Satellite Attitude Dynamics and Control
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Lecture - 67
Satellite Attitude Control using Magnetic Torquer (Contd.)

Welcome to the lecture number 67. So today we are going to discuss about the stability of the magnetically actuated satellite system.

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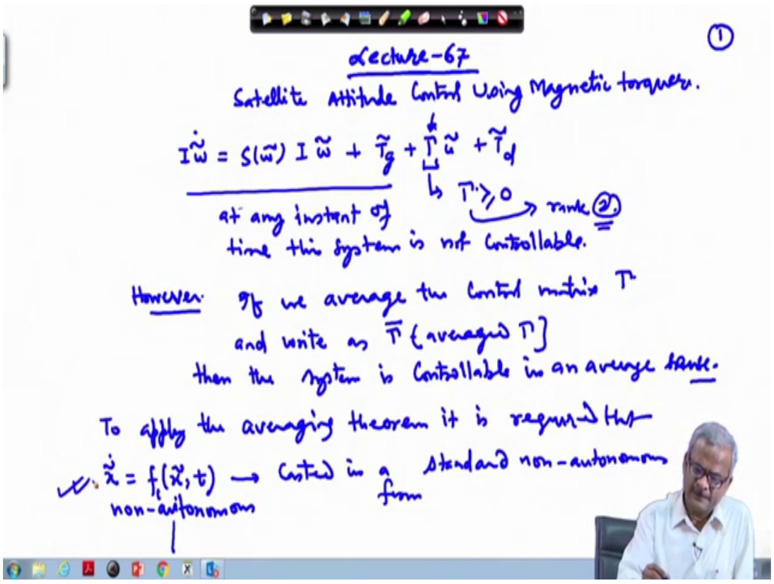

So, in that context what we require that we need to apply, as I have told you that in the system dynamics this matrix is similar that is this is of rank 2. And therefore, as a at any instant of time, this system is not controllable this is what we have observed in the last class.

However, if we average the control matrix gamma and write as gamma bar is the average gamma, then the system is controllable in an average sense. So, to apply the averaging theorem it is required that system dynamics which can be represented as x tilde equal to f x tilde times t; where why we are writing this because your this part is time dependent ok, therefore we are writing here as this. It is required that this be casted in a standard this is known as non-auto system in a standard non-autonomous form.

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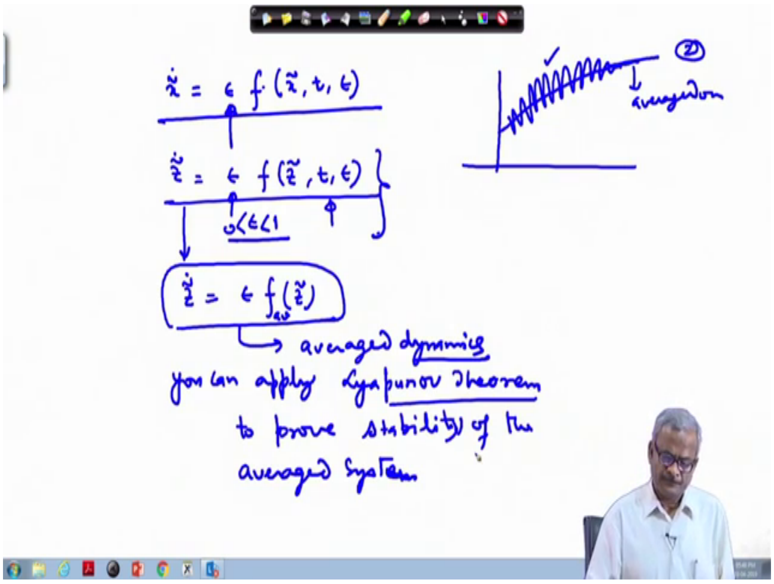

So, this standard non-autonomous form it looks like x tilde equal to f x tilde t epsilon. So, why this epsilon is appearing this equation can be changed, here let us write this as f 1, so that we use f here. So, the previous equation this equation can be changed and re casted in terms of other variables. Let us say that instead of writing here we write as z tilde epsilon f z tilde t epsilon. So, we can cast it in a new format where the epsilon which is a quantity lying between 0 and 1 it appears like this.

If it happens, so in that case the dynamics becomes lower than the excitation. So, for the averaging as I have stated you earlier, that if the dynamics is slower as compared to the excitation what you see here that excitation is very fast and because of that the system response is like this.

So, we can replace this trajectory by this averaged one, this is the averaged one this is shown like this ok. And there after once we have casted here this in the non-autonomous; a standard non autonomous form so, this can be sent changed to a standard autonomous form which will appear like this ok. And this will be your you can write as f averaged instead of f here this will be a f averaged and this will be in terms of z tilde ok. So, this is you are giving the average dynamics and then you can apply the Lyapunav theorem to this to prove stability of the averaged system.

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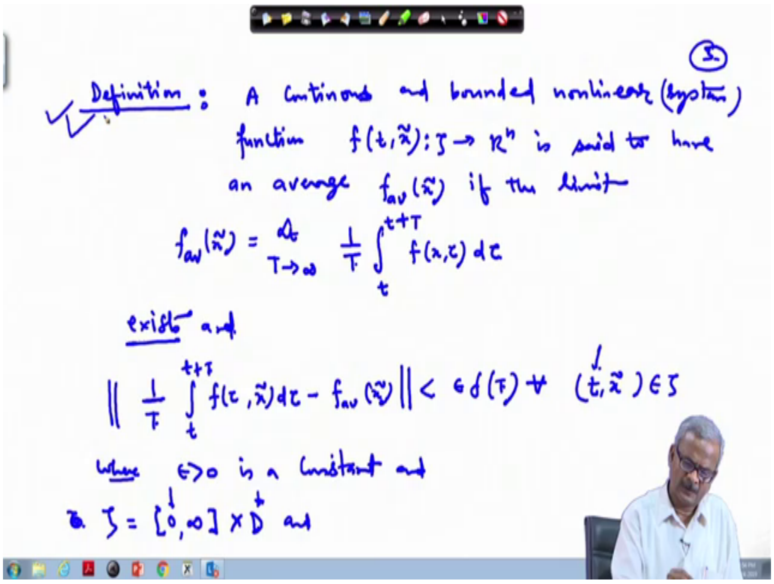

So, definition of the averaging theorem it states like continuous; how do we define the average system non-linear system or the non-linear function this exist. So, this is the averaging of this is the way of averaging any function. So, this is averaging in time domain it can be averaging in a space or whatever it depends on the type of function. This is stands for the t here this particular interval 0 to infinity and this D is the domain for x tilde.

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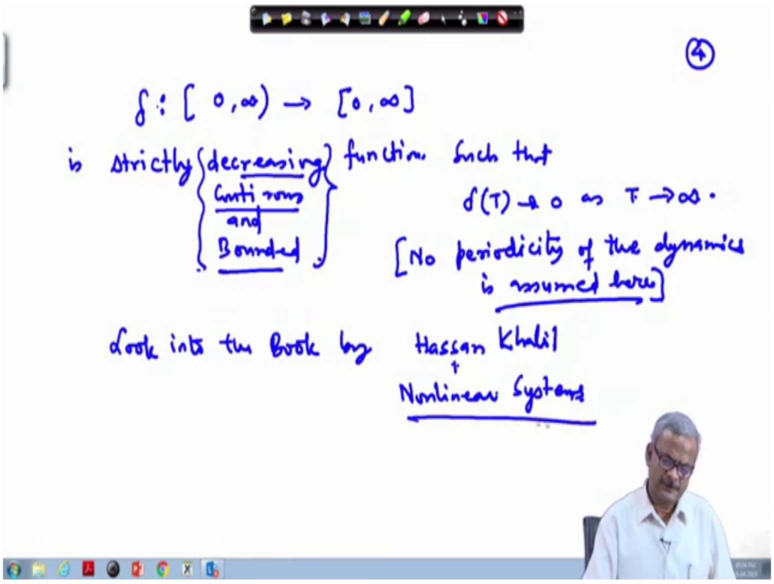

And which map delta map from 0 to infinity and is a strictly decreasing continuous and bounded function ok. So, these are the three qualities strictly decreasing, continuous and bounded. Such that delta T tends to 0 as T tends to infinity. No periodicity of the dynamics is assumed here ok, so this is the basic definition. Now, consider the proposition you can look into the book Hassan Khalil double s or single s is there I do not remember, but this is Hassan Khalil by on non-linear systems ok.

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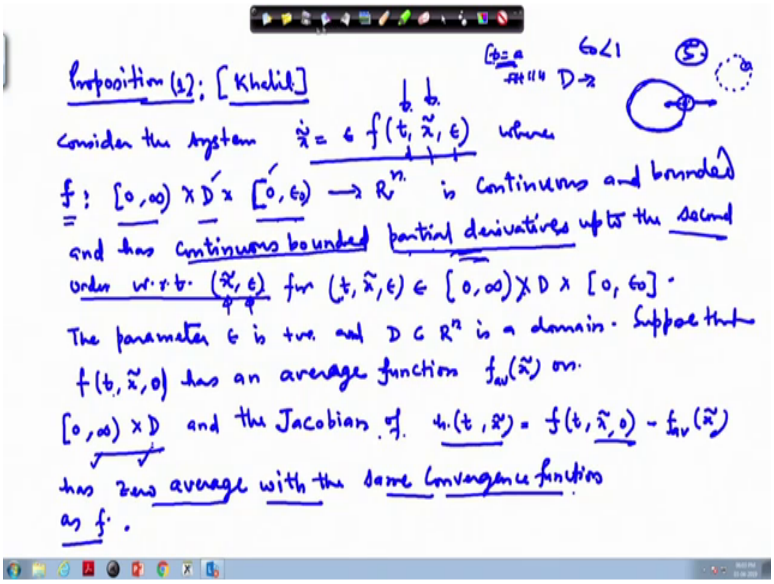

In this book you will find this definition, more over this proposition which I am stating here this is also given in Khalil. So, these are the basic things required to prove the system stability. So, proof is given there you may not be aware of all these things it is quite possible that you are not going through the Lyapunav stability analysis for the non-linear system, you might have done it for the linear system, but still has not effort I am putting it here. So, that this topic gets completed and I will put this proof as the supplementary material.

Now, consider the system where f is a mapping from 0 to infinity this is about t and then the domain D for x tilde, the epsilon 0 this is less than 1 and it maps into R n. So, the function f it maps from these are for the three items which is appearing here three variables, this is basically vector x tilde what t as a scalar epsilon is also a scalar. So, the corresponding three domains are here.

So, from there it is a mapped to R n, so n dimensional space, n dimensional real space. So, f is continuous and bounded up to the second order with respect to x tilde and epsilon. So, what we will do? We will change it to this one, if we are taking this f to have continuous bounded partial derivatives up to second order with respect to x and a tilde.

So, your domain must not be closed, if it is a closed domain, so that means, if I have a point here. So I cannot have neighbourhood of this on this side ok, here the neighbourhood is there in this domain, but this side we cannot have neighbourhood. But if the domain is open which is always shown like this, so you can take any point as close as to the boundary, but there will be always a neighbourhood of that. So, this is important in this respect ok.

So, the epsilon lies in this range and therefore, your system will be then differentiable with respect to or the will have partial derivative with respect to x tilde and the epsilon, so both this and this we make it open. Another proper way of defining this is that the systems the domain should be open, but in the case of the t it so happens that your start your t equal to t equal to 0.

So, always there is a starting time, so you will have the left hand side has the closed one. So, there after in the time at any instant of time you are looking into the differenceability of the system. So, therefore I initially inserted this large bracket here. So, otherwise if you look from this point of view, so both of them should be the open set and it is a call the open connected set.

So, those definition you should not go in to just look in to this part and then because some of the things made it beyond your reach right now. Suppose that f t x tilde 0 has an average function, f average x tilde on 0 to infinity and on D this is for t and this is for x tilde and the Jacobian. So, each is been defined like this has the 0 average with respect to Jacobian of this has 0 average with respect to the same with the same convergence has function, same convergence function as f .

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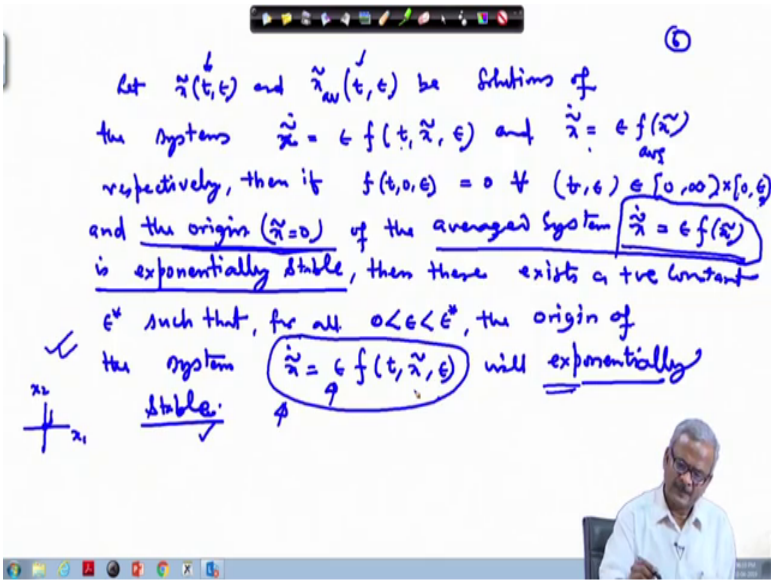

So, solution will be; obviously, contained this t, so therefore, t is appearing and this is for the averaged one. So, once you have average it, so this t will get eliminated and it will appear in this format. So, if at the origin this value is 0, for all t epsilon element of 0 to infinity times 0 to epsilon, 0 to epsilon 0 and the origin x tilde equal to 0 of the average system.

So, if the of the average system this origin is exponentially stable, then adjust a positive constant epsilon star such that for all epsilon lying between 0 and epsilon star, the origin of the system epsilon will be exponentially stable ok. So, if your average system what is telling that this theorem that if your average system is this one is exponentially stable then your axial system, this is a non autonomous system. So, what casted in this special format a standard format, then this will be also exponentially stable means it's a origin will be exponentially stable.

So, that implies if this is the origin and say this is the say this is x 1, x 2. So, in the neighbourhood of this the system will decay exponentially to the origin this is what it implies ok, but in the neighbourhood it does not talk about it is globally exponentially stable. So, these are the issues which we deal in the nonlinear system controls it. So, there is no time to discuss the details of all these issues ok.

So, even if you are not aware of the nonlinear control system or it's how to do the Lyapunav stability analysis, but if you take it for granted that this is the theorem and it goes like this ok. Then you will be able to understand some of the things which I am going to upload you upload on the internet.

So, here we have once we have got this, that this is once we average the system, so this is at least locally exponentially stable and then the theorem I will be providing you that says that the system will be globally stable. Now, already we have proved that the system is on an average, it is a controllable irrespective of whatever be the angular velocity of the satellite. Arbitrary high it can be anything less than infinity, so theoretically it remains controllable ok.

And therefore, on an average the system will also be it can be stablized, so this using this theorem ok. So, this is locally exponentially stable and also the system will be globally stable which will follow in the proof that I will upload, but not globally exponentially stable it is a only globally stable. So, anywhere you take the system, so it will remain as stable.

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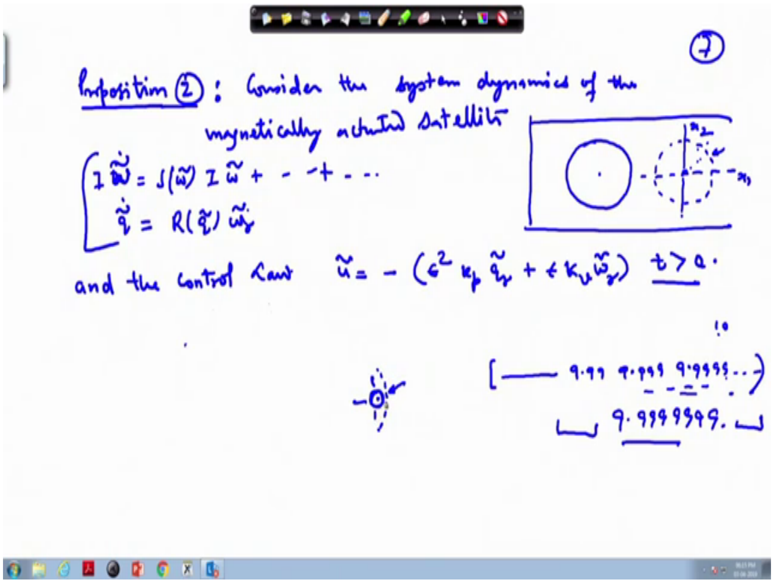

So, another proposition is the system dynamics we are describing in terms of I times omega tilde dot equal to s omega tilde I omega tilde and so on and the kinematics we have describes as q tilde equal to R q tilde times omega r tilde. So, this your system is given system dynamics of the magnetically actuated satellite and the control law minus epsilon square k p times q r tilde plus epsilon times k v times omega tilde r where t is greater than 0 ok. So, we were talking about the let me state again we were talking about the open domain, so this is your open domain.

So, in the x also x varies from interfaces space let us say this is the two dimensional phase space in x 1 and x 2. So, on the outer side this is you can see that here already you have the 0 ok and then the outer side on the boundary if you go on this side. So, let me state like this because this is an important point I have a point here, so I will have a neighbourhood of this; this is never closed like if I take some interval on the right hand side let us say this goes as 9.99, 9.999, 9.9999 and so on.

So, this is never closed on the right hand side though the upper bound is there the upper bound will be 10, but that 10 is not part of the system this domain ok. So, you will always have wherever you go so if you take 99999999 something like this. So, still on the right hand side you will find points and also on the left hand side there are points. So, in the 2 d similarly wherever; however, close you go to this particular dotted line.

So, you will find a neighbourhood of this on the left hand side there will be there on this side and this side also. So, therefore, you can ensure that the system is differentiable towards near the boundary. Then therefore, this is kept open, this is the significance otherwise your system will not be differentiable on the near the boundary.

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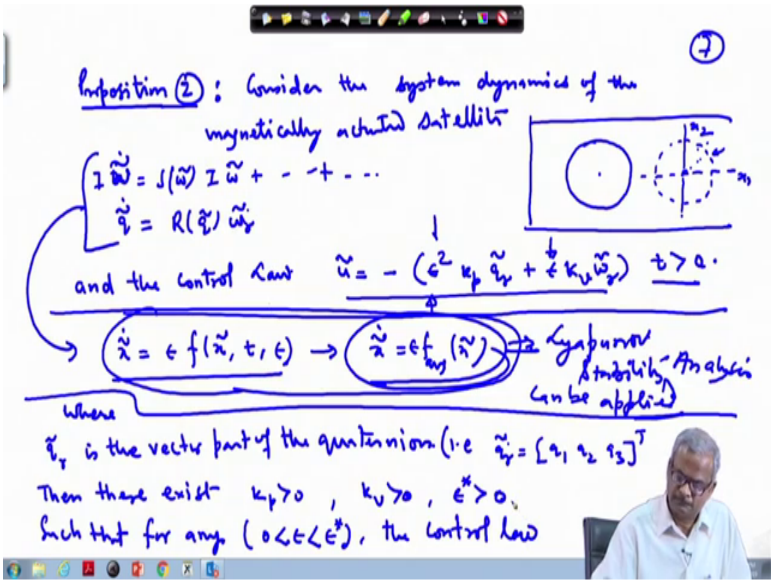

So, if we are given this system and the control this is the proportional differential control law only thing this one the extra term epsilon square and epsilon there appearing. So, using this law you can recast this system in this format x tilde equal to a epsilon and then this can be averaged to get a form epsilon f average x tilde and which can to which the Lyapunav a stability analysis can be applied ok. So, given this your t is greater than 0 where, so this you can remove this is not part of the statement of the theorem.

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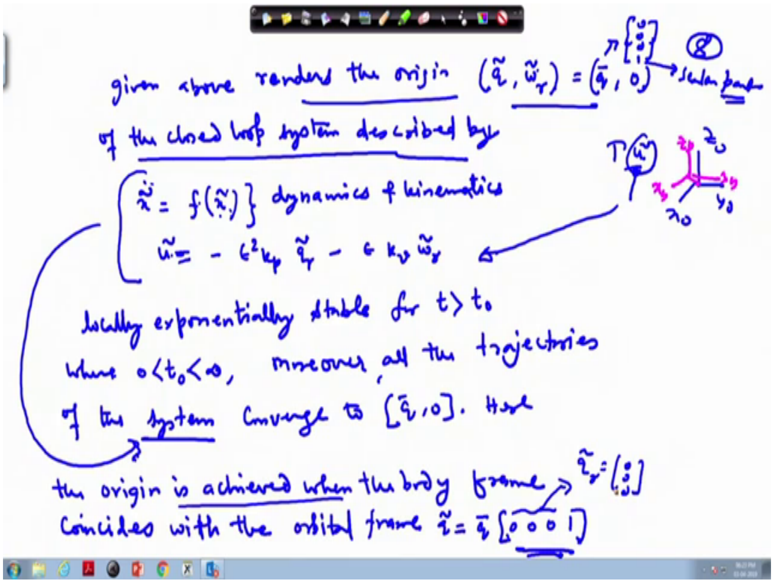

So, where q tilde r is the vector part of the quaternion renders the origin. So, this is basically your 0 0 0 the equilibrium condition of this origin. So, even it renders the origin of the close loop system described by x tilde dot equal to f x tilde and so here in this case you have average it out. So, averaging is appearing in the form of use a function of x, the state variables and therefore, this is appearing in this format ok

So, this includes your dynamics and kinematics and u tilde this equal to minus epsilon square k p times q bar tilde k v times omega tilde r. So, the closed loop system is described by this u goes into the you remember that we have the term gamma u. So, this gamma tilde u, u you are replacing by this. So, then it becomes a closed loop system.

So, that closed loop system is locally, exponentially stable for t greater than t 0, moreover all the trajectories of this system converged to q bar, 0. When the origin is achieved when the body frame concert with the orbital frame means they become parallel x 0 y 0 and z 0. And they are both at the same location I am just showing it differently and this is x b y b and z b it becomes like this. With the orbital frame that is q tilde this becomes equal to q bar 0 0 0 so, here q bar you can add 1 also, so this is the scalar part of the quaternion. That means, this simply implies that q tilde r this becomes 0 0 0 or equivalently q tilde r this will be 0 0 0 transpose. So, this is the origin of the system, so system will converge to this.

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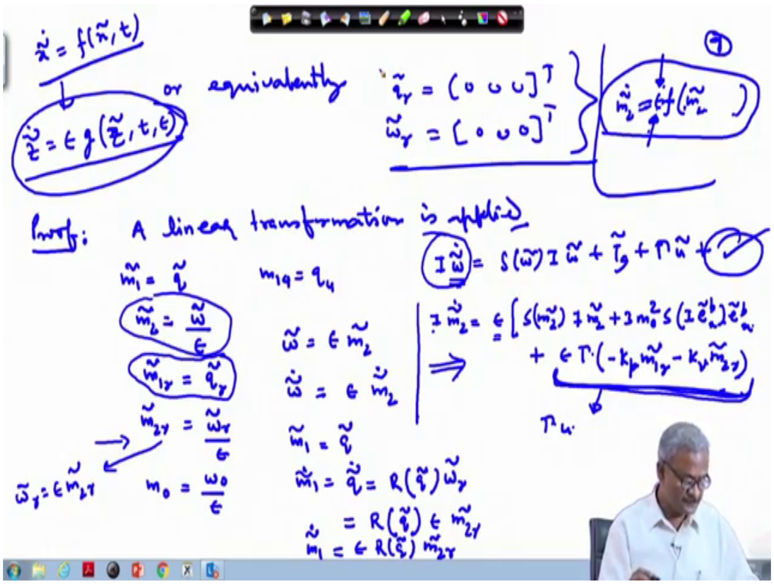

Now, the problem is of reducing your actual system this is governed by which is written like this to a format say which can be shown it like this z tilde, t, epsilon. So, we have to reduce it here into this format ok. And for that you need certain substitution, so I am not going through all the things just I will give you the initial part and then I will upload the material. So, a linear transformation is applied and we define m 1 tilde as q tilde m 2 tilde is equal to omega tilde divided by epsilon, m 1 r tilde equal to q r tilde business scalar part of the quaternion.

So, these are some of the transformation relationship we have to use to reduce our system into the required format; that means, you are given I times omega tilde dot S omega tilde I omega tilde plus T gravity gradient plus gamma times u tilde and this disturbance we can ignore here, that will facilitate simplicity of the derivation.

So, now, here what is required say this is omega tilde. So, omega tilde if you write here omega tilde is epsilon times m 2 tilde, omega tilde dot this will be epsilon times m 2 tilde dot, m 1 tilde like the m 1 is q tilde ok. So, this implies m 1 dot will be q tilde dot and which is nothing but R times q tilde times omega tilde r and R q tilde and omega r is from this place here in this place epsilon times m 2 r tilde. So, this gets reduced to epsilon times R q tilde times m 2 r tilde, so this is your m 1 tilde dot.

So, this way all the terms are to be worked out and if you do that, so your this equation this will get reduced into this format, you can try it yourself this exercise at least this is easy, 3 m 0 square instead of omega x square this become m 0 square and plus your epsilon times gamma k p times r tilde minus k v times m 2 r tilde. So, this particular term earlier times we have written it as gamma times u.

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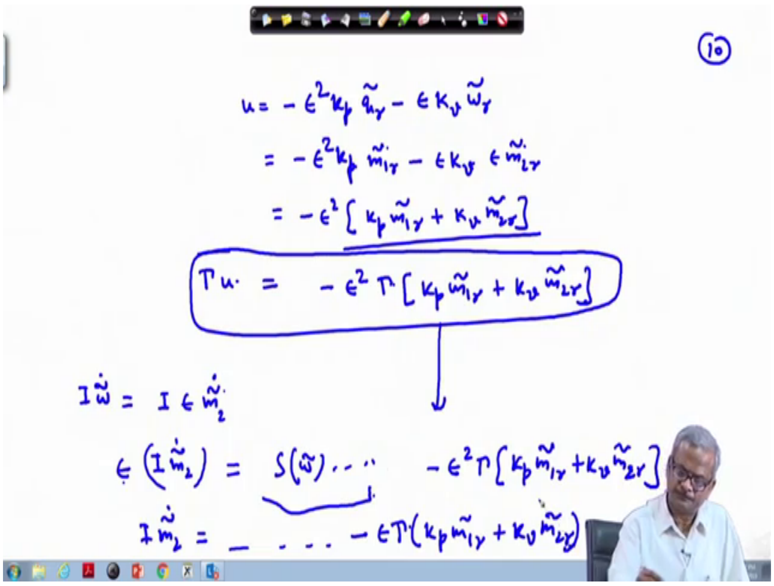

So, u your; u you have defined as epsilon square k p times q r tilde minus epsilon times k v times omega r tilde, q r tilde from this place, this is this part q r tilde equal to m 1 r tilde. So, this is m 1 r tilde minus epsilon k v times omega r tilde is omega r tilde is written here. So, this is omega r tilde this becomes epsilon times m 2 r tilde.

So, this is epsilon times m 2 r tilde ok. So, this becomes minus epsilon square k p m 1 r tilde plus k v m 2 r tilde. And then you are multiplying it with gamma so, this becomes minus epsilon square times gamma and k p m 1 r tilde plus k v times m 2 r tilde. And then your this particular part I times omega tilde this can be written as I times epsilon times here this is a epsilon times m 2 tilde ok. So, this is epsilon time’s m 2 tilde.

So, you can see that m 2 tilde dot, so this becomes and epsilon is taken out side. So, if we this is on the left hand side on the right hand side you have S omega tilde and all those terms. So, these terms need to be converted, but here I am particularly taking this term. So, this term appears as minus other terms I am not considering here for simplicity that will follow up in your the uploaded material m 2 r tilde.

See, if you see here this epsilon; this epsilon it will cancel and you will get this as m 2 tilde dot this equal to some other terms minus epsilon times gamma times k p m 1 r tilde plus k v m 2 r tilde. Now, go back and look here in this place is it the same thing. So, minus sign thing only here I have kept it here inside, here in this place I have brought it outside.

So, it is the; so this equation, so in this format what we see that, if I take this I on the right hand side. So, you can see that this will be of the form m 2 tilde equal to this will be a function of m 2 tilde and other variables which are involved, so and epsilon is here outside ok.

So, this becomes reduced; to this gets reduced to be a form where the averaging can be applied because here in this case this is a term which is less than 1 which is small and therefore, the it makes it cast the system into a form where the dynamics is wearing slowly as compared to the excitation. If we look in to the body frame; however, fast the as; you made the body rotate as fast as possible. So, that dynamics of the body itself getting accelerate it is a becoming very fast say its angular velocity has become very high.

So, does the excitation becomes high because of your magnetic field is in the orbital frame and from there you are converting into the body frame. So, that also changes in that case. So, that components of the magnetic field in the body frame will also change. So, here the excitation becomes also very high. So, if you speed up the body, so the excitation will also speed up ok. So, this averaging will always be of can be applied and therefore, this has got reduced to a standard format and there after it is just a matter of construction of a Lyapunav function ok.

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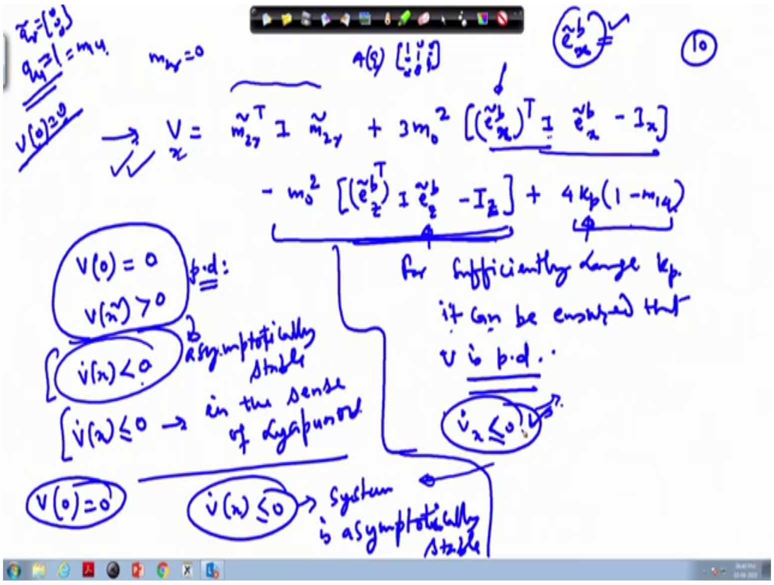

And if we do that then the system stability can be proved and so the Lyapunav theorem Lyapunav function can be chosen like this I am not going to derive this part I am going to upload this material. You see e tilde b x, this is the first column of your attitude matrix and e tilde b z this is the third column of the attitude matrix. So, basically this term is appearing for the gravity gradient consideration m 0 this part we have written.

Forming a Lyapunav function this is the Lyapunav function I have briefly earlier about this. Lyapunav function is a function where v 0 equal to 0 and v x tilde this will be greater than 0. So, it is a positive definite function. However, if we and if v dot x if this is less than 0, then we say that the system is asymptotically stable. If v dot x is less than equal to 0, so this is asymptotically stable and this is stable in the sense of Lyapunav.

So, I am not going into this theorem I am presuming that you know all these things ok, but if we apply LaSalle’s theorem. So, in the LaSalle’s theorem this condition that v 0 equal to 0 this is relaxed. And moreover v dot x less than equal to 0 this can also ensure that the system is asymptotically stable, you can look into Khalil for all these things.

So, here in this case for sufficiently large k p, v will be greater than 0; it can be ensued that v is positive definite. So, sufficiently large k p it can be ensured that v is positive definite and so we will follow rest of the things in the material to be uploaded. You can check that at the origin, so at the region m 2 r will be 0, so this term will vanish. At the origin your body frame coincide with the orbital frame and therefore your attitude matrix

A q will have it will be just an identity matrix. So, your e b x first column of this becomes 1 0 0. So, this can be reduced to a format and I is your inertia matrix.

So, this also get reduced to 0 you can verify, the same way this also gets reduced to 0 and at the origin because your q 4 equal to 1 and q r tilde this equal to 0 0 0 and this is nothing but q 4 equal to m 1 4 we have defined ok. So, therefore this also becomes 0, so that the origin it ensued that v 0 equal to 0.

And rest everywhere this is one negative term, but it can be ensured that if you keep k p equal to high. So, for irrespective of the state of the system this v can be made positive definite and for this the condition can be defined accordingly which will follow up in the materials to be uploaded. So, this way we have discussed about the system stability and it can be shown that v dot x this is less than 0. So, using the LaSalle's theorem, then it can be shown that such; it can be stated that such a system is globally asymptotically stable because there it happens that for any state it can this is satisfied.

Means anywhere you leave the system in the status space it will converge to the origin. So, what we have discussed about the magnetic attitude control. So, basically we have not gone into the design of the file or the magnetic torque curve because my objective is not to teach you the how to make the torquer and other things. But basics of the magnetic actuator, how does it work how to apply to the system and how the control is done.

So, this way it comes to an end and using the magneto coulombic system which is the using the charge also the system can be controlled. So, some minor changes in this is required, the equation remains same almost and then the proof of theorem remain same. And only thing in the dynamics part in the torque part, the equation the torque equation will differ that we need to derive.

So, if I find time I will do that otherwise we wind up here in this place today and we will continue this then we are left with the aero dynamic torque and solar radiation torque. So, these are basically taken as disturbance to the system ok. And if you are looking for a particular control that just using the aerodynamic torque you want to control. So, once your force is calculated ok, so the torque can be are calculated and using that torque, so if gravity gradient is project response because gravity gradient will always be present until and unless you are in the satellite is in the geostationary orbit the gravity gradient will be always present. And we will continue in the next lecture.

So, thank you very much for listening.