Satellite Attitude Dynamics and Control
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Lecture - 66
Satellite Attitude Control using Magnetic Torquer (Contd.)

Welcome to the lecture number 66. So, we have been discussing about the controllability issue and in that context I have because deriving of this proposition which was a stated. So, I was giving a proof of that so, that you become aware that how the system is really controllable.

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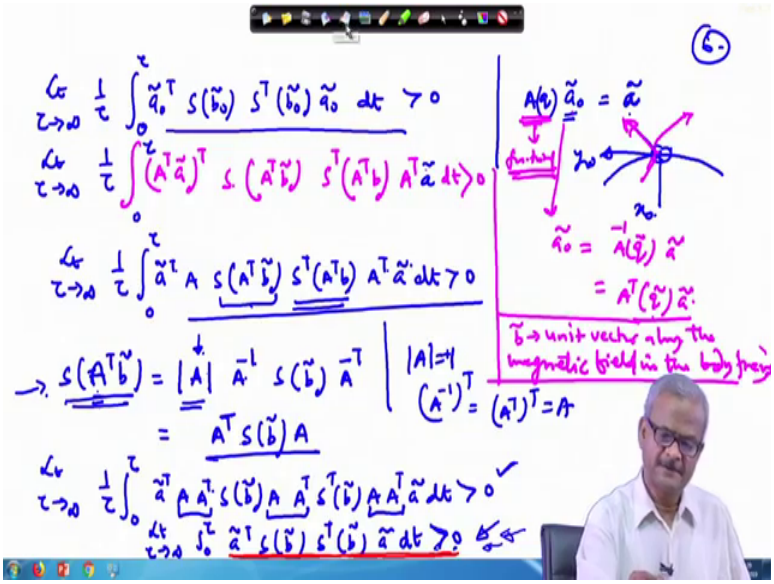

So, if we look back. So, we were working with this particular inequality. So, we showed that this inequality is valid then this is applied. Now we can rewrite this part this we can rewrite this equation again.

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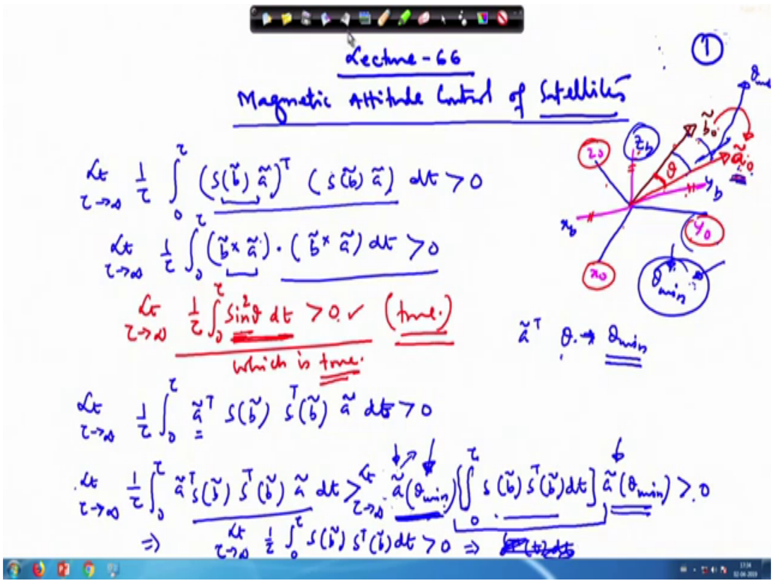

So, this can be written as the same way as we have written earlier S b tilde times a tilde transpose times S b tilde.

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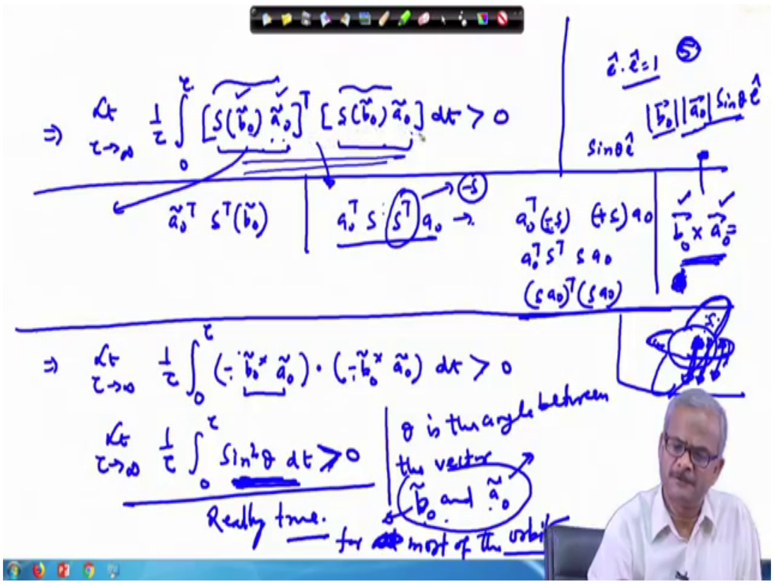

This is what we have done here. S b 0 times a 0 transpose S b 0 a 0 d t this is greater than 0. So, this quantity this inequality implies that this is going to be true and this implies that this is nothing, but b 0 tilde cross times a tilde bit minus sign. So, minus sign will get eliminated this transpose comes for the dot product and this is greater than 0 again we look here in this part. Now suppose that this is the orbital frame and I have the body frame here in this place.

So, this is your x 0, y 0 and z 0 this is the orbital frame and x b, y b and z b this is the body frame. Now in this frame b 0 tilde; this is a vector unit vector and in the same way we can say that a 0 tilde this is the unit vector these are referred to the orbital frame which is written by x 0, y 0 and z 0. So, angle between them at any instant of time is theta now if I am sitting on the pink frame which is the body frame and this is rotating.

So, irrespective of its rotation this vectors they remain same ok, if it is rotating this angle is not going to change this is a physical quantity which will not vary. So, either you are in this frame or either in this frame its component of the vector in the orbital frame and the body frame will be different component of the vector a 0 in the body and the orbital frame will be different, but this angle is not going to change this angle is going to remain the same ok.

So, therefore this angle here this is nothing, but sin theta this is a vector. So, as earlier I have told you this can be reduced into sin square theta d theta 0 to tau 1 by tau and limit tau tends to infinity. So, this quantity is going to be greater than 0 and which is true because this theta there will be only a few instances where b 0 just concedes with a 0, if a 0 is an arbitrary vector you have chosen it can be chosen in any direction. So, rarely it will happen that your b 0 goes and concedes with this even if it concedes.

So, only for that particular instant sin theta will be 0, but for other instances it will not be 0 and therefore, this summation or the integration here of sin square theta this is going to be greater than 0 and this is true ok. So, this says, now what we have started with we have converted here in this format.

So, this is our requirement ok. So, this is we can work on it little bit of some are arguments are required. So, let us say situation where theta comes very close to this or it is a nonzero value of theta which we write as theta min and say this vector it comes very close to this somewhere say here and then this angle is theta min and the, so this in this particular equation a transpose that we have written we can write it as we can take up that equation again.

So, limit tau tends to infinity 1 by tau 0 to tau a tilde transpose. So, this a tilde transpose I can replace it by. So, this is greater than 0. So, what I am going to write that this will be greater than 1 by tau limit tau tends to infinity, this quantity is greater than 0. So, your a tilde is in a tilde 0 was an arbitrary vector. So, in this body difference frame obviously, this quantity is never 0 a tilde 0 this is an unit vector.

So, this is an unit vector in the orbital frame is 0. So, the a in the body frame also remains an unit vector it is not going to be 0. But let us select the condition that where a 0 comes quite close to b 0 or b 0 comes quite close to a 0 and this condition we are writing that a tilde in that condition theta becomes equal to theta min. So, we are replacing with this and this becomes a constant because this is a for a particular theta min once the theta assume some minimum value other than 0.

So, this we are writing as theta min theta is not 0 it is not just going and overlapping with this in that case we are not considering. We are considering one case where b 0 comes close to a zero, but that value is we are writing as theta min not as 0 ok. So, for that value a tilde I can take it outside this is a vector. So, it will have certain components along the body axis ok. So, corresponding to theta that value can be taken outside because then you are selecting a particular value of theta.

So, automatically then at that time this becomes a constant vector and it can be taken outside and then this implies that this implies 1 by tau from this part you can see that this implies that limit tau tends to infinity 1 by tau 0 to tau here this quantity is greater than 0 this is an arbitrary vector at a particular instant and therefore, and this is greater than 0 therefore this implies that this quantity is going to be greater than 0 and this quantity. What we have written this quantity we have written as gamma t d t we will write in on the next page.

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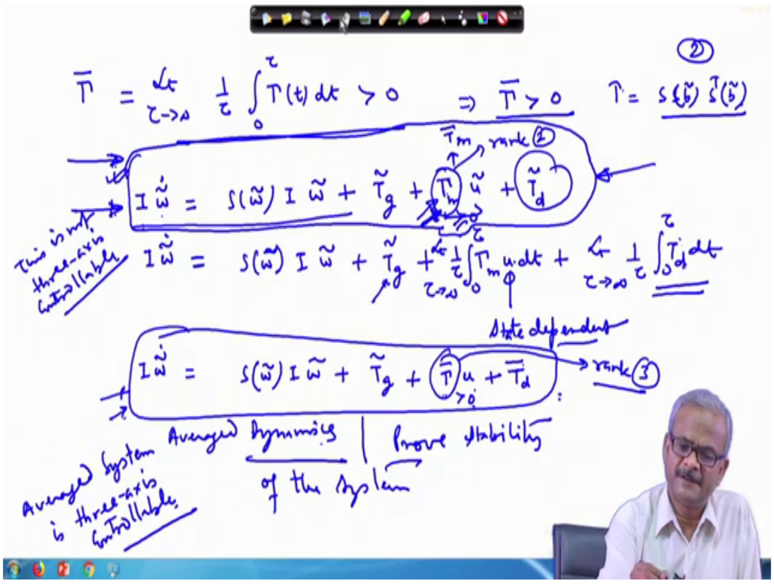

So, we can write here gamma t d t 0 to tau 1 by tau this will be greater than 0; that means, the quantity we have this quantity we have defined as gamma bar. So, this implies that the gamma bar quantity this is greater than 0. So, the control system we have chosen I times omega dot equal to S omega tilde T gravity gradient plus the moment part we resolved as T gamma where gamma is nothing, but S times S b tilde times S b tilde transpose ok. So, this is your magnetic torque T m these term we have written like this; and then you have of course, the disturbance term and once we are averaging it out.

So, this is these are not explicit function of time. So, as we have in indicated that I have indicated this is the generalized once we do the generalized averaging. So, these terms remain intact and we can write them. So, omega tilde I times omega tilde and T g this also remains like this because the gravity gradient term it does not depend on the time factor it is not explicitly dependent on time and this we have written last time.

So, this part will get averaged and this becomes gamma magnetic u d t 0 to infinity 0 to tau 1 by tau limit tau tends to infinity and this part also it remains limit this is T now if this is a state dependent. So, in this case this gets reduced to S omega tilde I omega tilde plus T g tilde and plus u can be taken outside if it is just a state dependent. So, we can write this as gamma bar u plus this is the averaged torque due to the disturbance.

So, this constitutes your average dynamics and this is used for proving the stability of the system if the in the original case where it is a written like this which is non average. So, this matrix is similar here gamma bar this is greater than 0 while here it is greater than equal to 0 ok. So, this is a similar matrix and this is a non similar matrix its a positive definite matrix ok.

So, because of this system was your original system is not three axis controllable because this matrix is of rank 2, If all the three actuators are working you will get only 2 axis control at any instant of time. So, this is matrix is of rank 2 while this matrix is of rank 3 and therefore, the full three axis controllability will be there for the average system.

So, average system is average system is three x controllable while this is not. However, while you design the control so, at that time we in actual situation what you will do you will be working only with this equation not with the averaged one averaged one is used just for proving stability of the system this is a necessity in control system. Until and unless your system is stable the control you are using it makes the system stable. How you can rely on that?

Your air craft is 1000 of [FL] rupee and if you are not aware that your system is controllable and its a say the system is not stable, it will lose a stability in certain region pilot is not able to hold the aircraft. So, it will crash your 1000 [FL] rupee will go pilot will also go similarly in the case of the satellite. So, the stability proof is mandatory for any control system we cannot skip it.

However, while we are simulating the system so, we are doing working with this with the original system.

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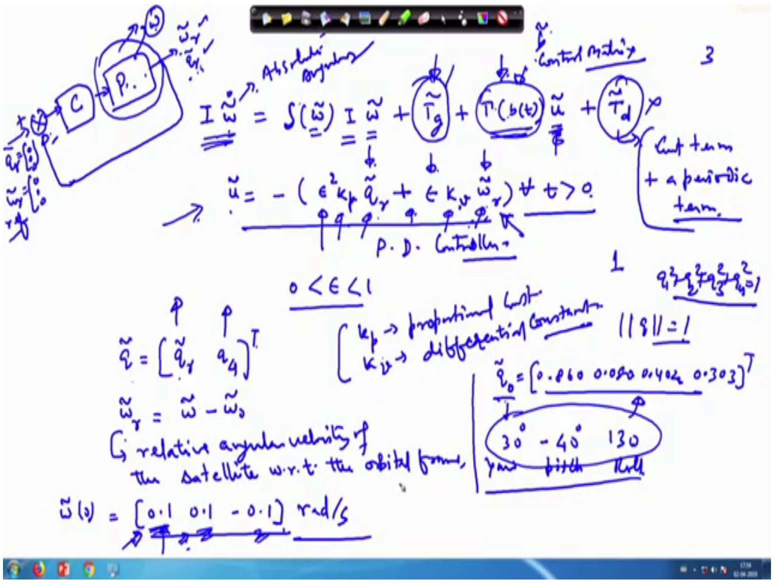

So, we take the original system. So, our original system is I times omega tilde no averaging here in this case this equation we are picking up plus T gravity gradient plus gamma. Here, in this case this is a similar matrix and also function of time or better we can write this as gamma b t time's u plus T tilde disturbance and this can be modelled as a constant term plus a periodic term ok.

So, what we need to do? We need to say I propose a control which looks something like again this is given in this paper the control which has been chosen and used to prove the stability of the system. So, this is simply a P D control ok, but it has been put in a special format to prove the stability of the average system ok. So, u tilde it is a chosen as minus epsilon square k p times q tilde r for all t greater than 0. So, this is your P D controller and why this epsilon has been chosen? As I post the supplementary material on proof of stability of the system you can look into that. Here, epsilon is a small quantity line lying between 0 and 1.

So, this is just dependent on the state this is q r omega r omega r and q r they are the quaternion representation just like you remember that q tilde we have written as q tilde r times q 4 transpose this is the vector part of the quaternion and this is the scalar part of the quaternion or say in the terms of parallel parameters both are equivalent. So, I am taking liberty of writing telling the quaternion similarly omega tilde r this is your relative angular velocity omega tilde minus omega 0 tilde.

So, relative angular velocity of the satellite with respect to the orbital frame orbital frame k p is your proportional constant as in the your P D control you use this symbol and k d is the k v is the differential this is the differential constant. So, this control is given initial condition will be given to you or either you choose yourself; that means, the initial what is the initial quaternion q tilde 0.

So, you can choose some arbitrary value let us say that. So, that the magnitude remains 1. So, I will put one value from the paper right this is 0.860 0.080 and 0.402 and 0.303 these transpose. So, this is your initial quaternion means the initial attitude of the satellite with respect to the orbital frame and similarly omega tilde 0 we can choose as 0.1 0.1 and minus 0.1 ok or any other value you can choose here such that this q this is a vector.

So, this magnitude this is equal to 1; that means the q 1 square plus q 2 a square plus q 3 a square plus q 4 a square this equal to 1. Now, you just say unit quaternion this rad per second this is quite large angular velocity. So, using the proof which I am going to upload so that implies, that you can not only control for this you can control even for whatever the value of the angular velocity you want to take you can take and you can control this satellite.

So, irrespective the of the angular velocity, aircraft space craft can be controlled it can be stabilized and this vector this quaternion vector it corresponds to as far as remember perhaps 30 degree 40 degree and 130 degree in yaw pitch and roll. So, you can convert it you can take this values find from here this quaternion's assume this in initial angular velocity and start the simulation assume that this is your control input which is a state dependent.

So, you have the block I have shown you in the last lecture this is the plant or the satellite from here you have certain difference value which see here in this case q r happens to be 0 0 0 and omega r tilde this also happens to be 0 0 0 and from this place this is the difference value and you are taking these are the difference omega r and q tilde r you are taking as the output from this then you are designing the whole controller I have broken into various parts.

So, this is plus and minus actuator and other things the actuator other things everything you can combine in this and here just you can put the controller. So, this is your controller structure very simple. So, given this initial state you will start this simulating here integrate this equation with the initial state omega is given. So, here omega dot you will get. So, as a result of this you will update it you will once you integrate it you will get omega at next time this is the kinematic equation we have already written.

So, from there you can update the attitude which will give you the quaternion. So, this way and here the control u is given. So, while you are working with this. So, at the initial state whatever the q r and the omega right in the beginning its there. So, k p and k v you have to choose appropriately. So, in this case in the paper it has been chosen from the lyapunov system analysis lyapunov stability analysis what you can do it by some trial and error by choosing different values whether your system is getting stabilized or not.

So, choose this values appropriately and then this will here enter into this place your at that particular location what is the value of the b vector. So, from there your gamma matrix will be known this is the control matrix. So, this matrix will be known to you. So, this is known this is known from this place t disturbance for the time being let us remove this is if it is not required gravity gradient term we have already written. So, it depends on the attitude of the satellite. So, and the moment of inertia of the satellite, so this can be written this is known to you this is known angular velocity these are the absolute angular velocity and while you are controlling in terms of omega r and q r. So, once you get omega.

So, you have to convert in terms of omega r and q r this plant will give you omega not omega r and q r. So, this needs to be converted into omega r and similarly q tilde r you have to get. So, using this scheme then you can propagate the system dynamics and as you keep on simulating it. So, you will see that the magnetic field it.

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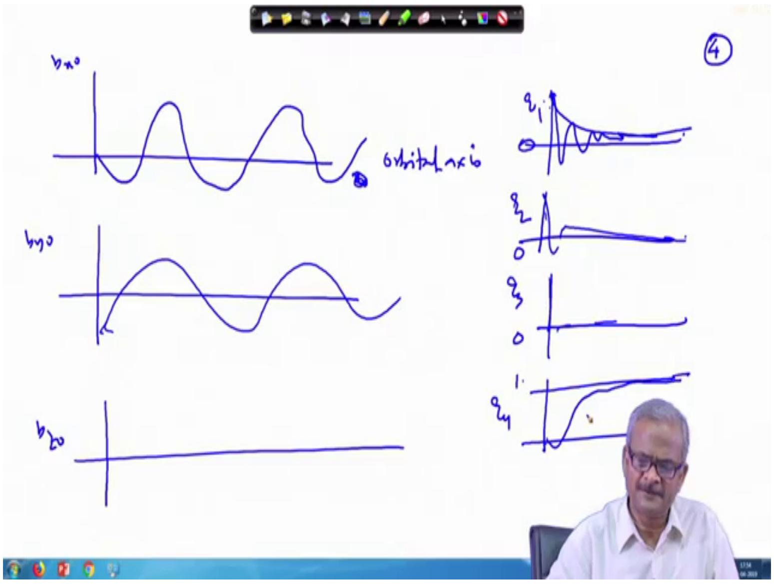

We have the while I work so I had the data from the I S R O otherwise we can use one equation which I will be uploading along with the proof materials which I am going to give you.

So, magnetic field may look like something like this, along the x y and z axis it will have of the body this is along the orbital axis it will be available to you in the orbital axis along the x 0 x 0 axis. So, this is b x 0 similarly you will have b y 0 and b z 0. So, some periodic pattern some little bit variation this is not just periodic it is not exactly periodic, but you can assume it to be periodic and in your control you will see that the your quaternion it should q 1, q 2 and q 3 they should go to 0 ok.

They are may be some initial oscillation, but this should go to converged to 0 this is the 0 value while the q 4 this will go to 1 ok. So, this is 1. So, there may be some initial oscillation and q 4 should look like this. Similarly your omega will also show variation you can calculate that how much torque it is being applied. So, many things if you experiment with writing in MATLAB code for this. So, you can get frequent with the system.

So, I will stop at this stage and I am going to upload rest of the materials because there is no point in proving the system stability here. Our main objective is to learn what we are really doing not just doing the mathematics. So, that part I will skip so.

Thank you very much for listening we will continue in the next lecture.