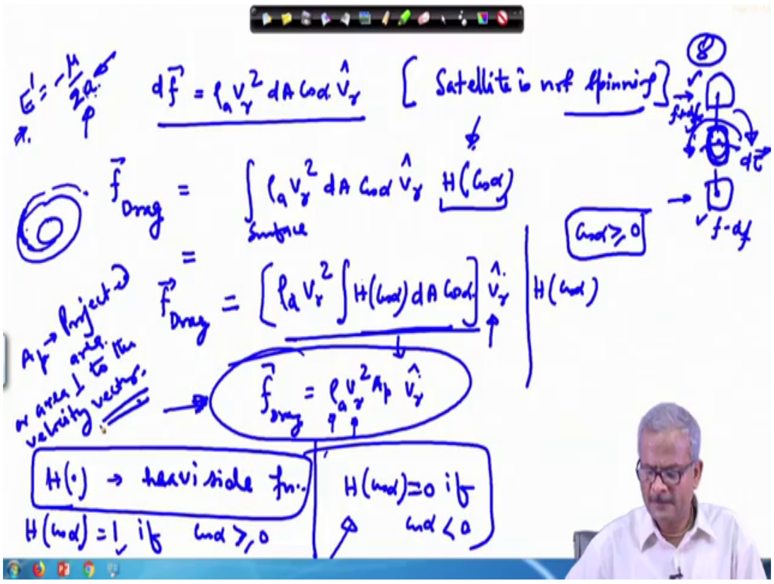

So, integration of this then for this equation we have written here. Ok over the whole surface this will give you the force acting on the satellite body. So, d f we have written as rho a times V r square d A cos alpha times V r cap. And therefore, f Drag and here we have assume that satellite is not spinning. If the satellite is spinning case will be different say this is the main body and here there is there are 2 solar panels and the satellite is rotating on this side. So, if the molecules are coming from this side.

So, you can see that relative angle relative velocity will be higher on this side and it will be lower on this side ok. And because of this there will be a differential torque applied. Here this is the force applied is a plus d f extra force and here the corresponding force will be f minus d f little less than that. So, this together this difference it causes a torque which is in opposite direction. So, this will be the differential torque ok. And this will cause the body to decelerate and spin if it is spinning like this so, because of this impact of the atmospheric molecules it is a angular speed will slowly decrease.

So, that part will come later on the lecture. So, f drag this becomes our rho a V r square d A cos alpha and V r cap and to take care of the where this cos alpha should be greater than equal to 0. So, for this we use this H function which called the Heaviside function. So, H cos alpha where H dot we write is as the Heaviside function h e a v i Heaviside function. If cos alpha is positive this will be 1; so, this quantity H cos alpha this will be 1 if cos alpha is greater than equal to 0 and H cos alpha this will be equal to 0, if cos alpha is less than 0. So, this characteristic can be included here in this place. But not characteristic this is the information the how the surface is distributed ok.

Now, whatever is not dependent on the surface distribution. So, this integration over the surface, so those things can be taken out side. So, we can write this is rho a V r square it is not depending on that surface ok. So, this becomes d A or we can also write first this part H cos alpha times d A remember this is either 1 or 0 ok. So, d A cos alpha and this times V r cap. So, this is f Drag. And drag takes place in the direction of V r cap. So, this is nothing but your projected area so this quantity totally turns out as projected area A p. So, this is rho a V r square times A p times V r cap.

So, this is the atmospheric drag acting on the satellite and this will tend to decelerate the satellite ok. And, it is square of the velocity remember this is very high. This is very low, but this is very high then ok. So, all the fragments of this Shakti mission where the satellite was shot down by India.

So, all those fragments because of this drag will slowly enter into the atmosphere it will decelerate those fragments, the speed will reduce and as this speed reduces it is a losing its energy and therefore as I we have written in the last time the energy per unit mass is minus mu by 2 a. So, it is the semi measure axis will as E prime becomes smaller and smaller ok, more and more negative. So, a is increasing a is decreasing and decreasing this is a semi measure axis.

So, a is decreasing and decreasing and therefore, this will become more and more and more negative and there will be a time once the orbit then enters into the atmosphere ok. So, it enters the atmosphere in the atmosphere this is the atmosphere level and then this is here it will a start burning once it gets into the dense atmosphere, it will start getting heated then it will as the temperature rises it will burn out or some of the fragments if it is not totally burn out it will fall on the earth surface ok.

So, we will stop here in this place and continue in the next lecture ok. So, we will this A p we can write here this is the projected area or area perpendicular to the velocity vector area perpendicular to the velocity vector ok. So, next time we are going to compute the torque acting because of this drag and there after we have to consider that case of the solar radiation pressure and then we will wind up. So, we will come to the end of this course.

So, thank you very much for listening for today.