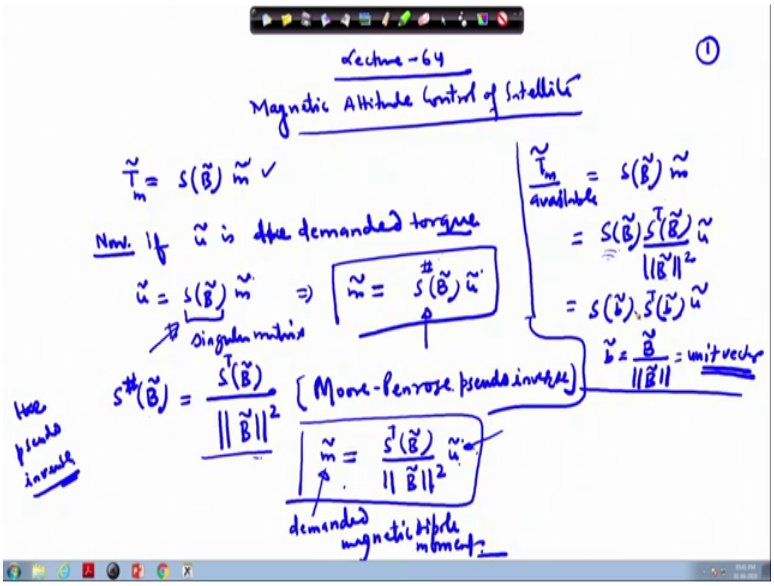
Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture - 64
Satellite Attitude Control using Magnetic Torquer (Contd.)

(Refer Slide Time: 00:19)



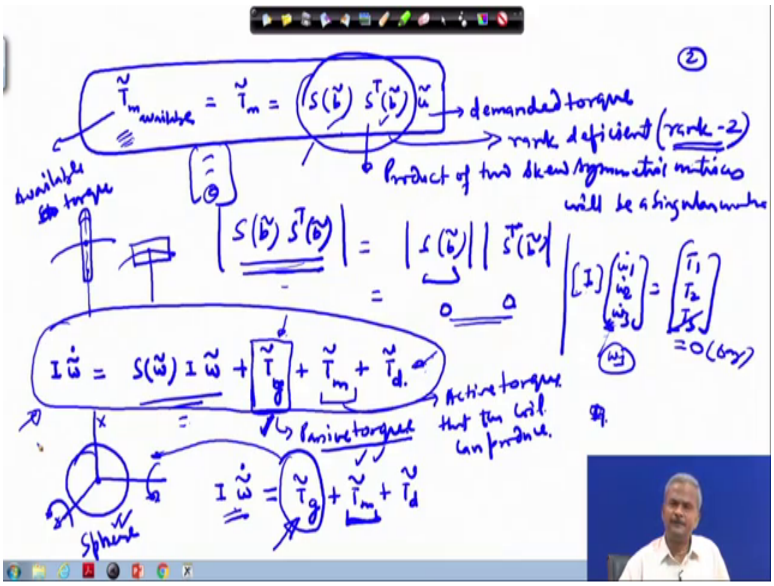
Welcome to the lecture number 64 we have discussing about the, we have been discussing about the magnetic attitude control, so we will continue with that. So, if you remember in the last lecture we have written T tilde this magnetic torque this will be given by S B tilde times m tilde. Now, as we were discussing that if u tilde is the demanded torque, then we must have u tilde equal to S B tilde times m tilde. Here this is a singular matrix, so we can find out this m tilde by taking pseudo inverse of this is skew symmetric matrix and we can write this as u tilde, S pseudo inverse B tilde u tilde.

And I will not go in to the derivation how to calculate this S pseudo inverse of this skew symmetric matrix S B was it will take a lot of time unnecessarily. And we will not we will lose basic the basic things we are trying to concentrate on. So, the quantity S tilde transpose B if yes this is hash basically this hash indicates the pseudo inverse ok. So, S B tilde this we can write as. The pseudo inverse is all the Moore Penrose, so for a singular matrix you can do this operation.

Now if we use this to write m tilde, so this becomes S transpose B tilde divided by B tilde magnitude a square and then this is u tilde if this is the demanded torque u ok. So, this demanded torque can be used to find, the demanded magnetic dipole moment and we can use this in this equation what your T tilde m this is the available this will be given by S B tilde times m tilde.

So, if I insert this here S B tilde m tilde if I insert from this place into this place, so this becomes S transpose and this can be written as where B tilde is nothing, but this is a unit vector and matrix notation is written. So, that you can check it is a very simple is this skew symmetric matrix; so, in that B is been divided by the B magnitude, so that will that is going to give you the unit vector.

(Refer Slide Time: 05:53)



So, the basic torque available equation this is the m available which will simply write as, this becomes S times B tilde times S transpose B tilde times u tilde this is what we have written here. So, this is the demanded torque and on this side you have the available torque, remember these two are skew symmetric matrices ok. So, product of skew symmetric matrices will be similar, will be a singular matrix because as we know that if we take the determinant of this we can simply write this as and this is determinant is 0 and this determinant is 0, so the product is also 0 ok.

So, this constitutes our basic equation, so in your the attitude dynamics equation we have written, so I times omega tilde this equal to S omega tilde times I omega tilde and plus T gravity gradient torque plus T the magnetic moment torque and plus T tilde disturbance which can be modelled because of the torque rising from the solar radiation pressure and the aerodynamic one ok.

So, in this equation as you can see this is the gravity gradient torque, this may be a stabilising, this may be DS stabilizing depending on the configuration of the satellite ok. As you have learnt in the gravity gradient chapter that if the satellite is here in this format, so this will be stable about this pitch axis. But if the same thing is here in this format, so this will be unstable about the pitch axis.

So, depending on the moment of inertia matrix your stability of due to the gravity gradient will be positive or negative means either it will be a stable stabilizing or either it will be destabilizing. However, so this is just a passive torque, this is the passive torque this is not in your hand for a small satellite this is not in your hand, so it will be always present. However, this is an active torque that the coil can produce.

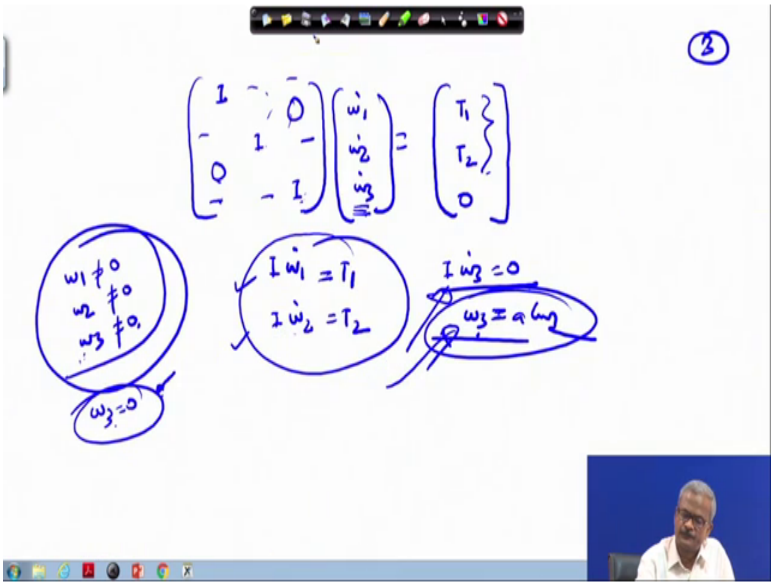
And as it is evident from this place that this is a product of two skew symmetric matrix and therefore, this determinant is also 0; that means, this is of rank deficient; that means, this is of rank 2. So, this simply implies that the torque will be available only along the two axis not along all the axis, these are the only two independent torques that you will be getting.

Now, the problem is then let us suppose that I have a sphere and if all the time the torque is available only along this axis and this axis. So, I cannot actuate along this because in the case of this sphere your dynamics this part will be 0, this will become equal to 0 and you will have only T g, T m plus T d. And as you know the gravity gradient does not make any difference to the spherical satellite get the moment of inertia along all the three axis it is equal, so only this part remains to work with ok.

And to have control so if you look here in this place I times this is a matrix and here you have omega 1 dot, omega 2 dot, omega 3 dot. On the right hand side I must have T 1, T 2 and T 3 because this is not effective here in this case of this is spherical satellite, this is the sphere.

So, if I have any of this T 3 equal to 0 suppose let us say this is 0, so in that case you would not be able to change this value omega 3 because and here let us say we write it like this I 1 will go on the next space.

(Refer Slide Time: 11:49)



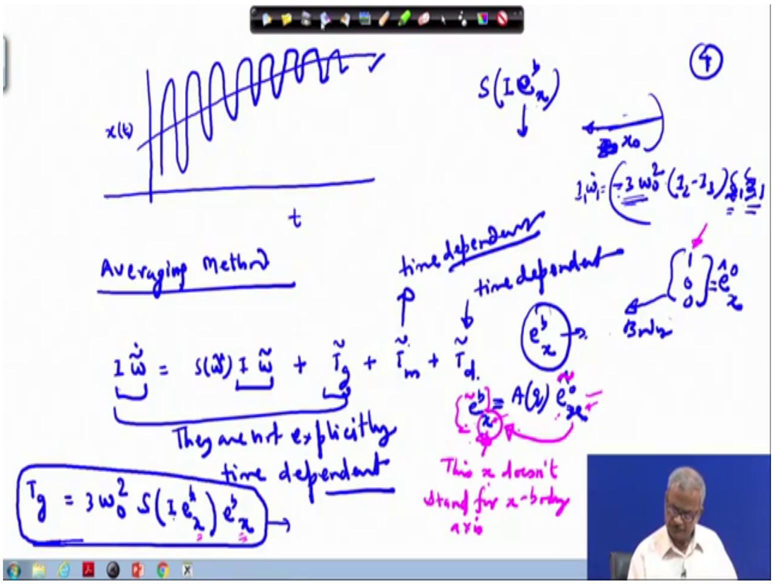
I times so this is I 1, I 2, I 3 which are all equal, so we will put this as I I and all other elements are 0 of diagonal terms and here you have omega 1 dot, omega 2 dot, omega 3 dot. On the right hand side gravity gradient is not working here in this case and disturbance torque let us assume that to be 0 for the time being, so, you have T 1 and T 2 and this part is 0. So, what we get from here? I 1 times omega 1 dot equal to T 1, I times omega 2 dot equal to T 2 and I times omega 3 dot equal to 0.

So, what this gives me omega 3 is a constant; that means, omega 3 you are not able to affect, angular velocity you cannot change along that direction. However, these two axis you can change the angular velocity. So, if you have to control the attitude of the satellite, so in many cases it may be required that you actuate the satellite along the all the three axis ok. If the satellite is rotating right in the beginning along all the three axis like omega 1, omega 2 and omega 3 they are all non-zero quantities.

So, without and you have to bring along the omega three axis say you have to bring it to omega 3 equal to 0, but there is no torque along this direction as seen here in this place. So, what will happen? We are not able to get the desired result, but if there is coupling means the of diagonal terms are there, then it is possible that using the two axis control you can get the desired result means you will be able to affect this omega 3 also, but for that case you need to have all this components here of diagonal term should not be 0.

But, here in this magnetic actuation case whether it is a spherical or whether it is a non spherical irrespective of that your T this is only effective along two axis the third axis it will be 0 and therefore this is all the time under actuated one, only actuated along the two axis. So, how to do the control? So, to sort out this problem control is done in an average sense. So, what we do, that we average out this dynamics this dynamics equation you can see here in this place we average it over infinite time period and then we look into the average dynamics of the satellite.

(Refer Slide Time: 14:53)



And this is based on the fact that if your, say the excitation is very fast. So, because of that excitation means the input you are giving to the system if it is very fast, so your system response may look like this.

So, this kind of system whose dynamics is looking like this because of the faster dynamic excitation, so you can average it out you can look into the average dynamics. So, this is the technique we are going to apply and this is called the averaging technique. So, your equation is I times omega tilde dot this equal to S omega tilde T gravity gradient plus T due to the magnetic and T due to the disturbance.

This may be time dependent, this is also time dependent; however, these are not explicitly time dependent. These three terms they are not and T g the gravity gradient torque it can be written as 3 omega 0 square. So, as you do as usual this is I is your inertia matrix in this skew matrix you have e times e B x it is a written like this. So, this is nothing, but your due to the gravity gradient this is your x 0 direction.

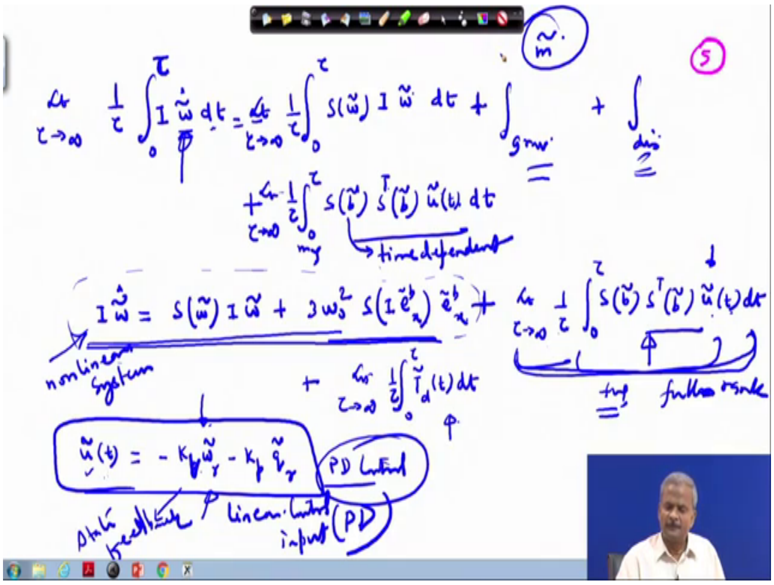
So, if you remember from the gravity gradient chapter that we have written this as I 1 1, so I 1 times omega 1 dot in the that the gravity gradient on the right hand side it appeared with a negative sign and then you had here I 2 minus I 3 and then C 2 3 and C 3 3, but in that case your Z direction was here along Z 0 was here.

So, if f 0 you are writing along this direction, so instead of writing here 3 3 I will write as 1 1, so this becomes 2 1 and 3 1, C 2 1 and C 3 1. So, this e b x this you are converting your along the x axis means 1 0 0 this is the unit vector which is your e orbital x axis unit vector along the x orbital axis, this is being converted to the body axis. So, if you operate by A q on e cap o x, so we get here e b x.

So, this is little confusing that why we are writing this is not along the x axis, this is just carrying this notation that this was along the x axis. So, for this the components along the body axis is given by this 1. So, this is e tilde we can write it here in the form of e tilde the matrix notation and these are the unit vectors. So, do not confuse this x that this is a long the x body axis, this is not this x is not along the or not does not stand for as we have done for the angular velocity, so in that case also I just removed that.

But in two place as we see that here if I write just e b and there also the e b is appearing without any tag, so it will be confusing in the derivation ok. So, therefore, we put a tag here x for this x.

(Refer Slide Time: 20:51)



So, we will apply averaging method to this, so if we apply averaging methods, so that implies that will be writing it like this omega tilde dot d t 0 to T this T or let us make it 0 to tau and 1 by tau this is the averaging this is the way of averaging limit tau tends to infinity this is called the general averaging ok.

So, on the right hand side similarly you will have S omega tilde I times omega tilde d t 0 to tau 1 by tau and this is limit tau tends to infinity similarly other terms due to the gravity due to the disturbance and due to the magnetic one. So, this is for the magnetic one some magnetic one we are aware of that the magnetic torque will be given by this already we have calculated and this T gravity is also available to us in the matrix format. So, this is your T magnetic.

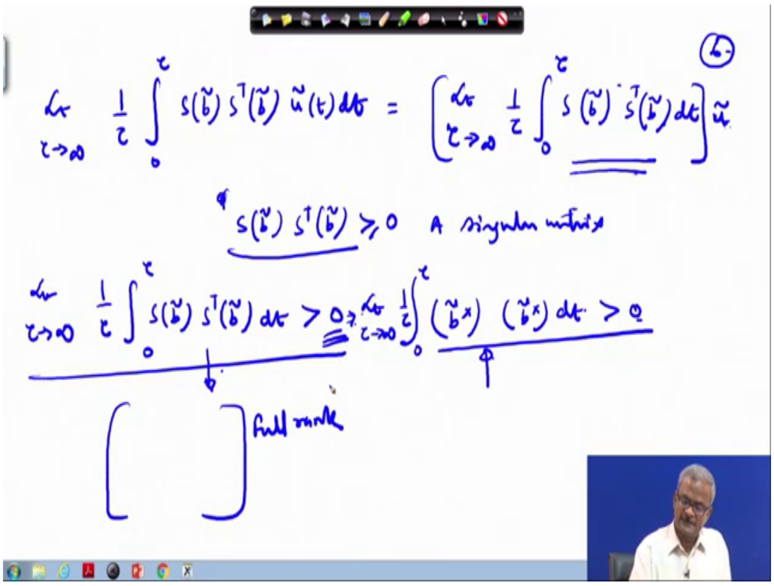
So, this is S b tilde times S b tilde transpose times u tilde 1 by tau 0 to tau here and limit tau tends to infinity other terms you can fill in here. So, what we observe that this terms they are not explicitly dependent on T and therefore this averaging gives you just I omega tilde. Similarly, this one give gives you and gravity gradient accordingly gives you S I times e b x from this place we get.

However, this term here this is time dependent, this B is time dependent and therefore, we just cannot do the averaging the way we are doing here for them. So, this will write as limit and then this is the gravity gradient term, this is the magnetic term and of course, the disturbance term. So, the disturbance term let us say this u tilde we have used the tau T. So, the disturbance term T tilde T limit tau tends to infinity this.

So, this is your equation, so we have to cop up with this and this term this is free from any integration this sort of integration, so this is very clear this part. Now, here in this part if your u tilde t suppose you design your u tilde t in a way k p times omega r tilde minus or this is k p times omega r tilde and k p times q r tilde means you are designing a proportional differential control means PD control.

So, if you use this PD control this is a linear control, but the system is non-linear this is a non-linear system we have not linearized it and this is your linear control input; linear control input or the PD control. So, if we have this kind of input here ok, so this can be taken out of the integration sign.

(Refer Slide Time: 25:43)



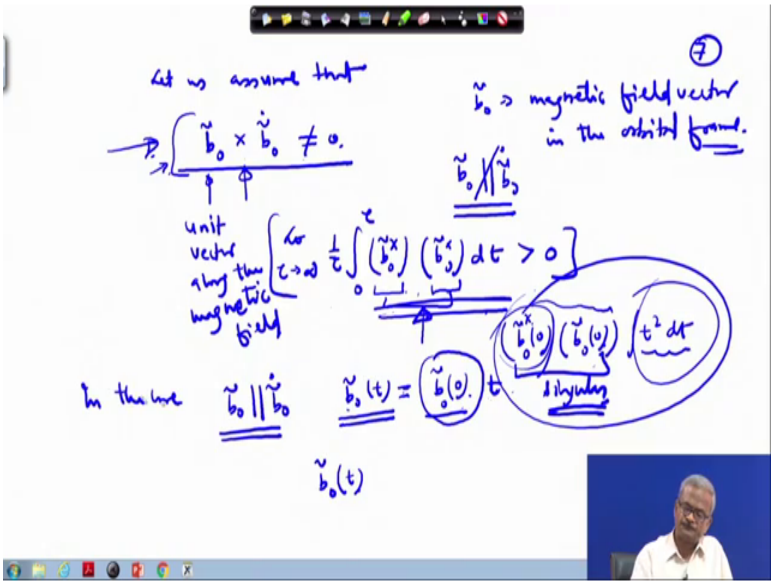
So, then this part gets reduced to ok, so the your 1 by tau for this case this will get reduced to limit tau tends infinity 1 by tau and this we just write as u tilde, we remove the T notation because we are taking the state feedback of course, here this is omega and this is omega r, so we need to convert the whole equation. But this is the simplification we get here.

Now, as you know that the quantity S B tilde S transpose B tilde this is; this is now singular matrix this is a this is the rank deficient ok, but if it so happens that if my integration S B tilde and S transpose B tilde d t, if this quantity is greater than 0.

So, this will simply imply that I am getting control matrix this is basically your control matrix which is positive definite means this will be of full rank, so this will be full rank if this happens like this. So, for that we need certain condition to full fill in the vector notation we can write this as B tilde cross B tilde cross minus minus sign that will become plus this is d t, so this needs to be greater than 0.

So, when this is possible this we have to explore, so using this magnetic attitude control because it is a quite simple and I am trying to show you that how to design a control and; obviously, for this we need the Lyapunav stability analysis ok. So, I will briefly introduce you I will presume that you know because those things are thought; those things are thought in the your basic control course ok. So, here this is tau 1 by tau limit tau tends to infinity ok.

(Refer Slide Time: 29:15)



So, for this purpose now let us assume; let us assume that b 0 tilde this is the unit vector along the magnetic field and this is a cross product taken with b dot 0. So, b 0 tilde this is the magnetic field vector; vector in the orbital frame. So, what it says that if b tilde dot is not that b tilde 0 is not parallel to b tilde dot?

So, in that case you will get here b tilde 0 cross this will be greater than 0 provided this condition is satisfied, why b tilde if this is parallel to this if it is not parallel only then this condition is satisfied in the case in the case b 0 tilde is parallel to b 0 tilde dot. So, what will happen? All the future b 0 t this will depend on the b tilde at time t equal to 0 this will be a function of linear function of this one.

So, these two terms then can be expressed in terms of b g b o 0. So, what we can do that this term can be taken outside this is this can be expressed in terms of some and then say if it is linearly dependent, so this will appear as something like, this is b 0 times t let us assume that this constant is not present.

So, for that case then your b 0 t this can be taken out of the, if that happens, so you can take it out because then you can replace this by b tilde 0 this is cross this will come outside and similarly b tilde this will come outside this two terms. And inside you will have t square d t and as you know this term is still singular because this is the singular term a skew symmetric matrix. So, therefore, this is singular, this whole thing is singular.

So, irrespective of that this term is positive or whatever or non zero this quantity is going to be this matrix will over all singular. So it is required that the basic assumption what we are doing here this is b tilde 0 cross b tilde dot this is not equal to 0. So, this will give you that this will ensure that this quantity is greater than 0 and now we can utilise this to model our controls ok.

So, then first what we have to prove that if this condition is satisfied so on; on an average, this term is going to be positive this is a average term, so in average sense if this will be full rank ok. And there for you will get three axis control in an average sense not at any instant of time still it remains under actuated, but in an average sense means overall long period of time this is acting like a three axis actuated system.

And this fact can be exploited for controlling the magnetically actuated satellite irrespective of whatever be the angular velocity of the magnetically actuated satellite ok. Earlier the there were problems with the magnetically actuated satellite, that if the angular velocity the theorem stated that if the angular velocity of the satellite it exceeds the angular frequency or orbital angular frequency ok.

So, in that case the satellite will cannot be controlled, but that problem has been overcome, so we worked on that and then we proved sum theorems and that has shown the result that this system can be controlled for any arbitrary high angular velocity. Provided you can provide that much of current in the coil because anyhow you have to produce this magnetic moment m tilde and this comes from flowing the current in the coil. So, we will continue discussing this in the next lecture.

Thank you for listening.