Satellite Attitude Dynamics and Control
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Lecture - 65
Satellite Attitude Control using Magnetic Torquer (Contd.)

Welcome to the lecture number 65, so we have been working on the magnetic satellite attitude control in Satellite Attitude Control using Magnetic Torquer. So, in that context we looked into some preliminaries and the basic requirements for doing the magnetic attitude control, the satellite dynamics and how the magnetic torque acts and what are the conditions under which magnetic torque may not be available.

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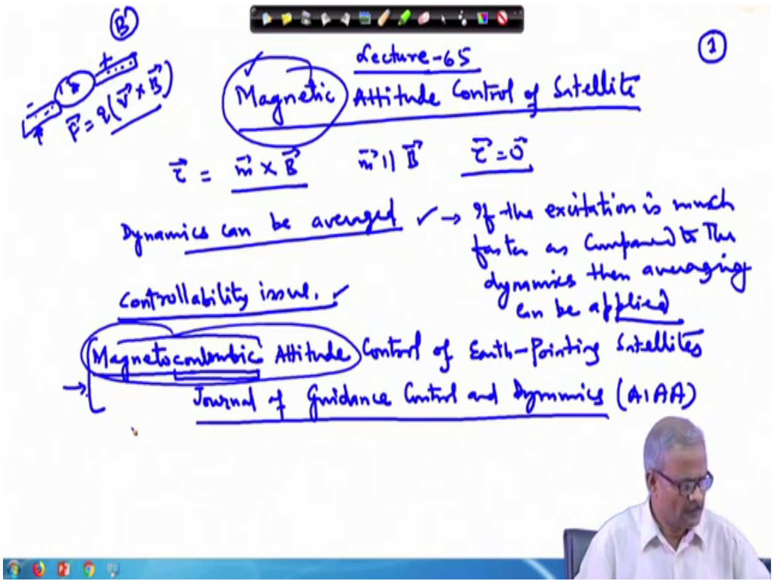

So, this will be directly given by this tau is the torque, so this m cross B. So, if m becomes parallel to B so in that case this torque vanishes. So, if we are given a satellite and we have to design a magnetic control for that.

So, in that case you will start with your basic requirement, you are looking for the inertial pointing or you are looking for the earth pointing satellite it is a circular orbit or is it a elliptical orbit. In the elliptical orbit this stability proof is difficult, in the circular orbit it's a relatively easy. So, and various other problems are there, so those things need to be sorted out.

But for most of the small satellite we small budget satellite because the coils making coil magnetic coil for the satellite it is a not a costly job. On the other hand if you use the CMG the Control Moment Giros or the reaction use for using the control, so they are very costly. And especially as the size reduces, so that fabrication of the system it becomes difficult. So, the in comparison to the size and the price the price will be on the higher side for the smaller ones ok.

So, in view of that the magnetic satellite attitude control, if your mission requirement is not very stringed means you have to point very milli or second or something like that. So, you should not worry and this magnetic attitude control will work for till 1 degree of pointing accuracy it can work very well.

So, last time we have been working with the I stated that the; dynamics can be averaged and why this averaging is required, so we will go through that and now already I was stated that if the excitation is; if the excitation is much faster as compared to the dynamics, then averaging can be applied ok. So, and then we have also looked into the controllability issue.

So, as you know from your basic control system that if the system is not controllable, so there is no question of it's proving its stability, you want to drive the system mistake to the origin of the system. So, in that context if you are putting the, basically you are applying control your system may be unstable. And if all the states can be affected through your control only then will say that the system is controllable this is in a very rough language.

So, for that now we are start and look into the controllability issue as we were discussing last time say and there is a paper on this that we worked out. So, I cannot upload the paper because it will be a copyright violation, but I will name you the paper and if you want to look into the other details it is available in this paper. Magneto columbic and this was published by Journal of guidance Controls and Dynamics.

So, volume number page number is not available, but if you give this title, so it will appear in and this was published by AIA. So, in the Google it will appear, but it is not freely available so, you may have to look for some library where the journal of guidance control these journal is available, so in that journal you will find it.

This is on magneto columbic, so this is magneto columbic means it Is a there is charge on the satellite say I have a satellite here and on the satellite surface charge is there on this side there is plus on this side there is minus. So, as the charge moves through the space so in the magnetic field, so this charge interacts with the magnetic field q times V cross B; this is the force acting on this charge ok.

Therefore, about the centre of mass of the satellite a net torque will be produced, here on this side there is minus sign and this torque can be used to actuate the satellite, so this is a new system. And, but the magnetic attitude control and this magneto columbic attitude control, there is a large portion of these two things two methods they are mathematically they are common.

So, referring to this one it will solve all your problems and some of the materials on the stability proof I am going to upload, I will not do here because that time is because the lack of the time. And more over it will be better that if I upload that material, but the paper I cannot upload because of the copyright violation. So, some of the proofs I am going to upload shortly maybe next week, this week it is not possible next week I will do all those things.

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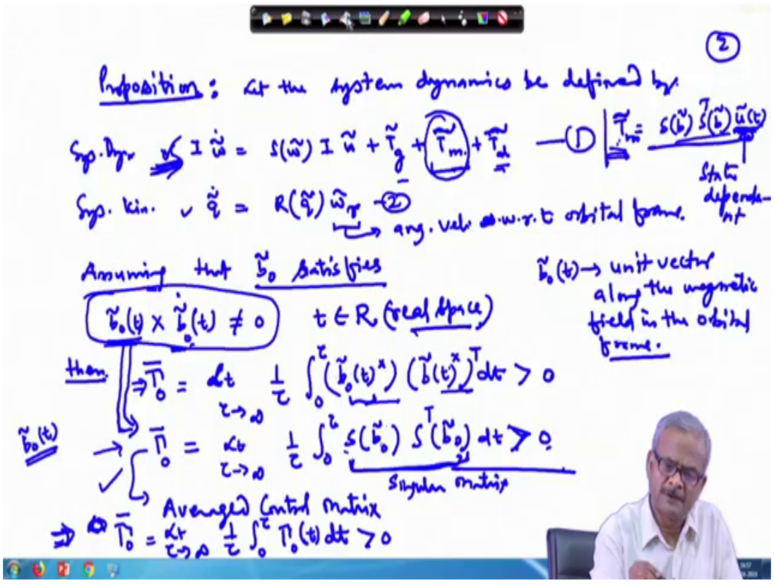

We have state a proposition; proposition it is a main theorem. So, these are the torques due to gravity gradient magnetic torquer and this is due to the disturbance. So, this is one equation and then you have the kinematic equation that we have written last time.

So, these two together it forms the, this is system dynamics and this is system kinematics; system dynamics and this is system kinematics, where omega r is the angular velocity with respect to the orbital frame. So, let the system dynamics be defined by these two equations system dynamics and kinematics. So, b tilde 0 assuming that b tilde 0 satisfies and these are function of t, so better we can write in terms of t R stands for the real space.

So, if this condition is satisfied as we were discussing last time. So, where b 0 tilde this is the unit vector along the magnetic field, in the orbital frame means the components are in the orbital frame. So, if this condition is satisfied then we can define a quantity gamma bar this I do not remember last time which notation I have used so I am keeping it gamma bar limit.

And this quantity we have written as this is transposed here, so this quantity we have written as S times b tilde which is 0. So, S this becomes transpose and obviously, this cross the way we have defined is, so this will appear with a minus sign b 0 tilde cross equal to minus S b 0 this are the minus minus for this two that becomes plus. So, I have omitted that part here writing directly this part. So, this d t will be greater than 0.

So, as I have explained you last time that this is implying that the future value of b 0 t does not depend on the past value of t in a linear, if it is depending on the past value of v 0 b in a linear manner. So, and I have shown you that this two can be taken out of the bracket and as this quantity is singular, this is singular matrix. So, therefore, this condition cannot be satisfied.

So, this gamma bar this is call the average control matrix; control matrix. So, and from why we are taking that average because here in this case for this part this T tilde m it was written as S b tilde times S b tilde transpose times u tilde T and if this is just state dependent ok, then if we integrate this with respect to T. So, this can be taken out of the u t can be taken out of the integration sign and we will get the term in let me go and show a write it on the next page.

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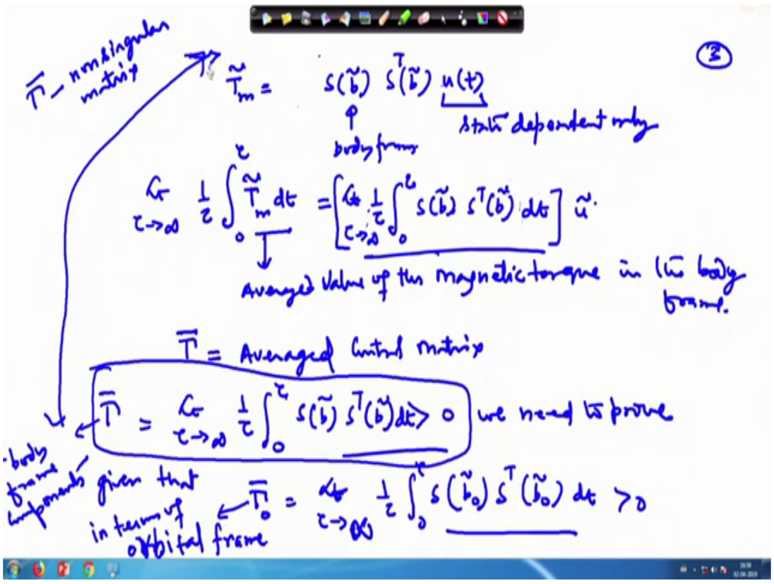

So, your T magnetic moment this term we can write in terms of S b tilde this is in the body frame b is in the body frame, S b tilde transpose d t this is u t. So, if this is only a state dependent means it is not a explicit function of time, then it if we average it out this torque so, it can written in this way this average value of this torque.

So, this is average value of the magnetic torque in the body frame and this will indicate by gamma bar. So, we have to prove that this quantity gamma bar which we can write as this is the averaged control matrix, limit this is your gamma bar we need to prove, given that gamma bar 0 which is been defined as limit tau tends to infinity.

These are tilde, this is d t here and d t is missing here so we need to put the d T also here in this place. So, this is your control matrix, this is in terms of orbital frame components and here this in the body frame components. So, based on the assumption that this condition is satisfied which implies that gamma bar we should write here 0; 0 here also in this place because this is in the orbital frame.

So, this implies that this will be true, this implies this and if this is given then we need to prove that this will be true. So, that means, gamma bar we have to prove that gamma bar is non singular. So, on an average we can do the control along the three axis for this system which is described here. So, this is the objective. So, the preposition it preposition this is proposition it states like this.

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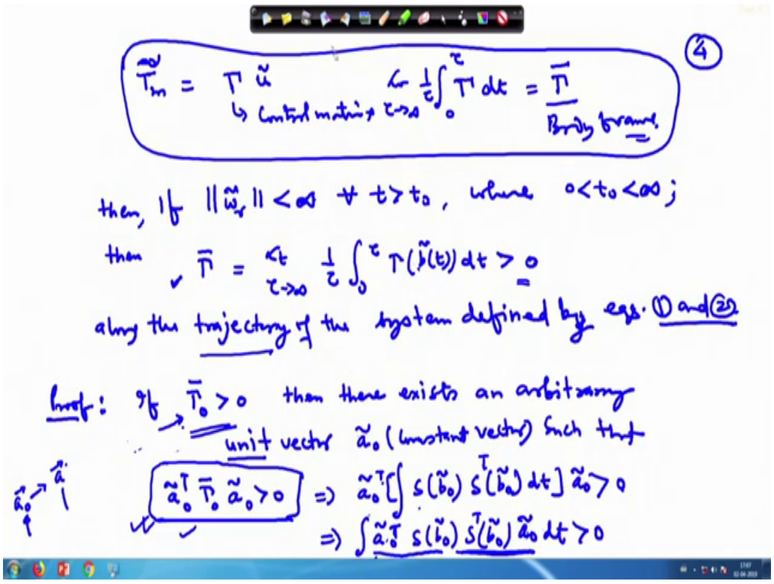

So, this particular part what we are doing that we have written this as T the magnetic torque, this can be written or in terms of tilde mat matrix notation, this can be written as gamma this is in the body frame times u tilde, here gamma is the control matrix. And then this is average, so once you average this, so this is gamma d t 0 to tau 1 by tau limit tau tends to infinity and this part we are writing as gamma bar, so this is also in the body frame.

So, this is the description, so let me complete the theorem now. So, in this proposition or the theorem, if this is given this part is given gamma 0 is greater than this so this implies I will write here in this place, that gamma bar 0 this will be equal to limit tau tends to infinity.

And this part we are writing as gamma 0 and this will be function of t, so t dt because b b 0 is a function of t it will vary with t, it is a time dependent part. And this should be, if this is give greater than 0 assuming that v 0 satisfies this condition, so this implies this and this implies this here ok. So, under that condition what we have to prove?

So, this is just for your information, the proposition I am going to complete on the next page ok. So, these are all for your information how the things are revolving. So, any angular velocity less than infinity for all t greater than t 0; t 0 lies between 0 and infinity, then gamma bar this will be the limit tau tends to infinity 1 by tau will be greater than 0 along the trajectory of the system defined by equations 1 and 2.

So, we have written the dynamics and the kinematics equation. So, gamma bar should be greater than 0 along that trajectory we need to prove that. So, proof is little mathematical and also logical and it is a very old standing problem it was more than six decades old. So, let us look into the proof I will try to be as sort as possible, otherwise you can look into the paper name I have mentioned.

If gamma bar is greater than 0; say it is a positive definite matrix, then there exists an arbitrary non-zero unit vector or simply we can write it because it is a unit vector. And therefore, this will be non-zero of course, so we simply write here as the unit vector. Then there exists an arbitrary unit vector we can name that vector as let us say w tilde 0. So, this is in orbital frame, w tilde 0 this is a constant vector arbitrary constant vector such that.

So, this is the basic property of a positive definite matrix because this will form a quadratic function and you will get it greater than 0. So, I hope that you know all these things from your matrix algebra ok. So, this implies that we can write this as omega tilde 0 transpose and gamma bar 0 is nothing, but S b 0 tilde times. So, this should be greater than 0, this implies this is true.

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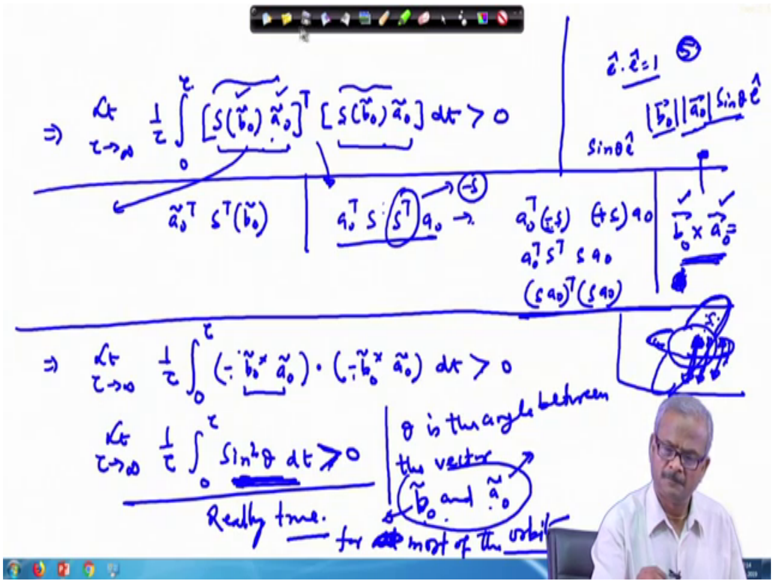

Now because this omega 0 is a constant vector, so this can be taken inside the integration sign. So, we write like this and that this will be equal to, so this implies omega 0 tilde transpose times S b 0 tilde b 0 tilde d t this will be greater than 0. Now we convert this omega 0 this is in the orbital frame, omega 0 tilde this is in orbital frame arbitrary vector, this is not the angular velocity vector or either we can change it. So, that the confusion does not arise instead of omega you can.

Let us make this as so that there is no confusion left ok. So, arbitrary constant here also we need to change ok. So, this is not the same thing as omega 0. So, this is an arbitrary constant vector. So, this property will be if this is positive definite greater than 0, so this property will be satisfied and this implies that this is true and this implies that this will be true because a 0 is a constant.

Now, a 0 this is in the body frame, so we need to convert this into the sorry it is a in the orbital frame, so we need to convert it into the body frame. So, a 0 must be change to a, so this is in the body frame and this is in the orbital frame. So, we do the matrix transformation the attitude matrix in terms of the quaternion it should known and there therefore, it can be changed to the body frame ok.

So, in the next step we have these two together we will club them together we write it like this S b 0 tilde times a 0 tilde transpose, this can be done if you take open it up, so this appears as a 0 tilde transpose times S transpose b 0 tilde. Here on this side you have simply this is written like this S transpose minus b times a 0.

So, take care that this part was written as a 0 transpose times S times S transpose a 0 this is written in this format this particular one ok. So, a 0 transpose I can put here as minus S and S transpose means this becomes equal to this S transpose is equal to minus S.

So, this minus S we can pitch here in this place and put it with plus sign and this remains S and the minus S we carry herein this place write it as a 0. And now minus S we can write as S transpose a 0 transpose S a 0, so this becomes S times a 0 transpose and this is S times a 0. So, using this manipulation we have written here in this place, so this quantity is then greater than 0, so this implies limit.

Now, this quantity transpose this quantity, so this is a vector, this is a skew symmetric matrix and this is a vector, so this matrix product once we are taking. So, this will also be a vector and this will also be a vector. So, we can the same thing can be written as minus b 0 tilde cross a 0, I hope you are by now comfortable with this notation dot.

Now, this is transpose these are two vectors, so this transpose gets converted into dot product. So, this is b 0 tilde cross times a 0 tilde and this is d t, so this will be greater than 0 and what this quantity is? This is nothing, but your b 0 cross a 0 ok. So, b 0 cross a 0 this we know that the angle between this two vectors, so this can be written as some resultant vector, this is a unit vector b 0 is also a unit vector and a 0 is also a unit vector.

So, product of them will be just a unit vector say some alpha or anything you can name alpha and so either you can write it like this b 0 magnitude times a 0 magnitude times angle between them let us say this is theta, so this becomes sin theta. And some vector which is perpendicular to both of them. So, unit vector in that direction and if we write it as e cap, so this will be this can be written like this.

So, as you can see that these are the unit vectors and therefore, this goes and simply you get a sin theta times e cap for this part. Similarly the dot product you are taking with this minus; minus sign will cancel out here minus sign will not stay.

So, therefore, you get this as sin square theta and e cap dot e cap that becomes equal to 1, so that goes also so sin square theta 0 to tau 1 by tau, where theta is the angle between the vector b tilde 0 and a tilde 0. This is the angle between these two vector this vector is arbitrary and this vector is varying with time in the orbit. So obviously, the angle between these two vector it is not going to be 0 until unless the orbit is such that if you take an equatorial orbit.

So, only in that case your b vector will be always fixed in a particular direction almost fixed, so which will be here what I am showing in this direction it will be going like this depending of. If I write this as the South Pole; south magnetic pole, so it will be going here in the downward direction ok. So, this is perpendicular all the time here in the same direction.

And therefore, the angle between b 0 and a 0 that will become constant and; and a 0 if I choose it to be parallel to b 0 in that case so this become 0, so this condition cannot be satisfied. Otherwise this b 0 direction in the orbit if you take any inclined orbit like this; this is inclined orbit this is equatorial orbit, so in this inclined orbit your b 0 it keep changing its direction.

And therefore, the angle between them will not be 0 between b 0 it maybe for a few values, but not for all of them and therefore you sum them, so this quantity will be greater than 0 all the time. So, this is really true for most of the orbit; most of the orbits.

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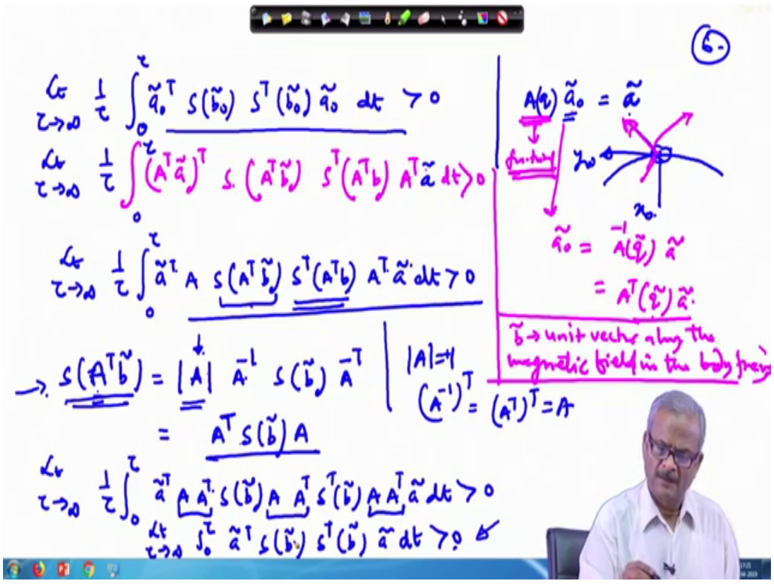

So, from here onwards we have to now prove that for the details of the writing you can look into the paper if you get it. So, now we can write, so what we have got that a 0 tilde transpose times and a 0 is a quantity which can be written in terms of so, if I operate on this with the attitude matrix this will get converted into the body frame.

In the body frame as the body is rotating, so you can see that here this is your orbital frame we have chosen this direction to be x and this to be y 0 and z 0 inside. So, your body frame is rotating with respect to this. So, your body frame is here and it is a continuously it will be changing its direction and this attitude matrix, so therefore, this will be a function of time.

So, we can see from this place that a 0 this can be written as A q tilde inverse a tilde and this becomes inversely nothing, but this a transformation matrix. So, we can replace all this thing here, so this q I will drop and just I will use this T to indicate here. So, this quantity here can be written as A 0 becomes and S b 0 similarly you can write in terms of A transpose b tilde where b tilde is the unit vector along the magnetic field in the body frame.

So, b tilde is the unit vector along the magnetic field in the body frame ok. So, we expand it and here we have; obviously, the limits are there 0 tau ok. So, if we rearrange it this a tilde transpose A times, this is a tilde, this will be greater than 0 and then we can use the matrix property and rewrite this particular term.

So, if we use this part there is a general you can say I can write it like this; this is a matrix identity which has a general form like this, determinant of matrix A times A inverse. So, as you know for the transformation matrix this is a generalized this is a skew symmetric matrix and this may be your general matrix.

But here in this case we know that A is equal to 1 for the rotation matrix because it is a rotation matrix, so this equal to plus 1. So, we give it plus 1, so this becomes 1 here and A inverse is nothing, but A transpose and. So, this becomes S b tilde and here this is nothing, but A inverse transpose.

So, this A inverse transpose is A transpose whole transpose, so this becomes A. So, this gets reduced into a very simple format. So, we can write therefore, a tilde transpose A times A transpose S b tilde times A, similarly we can expand this and if we expand it of that will appear as we have to take transpose of this quantity this is the transpose of S.

So, if we take transpose of S, so again we will have if we take transpose of this, so this will be A transpose S transpose this is b tilde and here we will get as A and there after we will have A transpose a tilde d t and this will be greater than 0. So, you can see that these are identity matrix and therefore, our case this gets simplified to tau tends to infinity; 0 to tau a tilde transpose this will be greater than 0.

So, you can see the simplification, it gets reduced into the format where instead of b we got here b 0 and we got here b 0 in this place also which we have written here in this place. So, this is exactly the same form except that instead of a 0, here your a is appearing and instead of b 0 here b is appearing ok. So, we will continue developing this, so in the next lecture we are going to work out this part.

Thank you very much for listening.