Satellite Attitude Dynamics and Control

Prof. Manoranjan Sinha
Department of Aerospace Engineering
Indian Institute of Technology, Kharagpur

Lecture - 62
Simplified Control Gyro for Satellite Attitude Control

Welcome to the lecture number 62, so we have been discussing about the variable speed control double gimbal control movement gyros. So, in that context we have discussed about how it is dynamics can be modeled. So, we have not gone into the its controls will look into the Simplified Control movement Gyros and its applications for the Satellite Attitude Control. So, if you are interested in going into all the detail controls of the double gimbal variable speed control movement gyros.

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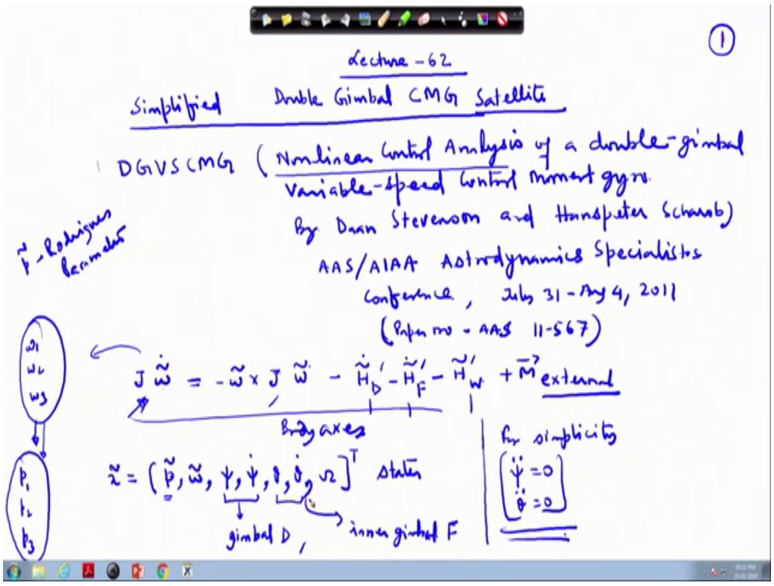

So, double gimbal, variable speed, control movement, gyros. The paper I have referred to a paper, so you can read that paper. This paper name is nonlinear control analysis, this is by Daan Stevenson and Hanspeter Schaub.

So, I am not going into the non-linear control what so ever for this was a quite detailed and we do not have that much of time to afford here and this is was published in AAS and this is paper number, thus so if required you can refer to this paper. So, it does not give you so much of details I explained, but after lot of effort you can work it out ok.

So, in this context little bit whatever was remaining last time, so we will cover that today. So, we have derived this equation or we have written in terms of J, so this is the equation we have written and all the quantities were converted into the body axes. So, where this J and all this quantities they are listed in body axes.

So, this equation can be propagated along with the kinematics. So, x tilde you can define as say the p tilde which is we p tilde is the Rodrigues parameter; parameters and its whatever the initial values are given, so initial value it is given in this paper ok. So, if you are referring to this paper, so you will get those parameter values or either you can assume on your own it is not a problem. So, p delta omega tilde psi; psi dot, theta theta dot and capital omega which is the wheel spin rate transpose. So, this forms the states or the state variables.

So, use the Cartesians as we have developed during the first 15 lectures, use those Cartesians and from there and in your assignment the Rodrigues parameters have appeared. So, the same equation goes here in this place, so those Cartesians can be converted to the Rodrigues parameter or directly the other angles you can convert to this and thereafter you can use this equation to propagate the system states.

So, from this place you get because this is the dynamic equation, so once you integrate it so you will get omega 1, omega 2, omega 3 at different time steps. This can be used to get your updated value of the p q updated value of the Cartesians or the Rodrigues parameters. So, p 1, p 2, p 3 this can be obtained from this place using the kinematic relationship and then you have the psi psi dot theta theta dot.

So, this is for the gimbal outer gimbal D and this is for the inner gimbal F. So, for simplicity you can assume that; for simplicity you can you can assume that psi double dot this equal to 0 and theta double dot this equal to 0. So, the equation will gets simplified and thereafter you can integrate this equation to get the corresponding result.

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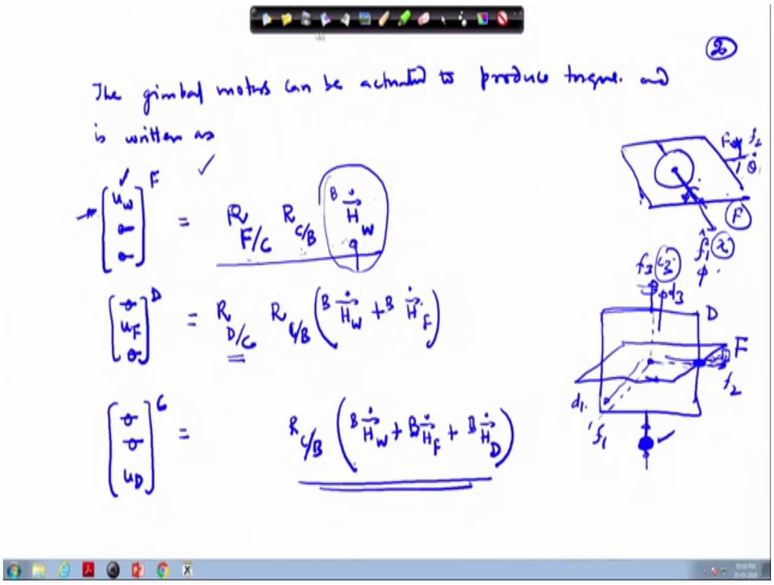

So, whatever is not will not appear in the or may be some of the materials I will post as the supplementary material which you can refer to because it is not possible to cover everything in the class. I have tried to explain you even the mathematics in the class which I could have put as the supplementary material, but seeing the complicacy of the system I have done that.

So, the gimbal torques they are written as u w 0 0 we can write like this in the F frame. So, your angular momentum of the body or here in this case the wheel, so the wheel you are trying to its angular momentum in the absolute reference frame this is changing, but everything has been converted to the body frame so this is written in the body frame. So, we need to convert this suppose we have this, so we need to convert it from B to C and then C to F ok. So, this will gets converted into this is the angular acceleration basically sorry this is the rate of change of the angular momentum.

So, the angular momentum change will be signified by the corresponding torque acting on the system. So, at any instant of time whatever the torque is acting on the system, so this is the wheel. So, that is acting in the F frame as you your noticed that the your frame we have shown it horizontally in the beginning and from here to here your wheel is there and this is the, this is your frame F and this will be torqued like this. So, once it’s been torqued, so the corresponding changes you will get sorry this is for the frame f which you are torquing here in this place, but here we are dealing with the wheel, so wheel is been torqued about this axis ok. So, here we have written as in this case the F 1 cap ok.

So, this is been torqued about this axis, so it is referring to that. So, in the F frame this absolute angular momentum it can be converted into the F frame ok. Rate of change of angular momentum which is nothing, but the torque, so this is been converted through the appropriate transformation matrix which we have written earlier and once you convert it, so this comes. So, this is along the x direction this is the x direction or the one direction of the F frame.

So, this is acting this motor torque will be acting along this direction, so this how it's a written. So, as a result of this H W will change in the change in this frame in the F frame ok, besides you have the now this frame because of the rotation your angular momentum of the wheel will also change because of this rotation about the f 2 axes you are rotating the frame about the f 2 axes.

So, if it rotates so as a result then we have to write that the wheel angular momentum, it will change because of that also. So, that has to be taken into account. So, that forms H dot wheel and of the body frame and then this needs to be converted from the C to body and then. So, we have the outer frame here which we have written as the D frame and then we have chosen the inner frame which we have written as F frame and in this direction we have chosen f 1, f 2 and f 3 and also d 1, d 2 and d 3 has been chosen along this direction; so, d 1 d 2 and d 3.

So, the rotation we are this is f 2 and d 2 they are coinciding, but we are giving rotation about the d 2 axes ok, which is attached to the D frame and in the D frame your motor is fixed which will rotate this frame the F frame ok. So, therefore, we do not need to convert up to the F frame and therefore, this will be given as 0 and this is converted to the D frame, along the same line then you will have 0 0 u d.

So, this frame also is being actuated, so one motor is attached at this point. So, the outer frame will be rotating it will accelerate. So, this we need to convert also. So, for that, so your wheel angular momentum this changes because of the rotation of the external motor also. So, that also we have to take into account, so that for we write.

So, these are the things which are not be accounted for and B H F dot and another one B H dot for the D frame and this has to be converted from C to B ok, thereafter we do not need to covert it because this motor is if we see here we have written this as the u D u D this is in the C frame.

So, we need only to convert till the C frame because this torque is applied along the c 3 direction, this is the c 3 direction also we have chosen and c 3 is a frame which is fixed in the satellite body ok. So, only one conversion is required and we get this equation. So, these are some of the ideas I do not want you to go into this until unless you do a professional level work where you are to publish research paper and other things ok.

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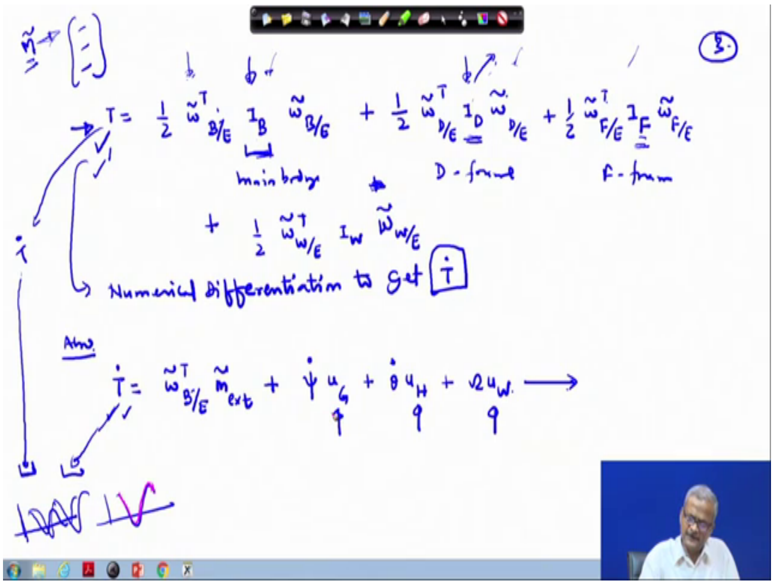

And if you are simulating the equation of motion, so we you need to verify that your equation is working alright. So, how to verify this? So, for that you need to write the kinetic energy. So, kinetic energy will be written as, so this is of the main body ok. Then you have to take into account the outer gimbal and then you have to take into account the inner gimbal which we have written by F.

So, because the left hand side is a scalar quantity it is not a vector quantity, so directly the body component of the inertia you can chose ok. So, if you are choosing along the principle axis, so corresponding you will get this all quantities as a diagonal matrix I D I F. So, and the things will get simplified and if you are aware of the corresponding, the angular velocity with respect to the inertial frame, so the work becomes easy.

On the other hand for the if we write the same thing for the torque in the torque equation because we have to write along a particular axis, so in that case if this torque will be a vector ok. So, along the 3 axes then you have to write and all of them are oriented along different axes and it is a moreover changing with time. So, for this we reduce all of them to the body axes and then work. So, here I am not giving any tag for the body axes if I am not giving any tag, so this implies that this is in the in their respective body axes ok.

So, this is for here also I will write this is for D frame, then F frame and plus whatever the extra terms are remaining. So, 1 by 2 wheel ok. So, calculate this quantity and then take its numerical differentiation do numerical differentiation to get T dot and also T dot will be equal to. So, these are the torque applied by the corresponding motor and the kinetic energy will change due to this.

So, T dot we get from this place if your T dot you are getting from this place and the T dot you will get from this place, so both of them should match ok. So, this T dot and this T dot they should be identical, they should be equal means if you do the plotting for the curve for T dot from here and T dot from this place, so both of them should overlap they should not do like this, they should exactly lie over each other ok.

So; that means, they have to be like this they are lying over each other exactly I have just shown it little bit different, so that both the curves are visible ok. So, this is the way to verify the equation of motion that you have done correctly and thereafter the process of the control starts, which we are not going to discuss in this class.

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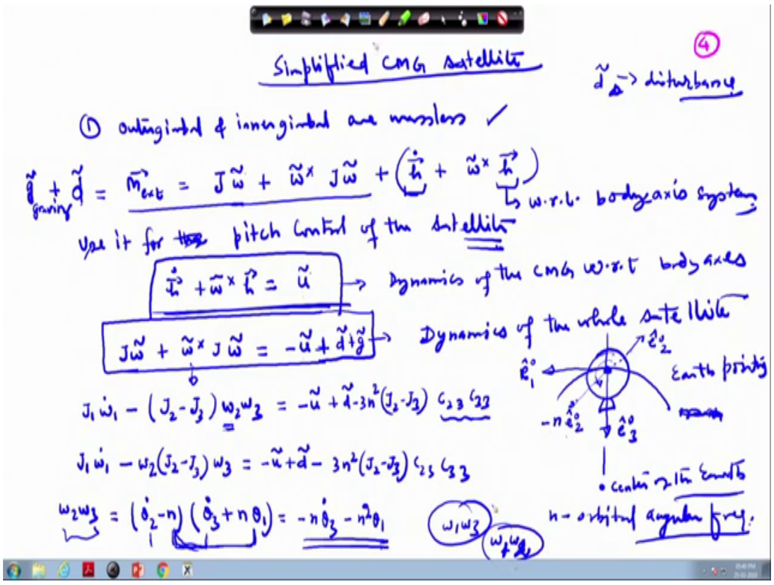

So, under the assumption: that outer gimbal and inner gimbal are mass less. So, therefore, their motor will also not be considered the outer gimbal and inner gimbal are mass less. So, under this assumption we have simplified the equation of motion as M external this equal to I times or the J times, where J is the angular momentum of the moment of inertia of the whole body. This h is with respect to the body axis, this we have derived earlier body axis system; body axis body axis system.

So, so we are going to discuss this for use it for, the pitch control; pitch control of the satellite. So, the equation of motion we have already developed and so if you expand this and assume this to be equal to 0 the external moment ok. So, only the right hand side will count and therefore, and there after what we have done? That we have written this quantity as, h dot plus omega tilde cross h this we have written as u tilde or minus u tilde accordingly the notation you chose, so the accordingly the things will differ.

So, you your equation then looks like J omega tilde plus omega tilde cross J omega tilde this equal to minus u tilde and besides M we M external we put let us say that we are just putting at the d tilde with the d tilde is nothing, but the disturbance torque this is the disturbance torque rest other disturbances are not acting on the system.

So we can write it like this and plus plus d tilde where d tilde is the disturbance and this equation together. So, this is for the dynamics of the CMG with respect to the body axes and this is the dynamics of the whole satellite ok. And; obviously, we can write this as J

1 times omega dot minus J 2 minus J 3 this part on the right hand side we will have the minus u tilde and plus beside this the in the external torque we will take into account your the gravity gradient torque.

So, here let us make one more addition which is g tilde which we have written as the gravity gradient torque. So, for this reason we will add here one more g tilde ok. So, this becomes g tilde equal to, so we have here d tilde and minus J 2 minus J 3 3 n square. So, this is the way we have written this is for the motion along the 1 axes and if we are looking for the attitude control with respect to the or vital difference frame.

So, this is your e cap o 1 here in this direction e cap o 3 and inside going this e cap o 2. So, in this direction you have the n vector n times e times o cap is the vector here, so in this direction this becomes. So, therefore, if we want that our my satellite is earth pointing; earth pointing, that is there is a camera here and this camera is always pointing towards the center of the earth; center of the earth.

So, if it is doing like this so; that means, my space craft also will be spinning at the rate n, where n is the angular frequency of the orbital angular frequency n is the orbital angular frequency ok. So, this way you can look that, this omega 2 it can be written as. So, the satellite is not rotating along the 1 and the 3 axes only rotation is taking place about the 2 axes which is opposite to the it e e cap o 2 axes ok.

So, this can be written as omega 1 dot and minus this omega 2 then becomes n J 2 minus J 3 and minus minus this minus sign, so that will make it plus omega 3 and this will be equal to minus u tilde plus the disturbance minus 3 n square J 2 minus J 3 and this quantity if you remember we have written this as theta 1.

So, for small angle approximation if the system is disturbed from the equilibrium condition, so you can replace it later on also it's not required at this stage, but this part is if there is a perturbation in omega 2, so we have to take into account that also ok. So, may be for the time being let us write this as the omega 2 only omega 2 times omega 3 and this will put as a minus sign.

So, we have developed this equation earlier you if you remember this we have done in the gravity gradient, then the spin stabilization and thereafter we have also this type of equation we have worked in the this gyro state satellite say number of times this has been repeated, so I hope that by now you remember all this things ok.

So, omega 2 times omega 3 this can be written as omega 2 equal to theta 2 dot minus n and omega 3 equal to theta 3 dot plus n times theta 1 and all this second order terms will be ignored, so we look for the second order terms. So this multiply together forms second order term, this; this multiplied together forms second order term this and this multiplied together it does not form second order term because this is minus n times theta dot 3.

So, this can be a approximately written as minus n times theta dot 3 and minus n times this term multiplied here, so this is minus n square times theta 1. So, another term we get here minus n square times theta 1.

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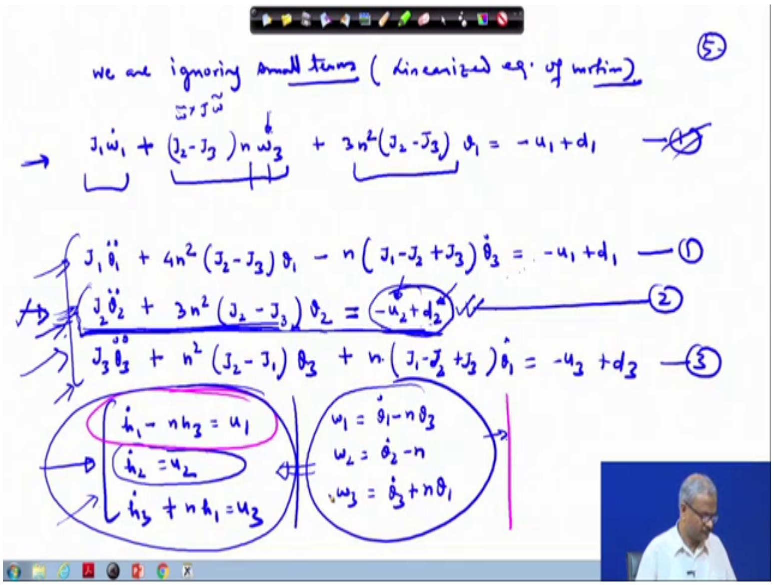

So, the first equation we need to reduce and put it in a proper format. So, what we are doing that? We are ignoring small terms and after ignoring it we whatever the equation we get that becomes our the linearized equation of motion. So, from here we get the linearized equation of motion. This is the gravity gradient term that we have brought it on the left hand side and this is the term omega tilde which is appearing from omega tilde cross J times omega tilde.

So, small terms we have neglected here later on we need to insert in here in this or either you need to replace the whole thing, if you want to write it directly in terms of the theta 1, theta 2 that can also be done at this stage ok. So, I have done it separately and so this exercise we have done in the spin stabilization part. So, I am not going to develop it completely either you write it like this or what we have done we are on the previous page you can utilize this ok.

So, after using this kind of formulation, so we can reduce the whole system to J 1 times theta 1 double dot plus 4 n square J 2 minus J 3 times theta 1. So, I am here assuming that you have gone through the earlier lectures. So, look back into those lectures and so you will get this equations and we have seen that in the spin stabilization that the motion about the 2 axes, it gets decoupled from the other axes ok.

So, independent of the other axes 1 and 3 axes we can control the 2 axes motion which we call as the pitch axes motion. So, these are the linearized equation of motion which you can check from the spin stabilization part or either just go through the process I have told you; you write omega 2 omega 3 like this and similar the in the same way you can write for omega 1, omega 3 you develop it and omega 2 omega 3; omega 2 omega 3 already taken care of omega 1 and omega 2.

So, in the same way you can write where omega 1, omega 2, omega 3 I will just write now I will write it now let me finish this part. So, J one minus J 2, third equation and rest your the equation of motion for this one, the gyro dynamics with respect to the body axis. So, here only the wheel motion is being considered and wheel is rotating on it's own axis and besides because of the gimbal motion it is rotating.

So,, but the gimbal moment of inertia has been ignored and therefore, the equation gets simplified and it can be written as the linearized equation of motion for this can be written like this h 2 dot equal to u 2 and h 3 dot plus. So, if you expand the previous equation this equation and linearize it this part ok, so, you get the this equation.

So, do it as a self exercise, where we are assuming that omega 1 equal to theta 1 dot minus n times theta 3 omega 2 equal to theta 2 dot minus n and omega 3 equal to theta 3 dot plus n times theta 1 ok. So, using this then you can expand and you can check all the terms ok, so you will get all these equations.

So, here what we are interested in doing the pitch control ok, so for the pitch axis we can provide a control because this is free from this is independent of the other 2 axes, it is not coupled with the other 2 axes, this axis and this axis they are coupled, but this one the second axis is decoupled from them. So, therefore, the control design for this can be done separately.

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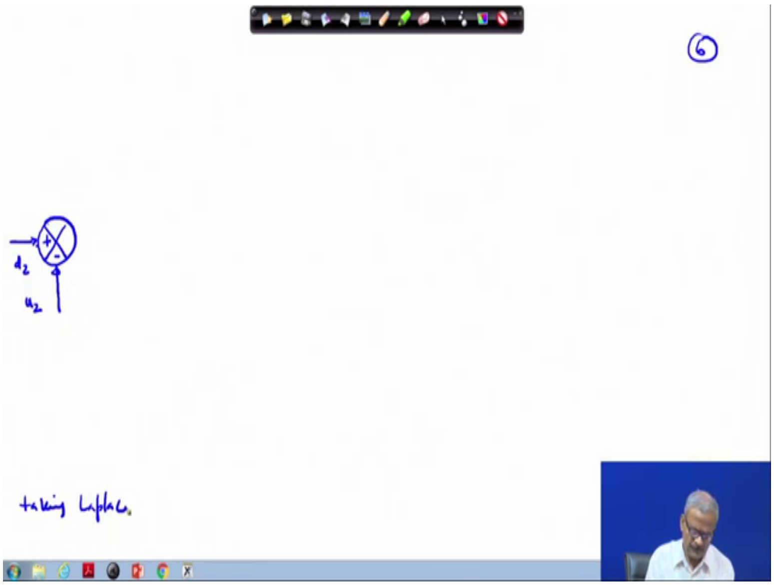

And a typical block diagram for this it appears like this is d 2, the disturbance and here this quantity is u 2. So, look back here in this d 2 minus u 2 in this equation d d 2 minus u 2 this is going as the control input to this system ok, so this is just like the torque applied to this part ok.

So, left hand side consist of the motion variables and right hand side purely the input to the system out of which d 2 is the external input and this comes from the control movement gyros and because of this the satellite attitude will change here in this case this is the pitch attitude. So, this for this if we take the Laplace transform equation 2 can be taking Laplace transformer or.

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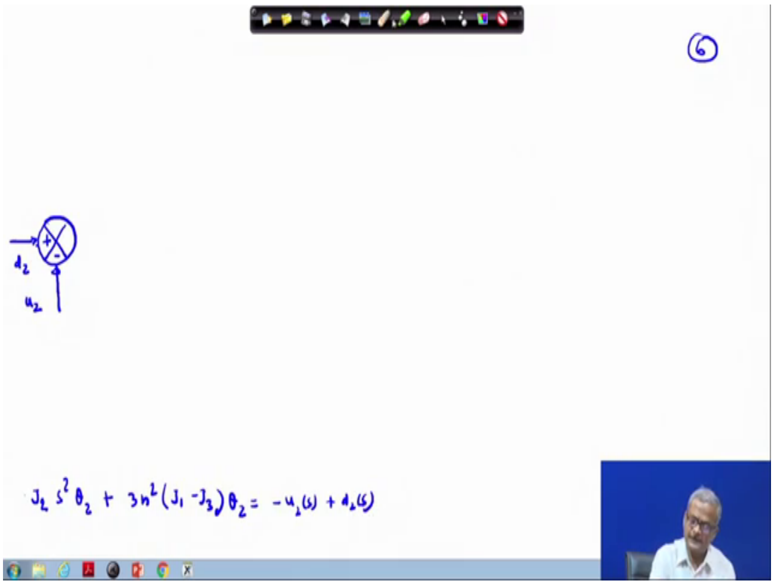

So, what we are going to do? We have to do the S domain representation for that. So, the we can convert this into J 2 times directly I am writing here J 2 times S square theta 2. You take the Laplace transform of the equation 2 and this is 3 n square J 1 minus J 3 times theta 2 these are all in the S domain I am writing here theta 2, but indicates this is a function of S which I am not writing here for clarity. So, the right hand side will be accordingly this is u 2 S and this will be d 2 S, but again this S notation I will delete from this place.

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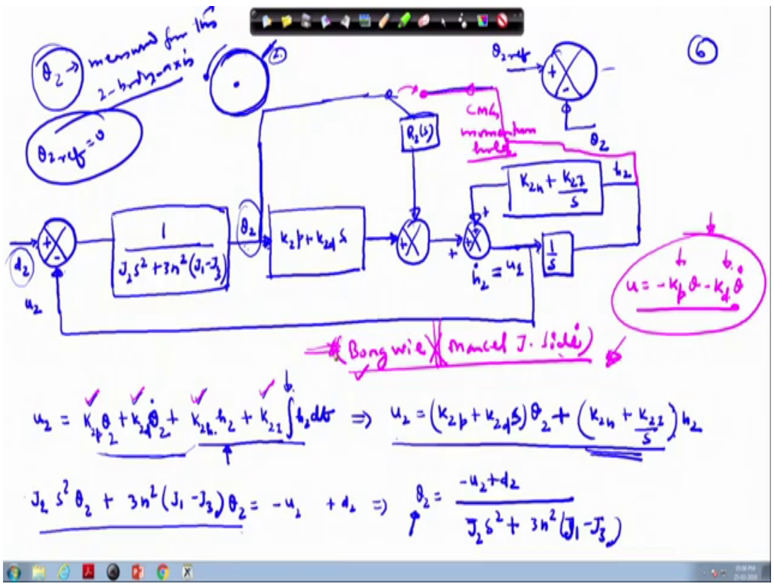

So, if you look for this ok, so this can be written as theta 2 equal to minus u 2 plus d 2 divided by J 2 S square plus 3 n square J 1 minus J 3. So, from here you get d 2 minus u 2 and this goes here in this place J 2 S square plus 3 n square J 1 minus J 3.

And after this you get this output we are all we have written in the S domain, so this is theta 2. Now, we can use a controller of the form say we can define u tilde u 2 which will actuate the system as k 2 p plus k 2 d times s, this is written in the s domain or in the terms of in the time domain we can write like this and thereafter we also add the integral part which is k 2 h times h 2, this is the your momentum of the angular momentum of your gyro along the 2 direction and k 2 for this the notations k 2 I we can write may be and h 2 dt.

So, if you take a Laplace transform of this, so this will get converted to k 2 p plus k 2 d times s times theta 2 or just theta here in this case we are written we have written in terms of theta 2, so write here theta 2 and for this part accordingly we have k 2 h plus k 2 I divided by S times. So, this is taking care of your as you know that the integral control is used for removal of the steady state error. So, you can use it for this system for this control for the CMG: Control Movement Gyros momentum hold or either you can use for the attitude control, so both are possible in this mode ok.

So, if we use this control, so this is the change control ok. So, now, you see from this place this is theta 2 is available, so this must be operated on by k 2 p plus k 2 d s and multiplied by this theta 2, so that gives you the one part of the u 2. And one more part we have to add to this; this is added with this particular part, so k 2 h plus k 2 I divided by S.

And this is fed here in this place and this operates on h 2, so this h 2 goes here in this place and you get certain output out of this. So, this output is nothing, but your u 2, so once both of them are summed up this is your getting summing up, so this is your u 2 and if you look back u 2 equal to h 2 dot ok.

So, this is the quantity, so h 2 dot this u 2 equal to h 2 dot. So, this you are integrating and this gives you the h 2 and the u 2 which is available here this is tabbed from this place, this is brought here in this place. If there is no disturbance then the this construction of this system is very simple; however, it requires choosing this values ok.

So, I am showing only this constants I have not shown their sign, so this proper sign is to be inserted if there is disturbance, so those disturbances need to be rejected. So, for that or disturbance rejection filter is added here. So, this part we are not going to discuss as a professional work or if you are doing some particular course on the control of the satellite, here we have done the dynamics part extensively only if the controls is involved, so on the MATLAB you can do all the control design for this ok.

So, remember that here theta 2 we are showing only. So, theta 2 is the angle measured from the theta 2 is measured from the 2 body axis, if we take the first rotation about the because your system this spacecraft is rotating, so the first rotation you will give about the 2 axis. So, theta 2 is measured from the 2 axis and thereafter you will give the 3 and 1, as we have done in the case of the spin stabilization ok.

So, theta 2 is the reference value of theta 2 this equal to 0 theta 2 reference and therefore, if you see here in this place we have not kept any the reference value because theta 2 otherwise you can formulate the block like this you can put here theta 2 reference, if you need some other value and then this goes as plus and then from here you can take this as minus and this comes as theta 2 and formulate this control block in terms of that ok. So, the same thing can be represented here in this format ok.

So, if you have to do the disturbance rejection, so this theta 2 have has to be tabbed and this is a tab which position you can change. So, this position here it goes for the control moment, gyro momentum hold I can take it from here it goes. So, this is the point, so this needs to be flipped into this position, so this is for CMG momentum hold.

So, this structure is simple, but you need to choose these values carefully and this may require a lot of effort and there are the sophisticated version of this also. So, you can look into the book by Bong Wie or by the Marcel J Sidi, this books name already I have given you one by Bong Wie another book is by Marcel J Sidi there are two different books.

So, many controller structure you will get there in that place and the satellite attitude control for the linearized satellite dynamics has been considered there not for the non-linear control ok. So, if you have to do the nonlinear controls, so what I will be doing one nonlinear control part? Nonlinear control means you are proving the stability of the designing control for the nonlinear system. So, you are control input may be in the linear format means you may be putting the control in this format let us say k p times theta and minus k d times theta dot.

So, this is just the proportional and the derivative control but if you consider the system without linearization. So, that becomes your nonlinear system and then proving the control for that that may be tough, but I will do that one case for the magnetic attitude control which needs little extensive calculation also, but I will do that, this part you can look into the book by Bong Wie and Marcel J Sidi, if you are interested in further simulating the satellite motion you want to launch your own satellite for your institute.

So, you go through this part and you can do the local control, but if you have to do the control in a wider sense means for a larger value of theta and other things other variable then you need to use the Lyapunav stability analysis and do the control design accordingly. So, here this equation that we are taking up here, this part one part I will show you show it to you on the next page.

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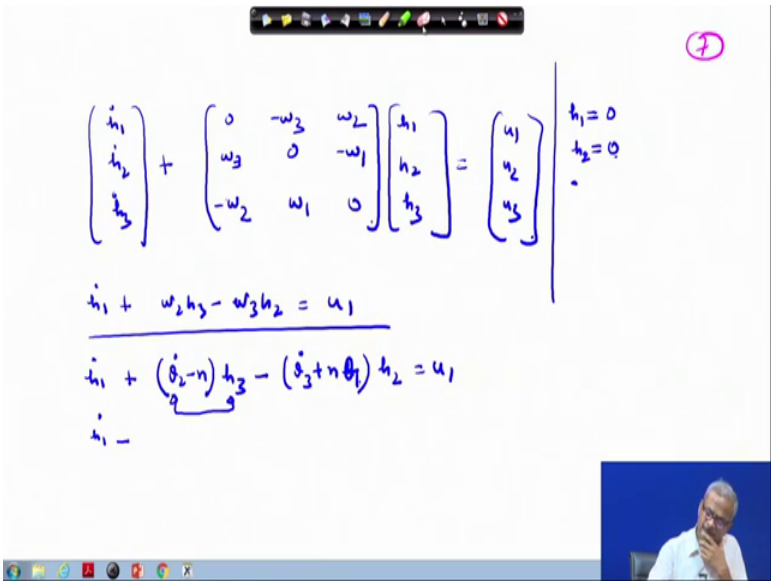

Say h 1 dot h 2 dot and h 3 dot this is for the CMG motion and; obviously, this is a simplified version because we are neglected the internal and the external gimbal mass, if we do not neglect it then the for the double gimbal variable speed control movement gyros whatever the procedure we have followed that procedure needs to be followed and thereafter the control has to be designed, so u 1, u 2 and u 3.

So, just let us look into the h 1 dot, so this becomes omega 2 times h 3 minus omega 3 times h 2 this equal to u 1 I will just do for one rest others you can do it yourself. Omega 2; obviously, this is theta 2 minus n and this is h 3 and omega 3 equal to theta 3 dot plus n times theta 1 times h 2 this equal to u 1.

So, here if you look this quantities are small ok, initially we do not have what are the conditions set that h 1 equal to 0 and similarly we have we can put h 2 equal to 0, this part this is part for the rotor dynamics we will h 1, h 2 these are let me explain you what we are doing, h 3 minus n times h 3 minus theta 3 dot times h 2 minus n times theta one h 2 times u 1.

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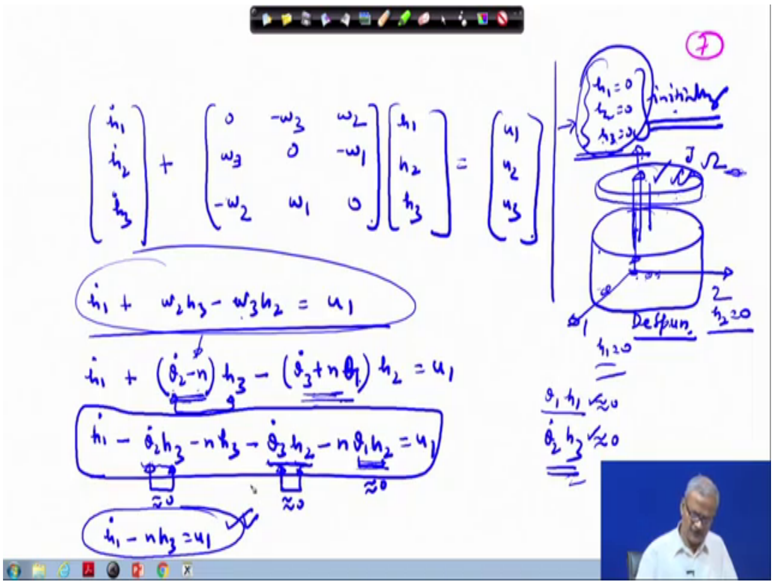

So, this quantity is and this quantity they are small, similarly here theta 2 dot h 3 this 2 quantities are small. So, they will be almost negligible, here also this quantity this will be almost negligible. So, this part will get 0 and we get here h 1 dot minus n times h 3 equal to u 1.

So, here initially your h 1 may be 0, h 2 may be 0, h 3 may be 0 this is the initially it may be, but as you actuate the gimbal ok. So, no longer your h 1, h 2, h 3 will be 0 ok, but the initial part it may be 0 in the gyro state say in the gyro state if there is a rotor and this is the main satellite ok. So, this rotor may be rotating let us say it is rotating and it is J is it is angular momentum of this one. So, in that case this is fixed along certain axis and for that already for the gyros state we have written the equations, so you can utilize this equation and reduce it.

So, for that part your objective will be to keep the satellite oriented along a particular direction ok. So, there it may be required that this part is despun and therefore, this is not rotating at all say accordingly we have to choose their angular momentum. So, let us say this is 1 2, so h 1 may be set to 0, h 2 it's set to 0 along the h 3 direction it's rotation is there, so this will appear if this is the h 3 direction.

So, for the satellite this will add there, but here in this case as you rotate the gimbal ok, so the h 1 h 2 h 3 which are initially 0 they will not be 0, but this together they constitute a small quantities and therefore, they are getting dropped and you are getting this simple linearized equation.

If you are considering the non-linear system, so you have to take this equation as a whole not a linearized one, this is the linearized part you are writing omega 2 equal to theta 2 dot minus n this is the linearized part for omega 2 ok. Similarly for omega 3 this is the linearized one you have replaced this omega 3 by this linearized value.

So, you can follow this notation and then where the product with theta or theta like the theta 1, h 1 this term occur this term occurs or either terms like theta 2 dot h 3 which has which is present here if it occurs. So, this kind of terms will be 0 almost 0, so they are getting dropped out. So, if you do this, so you get the sets of equation we have shown here in this place ok, so; obviously, this is based on using this values.

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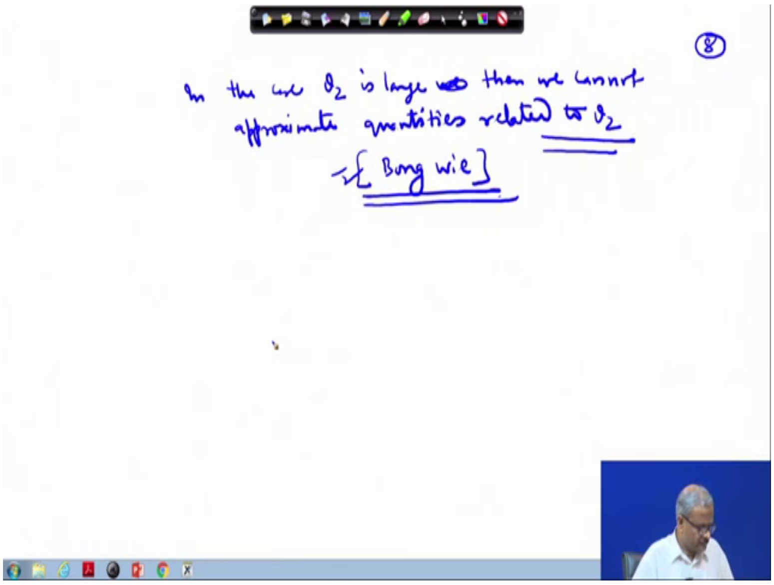

In the case; in the case theta 2 is large, then we cannot quantities related to theta 2. So, for that you have to write equations separately. So, for this part you can refer to Bong Wie ok, these are mechanical exercises which I should not do here ok, this can be done once you have learnt the basics, so you can do it anytime these are all little bit of mathematics required to work it out.

So, we stop here and we will continue in the next lecture.

Thank you very much.