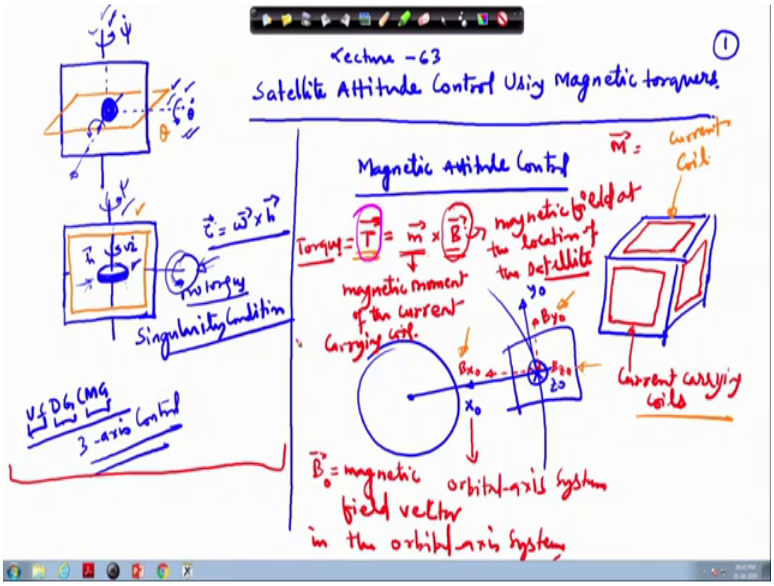
Satellite Attitude Dynamics and Control
Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture - 63
Satellite Attitude Control using Magnetic Torquer

(Refer Slide Time: 00:22)



Welcome to the lecture number 63. So, today we will be discussing about the Satellite Attitude Control using Magnetic Torquers. So, we have already discussed about the control moment gyros and if you remember we have done it for the variable speed control moment gyros. And also we have gone through a simplified double gimbal control moment gyros. So, single gimbal control moment gyros I have not done here.

So, I will be uploading materials for that. You can go through that material and it is along the same line moreover if I give you all the materials so, it will be much more easier for you to deal with that. Because we do not have that much of time to go through the extended part of the attitude control using the control moment gyros, especially the single gimbal control moment gyros.

So, today we are going to start with the satellite attitude control using magnetic torquers. But before this just one thing I would like to remind you that in the double gimbal control moment gyros; if you look this was the outer frame and then we had one inner frame and then we have taken axis like this and along this axis we have taken the theta dot ok, along this axis, we have taken the psi dot. Now if you see that if this frame this orange frame, it comes and coincides with the outer frame and in between you have the disc, here the disc is located.

So, if it so happens that this is the outer frame, and then the inner frame it becomes parallel to this means the theta becomes equal to 90 degree. If you rotate about this so, you are giving rotation by theta. So, if these two frames, they come become parallel to each other so that implies that your disc will be now along this direction. So, as you see in double gimbal here, this is speed we will assume to be constant. This is the rotation angular velocity of the rotational rate of the disk will assume it to be a constant; it is not varying.

So, if you apply any torque about this axis so, what we have discussed that I can apply torque along this axis and I can get output along this. I can apply torque along this axis I can get output along this axis in the simple double gimbal control moment gyros. So, here you can see the case that if your this is the gimbal here. So, if the frame has rotated such that it has become parallel to this outer frame; so, inner frame has become parallel to the outer frame. So, if you rotate about this axis by psi so, this angular momentum vector h of the disc or the fly wheel it is a; it will not change.

So, this is pointing here in this direction. So, this will keep pointing along this direction itself and therefore, the torque generation mechanism that we have chosen as omega cross h. So, here this will not be able to produce torque along this axis there will be no torque in this situation and this situation is particularly call the singularity condition.

So, in the right of that I am going to provide you so, there I will discuss some of these things but mainly I will concentrate there on the single gimbal control moment gyros, because those double gimbal control moment gyros we have already discussed ok. So, in the variable speed double gimbal control moment gyro, variable speed double gimbal control moment gyros so, in that case, if you vary this means capital omega dot is present.

So, what will happen that if you accelerate this disc ok; if you accelerate the disc, then you will be able to produce torque along this axis. However, the variable is speed double gimbal control moment gyros. It was meant for producing torque along the three axis like while rotating along this x axis you are getting torque along this axis, by rotating along this axis you are getting torque along this axis.

Similarly, if you change the angular velocity of the disc about this axis so, you also get torque along this axis. So, this is double variable speed double gimbal control moment gyros, this gives you three axis control; however, the associated similarities are there. So, if it coincides here say, here in this case; so, as you can see that no way you can apply torque along this axis ok. If the variable speed control moment gyros is there so, if you vary the disc speed. So, you can produce torque along this direction ok, but there is no way you can produce torque along this direction means it is a actuation capacity has got reduced.

So, these things I will address in the separate material that I will provide you. Let us say start with the magnetic attitude control for carrying out this kind of; using this kind of actuator so, here in the orbit you have the satellite. And in the orbital frame say, this is your let us assume this direction to be the x 0 direction ok, this to be the y 0 direction and going inside x y and z 0 is going inside this. It does not matter; I can put here x 0 and y 0 going inside and then z 0 here in this direction.

So, with respective of this configuration, your dynamics is not going to change. So, I will follow for this part this notation. So, with this assumption, now here on the satellite if this is suppose cubicle satellite so on this I can have three current carrying coils. So, these are current carrying coils and as you know that current carrying coils, it produces torque in the magnetic field.

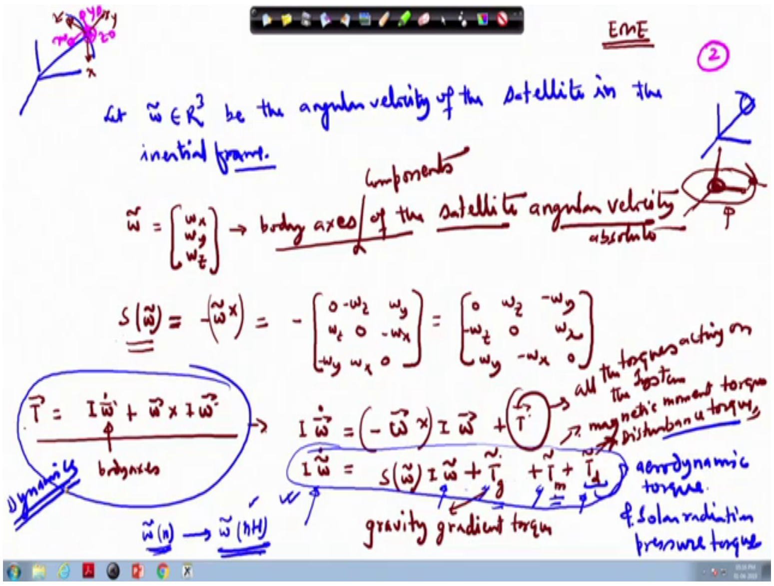
So, the torque produce tau or M, this will be given by; so here torque for the torque. Let us use the symbol T, because M is a particularly reserved for the magnetic moment or other things. So, it is better to use T here. So, this is given by m cross B, where m is the magnetic dipole moment or magnetic moment of the current carrying coil and B is the magnetic field at the location of the satellite. This is from your twelfth physics where the torque is given this is torque so, torque is given by magnetic moment, dipole moment times B.

Along the axis of the; these are the orbital axis system so, orbital axis system. So, along the orbital axis system, you will be given this B y 0 here along this direction B x 0. So, these are measured on board. So, this is B x 0 and going inside the B z 0. So, these are the magnetic fields components magnetic field component along the orbital axis and this we can indicate by B 0.

So, B 0 is the magnetic field vector in the orbital axis system ok. So, this part is not in your hand, if you are designing a magnetically actuated satellite which is quite often done for the small student satellite. So, in that case, you are not going to change this values B 0 y, B x 0 and B z 0. However, changing the current in these coils, these are coils current carrying coils already we have written.

So, you can change the current in this coil. So, as you change the current in coils and therefore, this torque T can be changed. So, if the torque can be changed and therefore, whatever the torque we require if we get that so, we will be able to reorient the satellite, but this is not as a straight forward. So, we discuss the various issues involved with this ok.

(Refer Slide Time: 12:20)



So, let us start with we have got the primary things and we will start with the basics which are required to deal with this topic. So, for going to this topic, we require three frames. One will be the inertial frame, one frame it is attached to the centre of mass of the satellite, one is our inertial frame, another frame which is attached to the centre of mass of the satellite which we have shown as x 0, y 0 and vertically going down this is z 0. And the third one with respect to this your body is getting reoriented.

So, it is a getting oriented and that we are writing as x, y and z. So, these are the three frames we require and inertial frame by definition this is your EME frame for whatever it can be. But here because we are dealing with the satellite attitude control, we are not dealing with the orbital mechanics. So, this can be conveniently taken say, if you are looking for a shorter time. So, if this is your orbit so, you can and here this is the earth. So, you can fix the inertial reference frame like here in this place, and then the satellite is located here; you have the orbital frame here and then the body frame is also there.

So, doing the mathematics part here does not require all this details how these are fixed and how they are oriented, so we go through the mathematics and learn how to solve this problem and what are the intricacies or the what are the complications of this magnetic attitude control. So, here omega tilde this we can write as omega x, omega, y omega z. So, this is along the body axis of the satellite, these are the components along the body axis; body axis component of the satellite angular velocity; inertial angular velocity or the absolute angular velocity.

We will define here one quantity S omega tilde, I do not remember what to the symbol I have used for earlier for S omega tilde, but here specifically I will write it for omega tilde with minus sign here. So, S omega tilde this means that this is minus and the diagonal elements are 0 here you have omega z and there used to be a minus sign here for omega tilde cross ok. So, this minus sign appears here and this becomes then omega y, omega x with minus sign and if you observe the minus sign inside so, this gets reduced to omega z minus omega y minus omega z 0 omega x and omega y.

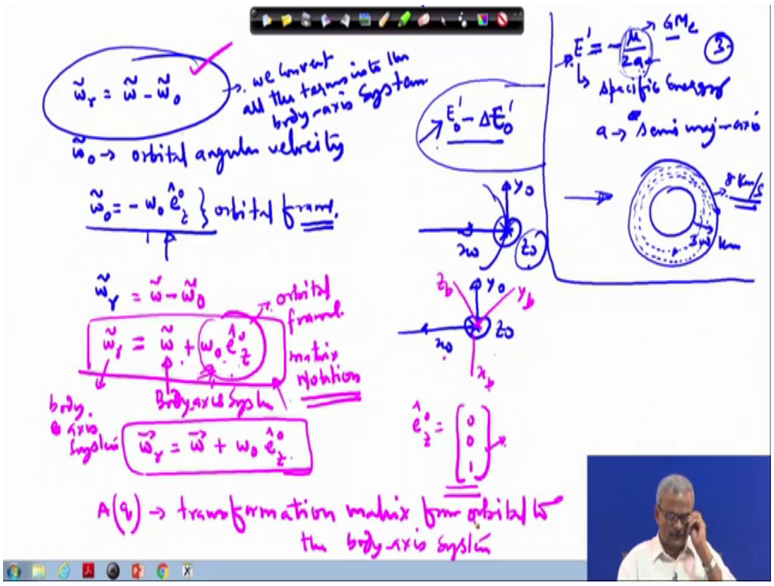
So, if we write like this so, the benefit of this will be that the equation of motion which earlier we have written as the torque acting on the system T equal to I times omega dot. This is in the body axis, then omega cross I omega this is the notation we have used torque equal to this angular dynamically equation.

So, here in this case we just want to modify this and we want to write it here in this fashion omega cross I omega plus T. This torque, this is the all the torques acting on the system and accordingly we can rewrite it. So, I want to write it here in this fashion so, there will be a minus sign here. So, therefore this quantity from here to here; this way I can replace in terms of this S omega tilde. So, this is S omega tilde and we will write it in the fashion in the matrix notation and this becomes T tilde.

So, T tilde we will bifurcate into T due to gravity gradient, T due to or in short we will write this as the T g and T due to the coil which is T m magnetic moment and T tilde due to the disturbance. So, this is gravity gradient torque and this is magnetic moment torque and this one turns out to be your disturbance torque. And in general the disturbance torque, it can be modelled as a periodic term and a constant term and this comes from the aerodynamic effect; aerodynamic effect means which we are going to discuss also after we finish this topic.

So, aerodynamic effect it comes from the some amount of molecules are left even at the satellite altitude, where say around the 300 kilometres. At 250 kilometres, there are number of molecules it is a very rare at atmosphere, but still they are there. So, if you remember in the recent event of the one missile short down the India has used one missile to sort a satellite.

(Refer Slide Time: 20:15)



So, in that case, we were talking about the satellite debris which was created there. So, once the missile has short the satellite. So, satellite has broken into many pieces. So, as it was announced by the Prime Minister and also by the ISRO that within 3 weeks the debri will be cleared. So, it is a because of the aerodynamic effect they are facing. So, because of the aerodynamic effect the satellite those small the fragments which we call as the debri. So, they will be losing their kinetic energy.

So, if the total energy because of the drag we will change ok. So, the total energy of the fragment it will continuously change and what we have not discussed in this course because it is a part of your orbital mechanics. So, E prime equal to minus mu by 2 a where E prime is the specific energy or energy of the satellite per unit mass. So, this is called the specific energy or energy of the satellite per unit mass, mu is the gravitational constant for earth. So, G time M e and a is the semi major axis. And for the satellite; obviously, this quality is you can say that this is negative the larger the value of a. The smaller value this will have E prime.

Now here in this case if your satellite is losing energy so, what happens that this a is starts shrinking. If a becomes small and small so, you will see that this quantity will grow in magnitude ok; that means, a prime becomes more negative. So, you have certain negative energy right now ok; say this is E 0 and it loose's certain another energy; certain energy this is the specific energy per unit mass and it is loosing this delta E 0 prime. So, this will become more negative because of this drag the continuously the satellite is losing energy and there for its orbit then start shrinking.

So, right now say this is in the 300 kilometres orbit. So, this fragment it will continuously lose its altitude. So, as the drag is observing its energy so, this will spiral down to the earth. And once it enters the atmosphere, dense atmosphere velocity of this fragment will be very high because at the 300 kilometre altitude, you will have around 8 kilometres per second this satellite speed approximately I am not giving you the exact value.

So, once it enters the atmosphere, this velocity will become very high and therefore this will start burning and it turns into access. So, therefore your problem of debri getting created in the space because of missiles shooting the satellite, so that problem will be over because of this phenomenon. So, here in this case, I was telling that this is the gravity gradient torque; this is the magnetic moment torque and this torque this involves your disturbance torque which can be modelled as a constant term and a periodic term.

So, this consist of it comes from arises from the aerodynamic torque and the solar radiation torque due to solar radiation pressure ok. So, now we define omega tilde of r this equal to omega tilde minus omega tilde 0 where omega tilde 0, this is your the orbital angular velocity. And the way we have used this direction we are using as y 0 here, this as the x 0, and inside going this is z 0. 25.34

So, the orbital angular velocity this is in the negative direction of the z 0 orbital axis. So, we can write this as minus u orbital along the z direction. So, this is in the orbital frame and we need to convert this because while we are dealing with the satellite dynamic. So, we convert all the terms into the body axis system. So, therefore, we need to convert this also into the body axis system; however, we can write here omega tilde r. This is the relative angular velocity means with respect to the orbital frame which we are showing here y 0 and z 0.

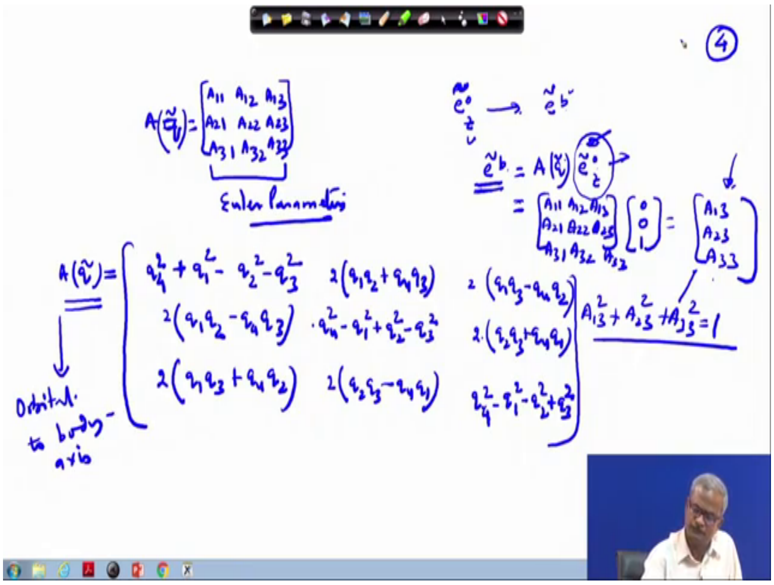
So, with respect to this, your body is rotating this is x body, y body and z body ok. So, this is rotating with respect to this so, the rotational rate of the body axis with respect to the orbital axis, this will be given by this is the inertial velocity minus omega tilde 0 as shown here in this place.

So, insert this becomes omega tilde plus omega 0 e cap o z. So, we are writing this in terms of the matrix notation. So, this one while we writing this format so, we must convert this; this is still in the orbital frame remember. This is not consistent still this is in orbital frame and this will be taking along the body axis and this will also be along the body axis system. So, we need to convert it into this format before, it becomes consistent.

So, it is a better it can be written like omega r equal to omega plus omega 0 times e cap o z. So, this is the vector notation we have used in this there is no problem. But while we convert here in this format; we write here in this format this must be converted to the body frame. So, as you know that e cap o z, the unit vector along the z axis this will be 0 0 in the matrix notation; we can write it like this.

So, this must be then converted to the body axis and that we can do using the A q which is the transformation matrix A q is the transformation matrix from orbital to the body axis system.

(Refer Slide Time: 29:54)



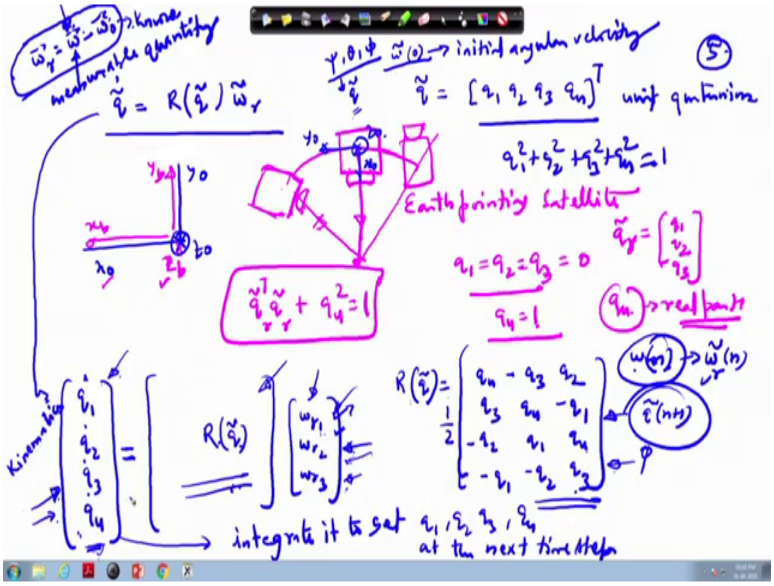
So, if we do this, we go on the next page and so, a matrix we can write as A11 ok. So, and this will be in terms of the quaternion or the Euler parameters, which I am going to write we have already done all this things; the basics we have done write in the beginning of this course. So, therefore, you can see that the body component of e cap o z; if we convert this into the body component and so, if we write here in the tilde notation. So, basically we are converting into the body frame.

So, what will be the this component along the body frame? So, if I put here z eta appears like this does in the body frame the z; unit vector along the z direction, but this is not the fact here. So, I will remove this part.

So, this is converted in here into this so, how we do that? So, e tilde o z we operate by A q; this is the A q, this is function of your the quaternion or the Euler parameters. So, this will get converted into e tilde b and this is 0 0 1. So, if you see this so, A11, A 12, A 31, A 32 and A 33. So, this will result in only the last column will be working and rest others will be 0. So, this one this is equivalent to it is a basically the unit vector along the z orbital axis, this will have components like this.

So, this will also be a unit vector means the A13 square plus A23 A square plus A 3 3 A square this is equal to 1. So, these are all the properties of the attitude matrix or the transformation matrix which we have already discussed. And A q tilde this is given by q 4 a square, the second column will be 2 times the diagonal entry here. This will be q 4 a square minus q 1 square, this is minus here and we have the last term here ok. So, this is your transformation matrix in terms of quaternion means you are going from this takes you from orbital axis system to; orbital to body axis.

(Refer Slide Time: 34:45)



The quaternion which indicates the attitude of the body axis q tilde which we are writing as q 1 q 2 q 3 q 4 transpose and this is an unit quaternion; that means, the q 1 square plus q 2 square q 3 square plus q 4 square this will be equal to 1. And if your body axis coincides with the orbital axis means this is a x 0 y 0 this is z 0. So, if your body axis x b is along this direction, y b is along this direction and z b is along this direction going inside the page. So, that we will refer to the this is the attitude. So, what we are going to do? We are looking for a satellite which can always point towards the earth. So, this we call as the earth pointing satellite. The other objective is to do this that my satellite should always point toward the centre of the earth.

So, initially it may be that your satellite is not here in this direction, but rather it is a pointing something like this. And you have to bring your satellite into a situation where all the time it is a pointing toward the centre of the earth. So, in this situation, once you the satellite comes in this format so, at that time, we represent the body axis orientation with respect to the orbital axis like this. And in that format your q 1 equal to q 2 equal to q 3 ok this is the complex component of the quaternion or the Euler. In the Euler parameters vector part, this also we call as the vector part.

So, this quantity will be 0. So, either you can use the Euler components or the quaternion; they are equivalent, but not the same as I explained you and q 4 will be equal to 1. So, the q r, we will define as q 1 q 2 q 3. So, this is your q r tilde as we have not shown here and q 4 is the real part of the quaternion. So, from this place, we can write this as q tilde transpose times q tilde r plus q 4 a square this quantity will be equal to 1.

So, this particular equation; this gives you q 1 dot q 2 dot q 3 dot and q 4 dot means you are not dealing with the attitude kinematics using the Euler angles rather you are using quaternions to deal with them ok. And in this format, it is a much easier to comprehend prove the system stability design the controls and so on. So, here your r matrix is there which is function of q tilde, and then omega tilde r this is the relative angular velocity so, this will write as omega r 1 omega r 2 omega r 3. Relative angular velocity means with respect to the orbital frame this is y 0 along this direction; this is x 0 going inside is z 0.

So, with respect to this, what is the angular velocity of the satellite this is given here. So, R q tilde this is given as and we have work it out earlier in our first 15 lectures so, this is q 4 minus q 3 q 2; this 1 by 2 here 1 by 2 time this. So, if you know the angular velocity of the satellite with respect to the body frame, then this can be updated. So, this can be integrated to get q 1 q 2 q 3 q 4 at the next time step. And how your omega r will be available? This is available from omega r equal to omega minus omega 0.

So, omega r, this is a measurable quantity using direct gyros these are measured and omega 0 this is the orbital angular velocity. So, this is also known. So, using this, then you can get the future values of the this satellite attitude with respect to the orbital axis. And of course, for that you need this and this omega the absolute angular velocity, it will be dictated by this equation which we have written here in this format.

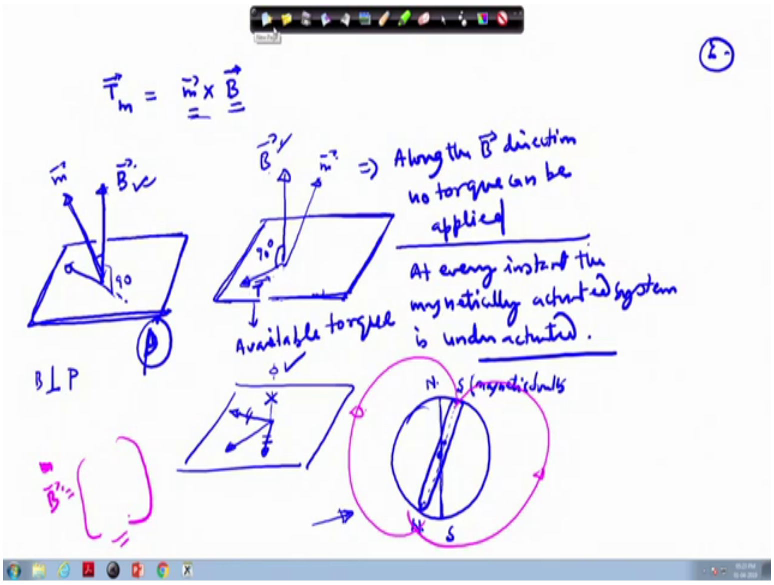
So, as you apply the torque, then omega you know at certain instant so, you can see that, you will get certain value of omega dot. So, if you integrate this equation so, you will get the omega at next time step means the next time step; let us say omega n plus 1. So, omega n is known, omega tilde n is known so, this will give you omega n plus 1. So, omega n plus 1 and this present value, you are also using to the present omega n you are using to get omega n this is the present value. So, from here you will be knowing this omega tilde n r and this you can use here in this place to get the future value of the quaternions.

So, quaternion at present will be available to you without knowing this. If the quaternion are available only then this matrix will be known ok. So, we are start with the simulation that some attitude is known to us say in terms of psi theta and phi this is the Euler angles and then we can convert them into the quaternions which we have already done. Once it is converted in terms of the quaternions so, this quantity is known to you, this is the R matrix as shown here.

Omega r will be known to you so, initial angular velocity also you will assume omega tilde 0, this also you will assume. This omega tilde 0 and this is different we will write it like this; omega tilde in bracket will write it 0. So, this is the initial angular velocity. So, this will be known to you and there for omega r right in the beginning will be known to you. So, this quantity is known, this quantity is known and therefore, q 1 dot will be known and this you can use to integrate and get the q tilde n plus 1 ok. So, this is how this equation is propagated.

So, this forms your kinematics equation, kinematics and this is your dynamics. So, this two equations are used to solve this system.

(Refer Slide Time: 44:28)



Now already we have discussed that the torque acting on the T due to the magnetic moment, this will be given by m cross B where m is the magnetic dipole moment of the coil and B is the magnetic field in the body axis. Because the equation which we are using here all the terms are written in terms of the body axis. So, we need the magnetic moment disturbance, the gravity gradient everything in the body axis.

Now, one thing we look into this place also let us say that B vector is directed here. This is a plane which we write as a P; this is the plane P. If m is the magnetic moment vector either here in this direction or either we can show it in another direction. So, m cross B will be line perpendicular to both of them; that means, if I draw a plane perpendicular to this B vector so, B is perpendicular to the plane P; some where your T is going to like this.

Another way to show it on the right hand side, we can show B like this and say the m vector is here so, m cross B. So, your torque will be lying along this axis perpendicular to both of them ok. So, this is the T is the torque available, what we can observe from this place that the torque is lying always perpendicular to the B vector ok. This is 90 degree here; this angle this is 90 degree. So, this simply implies that your torque is confined to a plane.

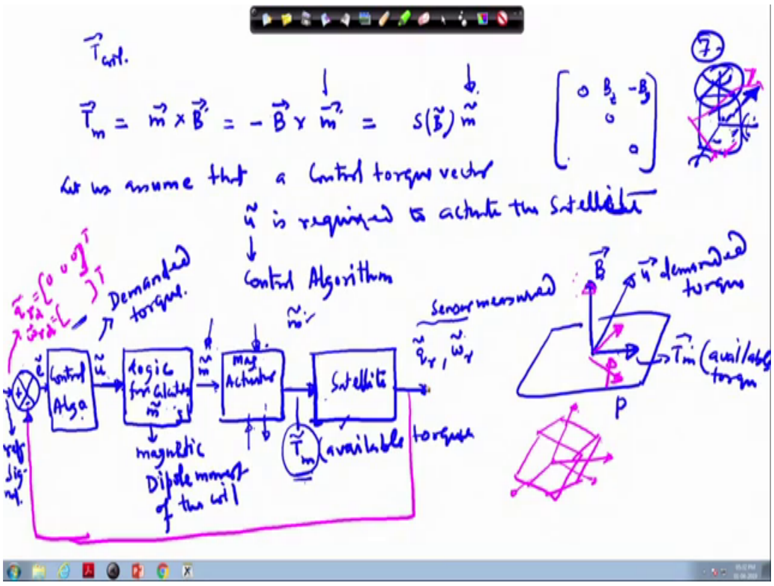
So, if this torque has shown here; this is the torque here so, this torque can be broken into two perpendicular components. So, it is a third component is missing means this implies that along the B direction so, this implies along the B direction; no torque can be applied. And this as the consequence that at every instant, the magnetically actuated system is under actuated because you are applying torque only along the this two perpendicular axis, the third one is missing, this is not present.

However, it so happens that as the satellite is passing through the orbit. So, this B vector it keeps changing. So, say you have the earth; so, earth geographic north is here and geography south is here in this place. So, this is your geographic north and south and magnetic north and south it makes certain angle with this.

So, let us say this is the south pole and this is the magnetic north pole; it is just opposite. So, you can assume your earth to be a magnetic dipole. So, a big magnet and there for because of this your magnetic field it appears like this. So, from north to South Pole, it say emerging from this and going here in this direction towards the magnetic south.

So, all along the earth; around the earth, this vector will be present and this can be modelled. So, we will look into that equation that how this B equation is represented this is not very important ok. So, B can be written in the form of equation also you can have the on board magnetometers which can measure this field and it can give it to you; however, this field keeps changing with time. So, right now, the issue I am not going to discuss this part; we will take up later on ok. What are our objective here is to solve problem of the magnetic attitude control.

(Refer Slide Time: 51:19)



So, the T; the torque due to the magnetic moment; T coil also we can write this as the T coil this is given by m cross B and which we can write as minus B cross m. And as per the skew symmetric matrix notation, we are using. So, we can write this as S B tilde times m tilde; where S B tilde is your skew symmetric matrix as defined earlier. This appears with a B z, here plus sign it will become plus and minus B y here in this place and so on.

Now, let us assume that a control torque vector u tilde is required to actuate the satellite and to actuate the satellite. And from where it comes? It comes from the control algorithm and this control block, we can show using this control diagram I will show it to you ok. So, here we have the plant which we call as a satellite; in the typical control notation this is your satellite and then this is your actuator. So, this is your magnetic actuator; input goes into the satellite and then the satellite output your q tilde r and omega tilde r will be available to you. These are measured through sensors so, sensor block I am not showing here.

And then we have the logic for calculating m tilde. How much m tilde is required? In this equation, how much tilde is required to produce a particular amount of torque ok? So, this is u tilde the control algorithm, this is the control algo, this gives you the u tilde. So, this is the demanded torque this is demanded. This passes through the logic unit and this calculates you are the magnetic dipole moment of the coil of the magnetic torque.

So, accordingly your system will then produce current in the coil and this will produce certain torque this will act on the whole satellite ok. So, along the three axes, then your torque will be available. But here in this case as already we have discussed that this is under actuated system. So, along one of the axis torque will be not available at any instant of time ok.

So, here in this place, this gives you m tilde; m tilde is input to this, and from there this is the output which is T m or this is the available torque. So, in a figure we can see that let us say this is the plane P as we were discussing earlier, this is the B vector and say this is the u the demanded torque. But we can have torque only in this plane.

So, this is T magnetic; this is the available torque. I cannot have torque more than this ok. So, this is indicating here that only along two axis of the satellite, torque will be available. So, this does not mean that say your available torque is lying here along this axis as is shown here. So, this can be broken along this, along this, along this ok; then you will tend to say that along the three axes, the torques are available, this is not the reality. The reality is that this torque T, it will be available only in a plane ok.

So, overall three independent torques are not available to you; three independent components will not be available to you as shown here in this place. You have only this torque here available and you can break along this direction and this direction. For the satellite it may happen that I have the satellite here and this torque happens to follow along this direction. So, I am breaking it along these three axes but in reality this is not three axes actuation because you will never have torque perpendicular along the B direction. So, this is always an under actuated system.

Then this feedback is taken from this place ok. This goes here and you have certain reference value q tilde, we can write as q tilde say r which is of course, here in this case this is 0 0 0 transpose; vector component of the quaternion and omega tilde. This is the desired, you can put here d; omega tilde r this will be the orbital angular velocity and here in this place, you have the operation. So, from this place, the error vector will be generated; this is plus, this is minus and here this is your reference signal; this is the reference signal.

So, based on this error, what is the error in the quaternion or the attitude and what is the error in the angular velocity? So, based on that system will produce certain torque; the control algorithm will generate certain demanded torque and this will effect to your logic calculator which I will show how to calculate m. And this m will be used for producing the; this is the demanded m.

Accordingly using your equation for the magnetic actuator, you can calculate the torque m tilde. And once this is available once you have generated this m so, this magnetic actuator it will interact with the magnetic field and produce T m which will actuate the satellite. So, this is basically loaded on the satellite itself. So, this is your control block diagram ok. So, now we will continue in the next lecture, we are stop here.

Thank you very much.