Satellite Attitude Dynamics and Control
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Lecture - 68
Satellite Attitude Control using Lorentz Force

Welcome to the lecture number 68. So, we have been discussing about the magnetic satellite attitude control using magnetic actuator. So, satellite attitude control can also be done using the Lorentz force. So, this technology is still has not been developed and we have been working in this area. So, I will introduce you to that also and proof of the stability another things, it is remains the same as it is in the case of the magnetic attitude control.

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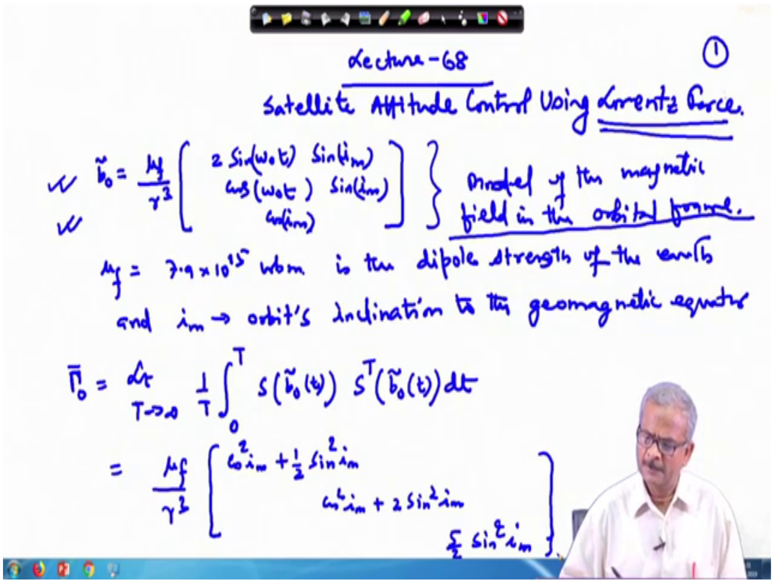

So, let us say start with this and for the magnetic attitude control, we will satellite, ok. So, magnetic field in the orbit, it can be described as it is a orbit inclination, ok. So, you can use this model in the this is the model of the magnetic field, in the orbital frame in our work, we have used data from the ISRO.

So, if and the quantity gamma bar, it will be given by limit. T tends to infinity, this you can take 0 to T or from certain interval to infinity also it can go, but one this remember that whatever we are doing here. Writing this is equally applicable to the satellite attitude control using Lorentz force and also to the satellite attitude control using magnetic torques.

So, in both the cases this particular part, it will remain the same. And this quantity will be given by cos S square i m plus 1 by 2 and 5 by 2 sin S square i m. So, this data can be utilized whenever you are trying to model. So, you can utilize this data.

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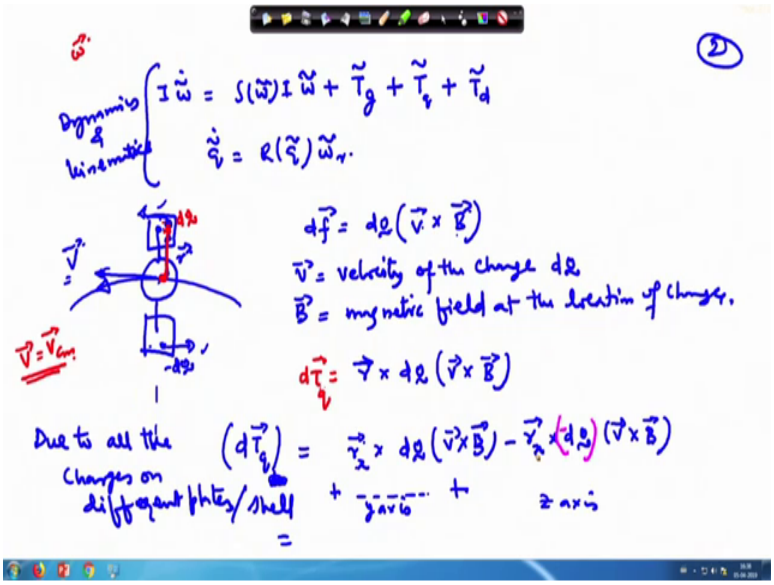

Sorry, this formula you can utilize. Instead of data you can use the formula you can generate the field in the orbit and use it for the control purpose. Now so, what I was telling that using the Lorentz force satellite or attitude control, it follows exactly the same principle. Only difference will be lie in the torque generated, ok. So, however, the equation of motion remains the same. Now here the gravity gradient remains same. There is no difference in this then this is the instead of magnetic moment. Now we will have the Lorentz force.

So, this will write as T q, this is due to indicate this is charge due to the charge. And then of course, we will have the T disturbance. So, this is the equation of motion and then we have q tilde equal to R q tilde time’s omega tilde r.

So, this is what we have written last time. So, this constitutes your dynamics and kinematics ok. Now consider that some satellite is going in the orbit and here this is the centre of the earth. And it is going in the orbit on this side V is the velocity. So, any charge which is placed in a magnetic field and moving with V, it experiences a force d f equal to change in the charge is d q.

So, d q times V cross B where V is the velocity of the charge. And B is the magnetic field at the location of charge, ok. So, you can assume that there is a charge here in this place and it is lying at a distance r. This is the vector from the centre of the mass of the satellite to this place we can show it by some other colour. So, centre of mass to this place this is your d q charge is located, and it is a moving with velocity V.

So, we will take the same velocity as that of the centre of mass. So, V equal to V centre of mass. Though the satellite is rotating with angular velocity omega a small correction can be added, but that does not matter much.

So, for simplicity we can take it this way. So, the torque acting on the charge, this will be given by or we can write it as d T due to q; d T q will be then r cross d q time V cross B. So, if we consider we have another charge here in this place with opposite sign.

So, your force on the charge will be given by the rule V cross B; whatever the direction of V is there and whatever the direction of B is there. So, it will be perpendicular to both of them. So, if we have a negative charge here in this place, say minus d q here in this place. So, just a opposite force will be experienced in a in this place. And that will create this two together will create this one. Suppose this is acting here and this is acting here. So, these two will form a couple.

So, this way we can have along the all three axis of the satellite a shell, or a plate on which the charge is distributed. So, if we take that then the total let us write it this way d T q along all the three axis. So, I will not tag it, I will keep it the same way. So, I will write here in this way. Due to all the charges on different plate slash shell, ok.

So, one is plus another is minus. So, they are in opposite direction. So, for them we can assume that the say one is located in r x direction. So, this will be r x cross d q V cross B, ok. And another one is V located here in the minus r x direction. So, minus r x cross d q V cross B and we have to put on minus sign here in this place to indicate this is the negative charge.

So, therefore, this becomes gets a plus sign the same way we can write for y axis and z axis, ok.

And if we add all of them so, this appears as distance between this axis along the x axis d between this T q charge, distance between the y axis, distance along the y axis between the charges and similarly along the z axis.

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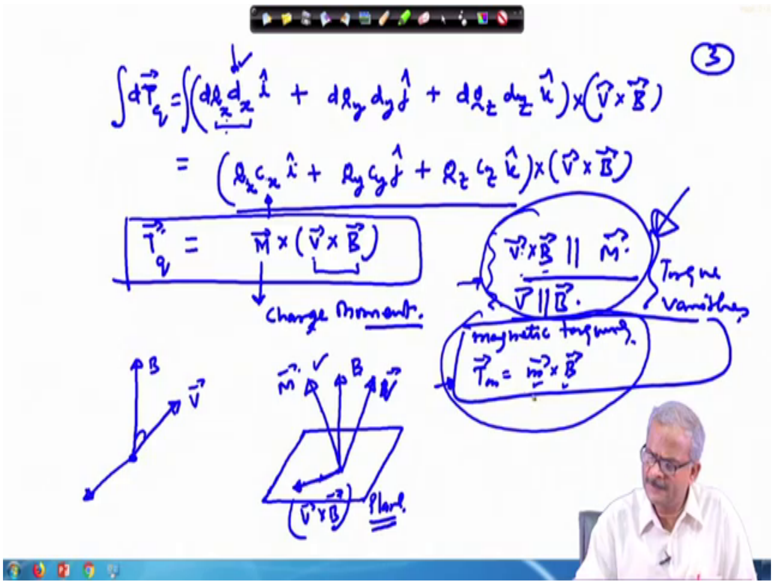

So, this d T q then we can write as let us say that the total distance is from what this is d along the x axis times i cap, ok. And d q which is multiplied there, we will bring this d q here in this place and this is along the x axis, ok. So, d q x times distance along the x axis this two can be added together.

So, r x; r x this 2 r x magnitude wise, we can write it as d x this is the distance between these two points and i cap this is the unit vector along the x axis. So, same way we will have and then V cross B, ok. So, V cross B, we can take it outside and here then d q y d y times j cap and d q z d z times k cap. And we take V cross B outside, because it is a common to all of them, integrate it.

If we integrate it over the whole body and so, this can be replaced as q x times distance along the x axis, which is the here in this case; you can consider it to be the centre of the charge times i cap plus d q y sorry, q y. So, we should put another notation here for this may be we can use c q y times c y j cap plus q z times c z k cap V cross B, here this is R cross is also there.

So, we have cross sign will appear in this place. So, this quantity from here to here, we call this as the moment of the charge and we write this as the M, capital M. So, moment of the charge cross V cross B and this becomes T due to the charge, or simply we write this as the charge moment. So, now, it can be easily observed that if this is the B vector and say in this direction we have V vector, so V cross B.

So, it will live perpendicular to both of them, ok and till the vector M. So, we will make it plane rather. Let us say this is the d and V is here in this direction so, V cross B, it will lie here in this plane, this is V cross B. And if M is another vector here so, M cross V cross B so, this will be perpendicular to both of them. So, somewhere as you can see that this is lying out of the plane M is lying out of this plane; this is.

So, perpendicular to this one V cross B and perpendicular to M also. So, we have to look for that particular direction where it is going to lie. So, accordingly it can be located. Now another thing we can see that whenever V cross B, this is parallel to M the torque vanishes or either whenever V is parallel to B then also the torque vanishes. However, in the case of the magnetic torquer, there the torque was given by m cross B.

So, it was vanishing only when m is parallel to B, but here there are two conditions. One the velocity vector becomes parallel to B or the magnetic moment of the charge or the charge moment, it becomes parallel to V cross B then also it will vanish. So, this two cases make it more constraint, ok. Here in this case this becomes more constraint as compared to this one, as compared to the magnetic one.

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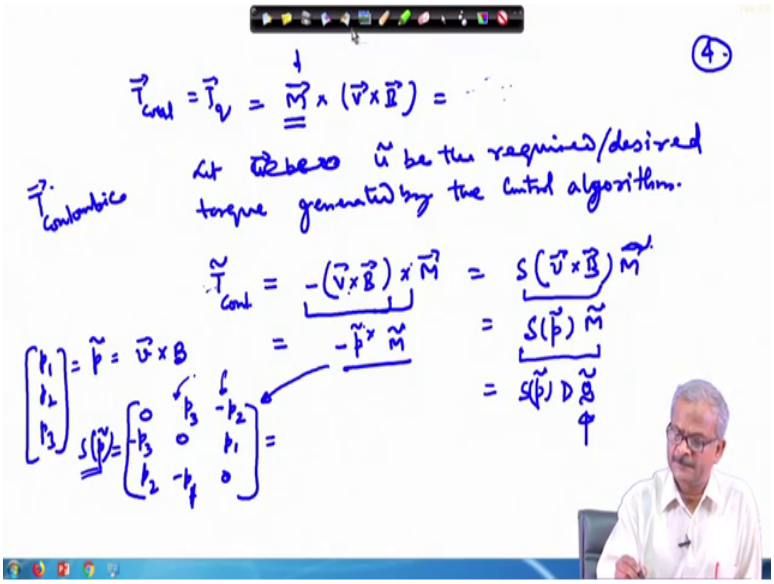

So, we have the torque then we can write as so, T q or this is the columbic or T q we can write it this way, this is T columbic. So, these we can write as already we are written M cross V cross B. Now let u be the or u tilde, you can write in terms of u tilde also, u tilde be the required slash desired torque generated by the control algorithm. So, from there how much M is required we can find it out.

So, we have let us write this quantity M cross V cross B we can designate this another vector. Let us say simple, you can assume we are own vector notations. I have used a notation whatever notation I have used I will tell you about that, but before that let me simplify and put this in the matrix. So, that we can do the matrix should inverse.

So, T tilde this due to the coulomb charges or the due to the electric charges, this will become minus V cross B cross M, ok. And this quantity we can write as S V cross B here from this place to this place, it can be already we have used it, this is the q symmetric matrix.

So, if V cross B, we put as in the note let us make this as some right now a vector. Suppose this I write as S p tilde and this is M tilde. So, p tilde is here V cross B p 1 p 2 p 3 and S tilde. Then this becomes 0 0 plus p 3 minus p 2 p 1 minus p 1 and then p 2 here minus p 3. So, remember that this is S B which means this equal to here this is p.

So, this simply implies that we are can writing this as minus p tilde cross M tilde, ok. And this stands for this one p till p tilde cross will be, we have to change the signs. So, here the minus sign will appear this will become plus and so on.

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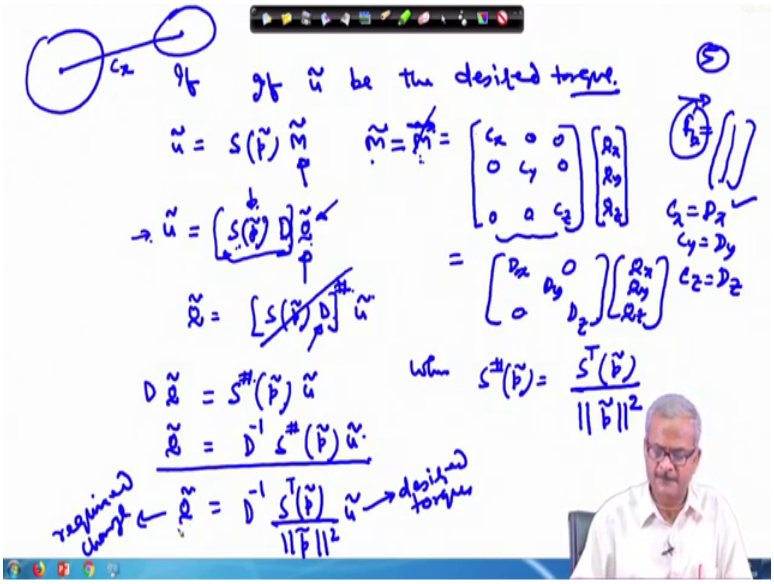

So, we can write u tilde equal to S p tilde time's M tilde, ok. And we need to decide this in order to find out how much charge on the various shells or the plates extra required.

So, M tilde is your M in the vector notation, we have written as q x, q x time c x and so on, ok. So, this we can write as in the matrix notation the same thing will become q x q y q z. And here you will have c x c y c z which indicates the distances between the various shells. I am not showing the joystick when analysis, but if we have a shell it is a and another cell in this place. So, this is centre to centre distance this is c x similarly c y.

So, M tilde can be written in this way and if you, I am not doing the formal change also. From the vector notation I am just converting it to the matrix notation, you can use the notation we have developed earlier which is the indicating f b this is a metrics and taking the basis vector here and then converting. So, unnecessary time gets wasted in that. So, you can just follow this approach.

So, this becomes S p tilde this is a q symmetric matrix and you have here let us write this as it will better to write this in the form of d x d y and d z, ok. On the left hand side this is the earth diagonal terms are 0, this is the way of writing. So, this is q x q y and q z.

So, capital D, I am using for distance instead of c x. So, c x equal to p x c y equal to I am replacing it and c z equal to D z. So, that you can indicates the distance. So, this becomes this is a diagonal matrix D and this is a vector which is you can write as q tilde. So, S p D T tilde this equal to u tilde. So, this is the charge you need to decide these are the things which are fixed, it depends on the V cross B. So, this things are not in your hand.

So, this part you can manipulate means you can keep changing with time. And therefore, this torque you can generate. So, this gives you the q tilde equal to S p tilde times D pseudo inverse times u tilde out of this d is invertible. So, we could have also written it another way like instead of doing like this, you can also do the same thing. Let us first remove this part. So, this becomes S pseudo inverse p tilde times u tilde, this will become equal to D q tilde, ok. This is taken to the left hand side.

So, I am writing it a this way then take D on the this side. So, this becomes T inverse S pseudo inverse p tilde times u tilde where S pseudo inverse p tilde already in the magnetic torque case we have written this. So, it follows the same formula, this is called the Moore Penrose inverse S transpose p tilde divided by p tilde magnitude square. So, this will get reduced to this is equal to q tilde. So, D if this is the desired torque you require this much of charge this is the charge vector.

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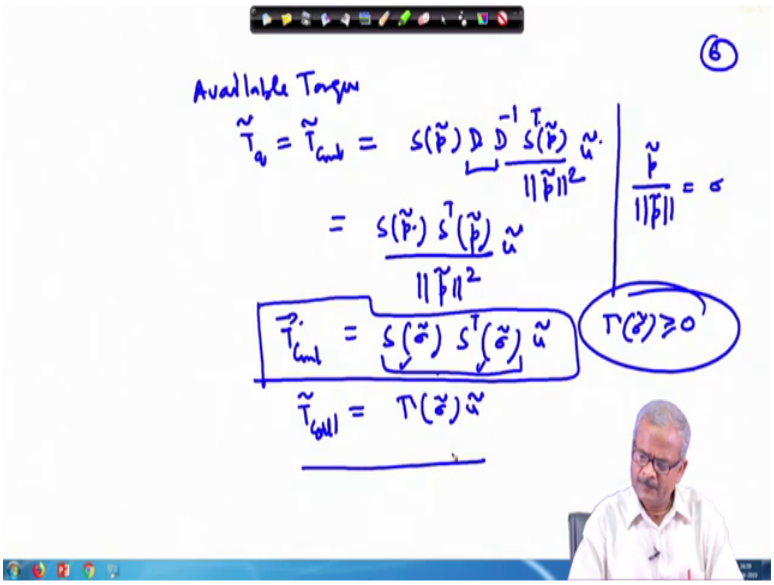

So, the available torque then T q tilde or T coulomb this can be written as we have use the earlier equation. This part we have broken up and this we have written as s p tilde and m we have written as D times q tilde. So, we need to replace here q tilde.

So, T coulomb then becomes S p D s p tilde times D and then replace this q. So, q we have calculated here in this D inverse is V transpose. So, this is D inverse S p tilde transpose divided by p tilde magnitude S square and times u tilde. So, these together constitute an identity matrix. Now let us write p tilde divided by p tilde magnitude as sigma.

So, then you will be able to write this as these are the unit vectors S sigma times S transpose sigma tilde times u tilde. You can, see that that torque due to the charges it has got reduced into the same format. As we have got for the where in the case of the magnetic torquer, this is exactly the same thing same part. So, this you can write as gamma sigma tilde times u tilde where gamma sigma tilde, this is skew symmetric matrix the same and this is the product of those skew symmetric matrix it remains similar.

So, this is the property of this matrix and rest of the treatment. It remains the same as we have done for the magnetic torque there is nothing no difference in this and that, ok. Only thing that there are two cases in which the torque is vanishing. In the case of the Lorentz force one this are the two cases and while for the magnetic torquer there is only one case, one case means M becomes parallel to B here V becomes parallel to B that also creates trouble that the torque vanishes and V cross B also if it becomes parallel to M then also the torque will vanish.

So, this is more problematic as compared to this one, ok. So, and the proof for stability controllability is already I have discussed and the same controllability issue also applies here in this case mathematically and stability issues I have not discussed, I have not detailed here in the class. So, I will upload material for that next week and there I will that is the those issues which are related to the stability. And the formula that will use, you can extend it to other places wherever the gravity gradient is used, ok.

So, we will wind up this topic in this place and then will start with. So, we end here, the law satellite attitude control using magnetic torque and then we will continue with the satellite attitude control using the thruster in the next lecture.