Satellite Attitude Dynamics and Control
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Lecture - 60
Satellite Dynamics with Control Moment Gyro (Contd.)

Welcome to the lecture number 60.

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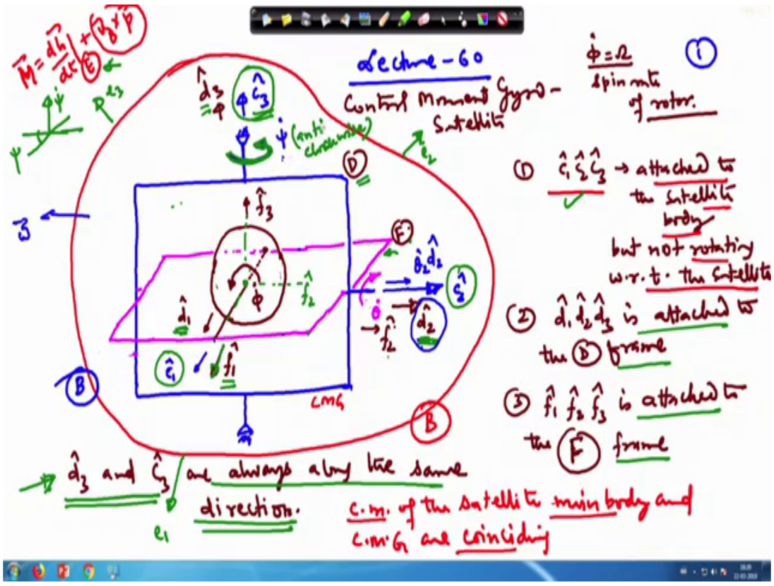

So, we have been discussing about the Control Moment Gyros. So, I have made a this figure for the control moment gyro in an advance. So, what we can see here that satellite is enclosing this control moment gyros and we are assuming that the center of mass of the satellite, main body and center of mass of the control moment gyro both are coinciding. So, center of mass of the satellite and CMG are coinciding satellite main body satellite main body and CMG are coinciding.

And therefore, the equation then becomes very easy to work with because if you remember that whenever we are differentiating this, so, we have got the term like if we have written M, the equation for the external moment, so that has appeared at dh by dt with respect to the E frame and plus v cross p ok. So, v 0 plus v 0 cross p, where p is the total linear momentum of the whole body.

So, unnecessarily this term we do not have to carry if we assume this ok. So, this will simplify little bit our the whole process of working with this and its always possible that you fix your the other components such that the center of mass and the satellite the rest of the satellites and center of mass both of them coincide. So, this we are terming as B and this is your CMG.

So, here in this figure c 1 c 2 c 3 as it is written in here this is attached to the satellite body, but not rotating with respect to the satellite ok. So, it is a fixed in the satellite and there is also the body axis which I have not shown here and say that body axis if I try to show here so that I can point out along this direction. Let us say that this is this are the 3 directions in which your this is your e1 direction body, e2 direction body and e3 direction of body; the small e 1, e2, e3; capital E always we have used for the inertial plane.

So, in this body then your c 1 c 2 and c 3 they are fixed. So, here is your c 1, this is c 2 and c 3. These are the fixed direction, they are not going to change. Thereafter we have chosen a frame which is attached to the frame external frame D ok, which I have shown as d1 ok, d2 which is going here; d 1, d2 and d 3. This is rotating along with the frame D. But there is 1 point to note; d3 and c 3 they are always along the same direction ok. So, this d3 and c 3 which is written here shown here ok. So, they are always along the same direction because this frame w frame shown by blue which is the D frame it is a rotating about this axis its rotating like this.

So, this rotating in anti clockwise direction; anti clockwise. So, this two are not going to they are always in the same direction. So, this way we have defined d 1, d 2, d 3. So, your d1 and d2 will change direction as the angle size changes. If this frame it rotates and comes to this place new place which I have shown your earlier also this is the whole place and this rotates by here this psi angle. So, this you are showing as psi dot ok. So, if it comes to this place you will see that this will change its position and this will also change its position with respect to the satellite body ok. Thereafter we have fixed 1 frame to the inner ring this is your inner ring. So, to this the frame we have fixed such that your along this direction you have f 1; f1 is coming here this is your f1 ok.

So, from this place f1 is being shown from his place then outward this is your f2 which is shown here. So, f2 is here f2 cap and similarly in the upward direction then your f3 is shown. So, this frame is rotating along with the pink frame which is here F frame ok. Now we can start all the things I have written here f1 f2 is attached to the frame F; d1 d2 d3 is attached to frame D and c 1 c 2 c 3 attached to the satellite body and its non rotating frame. It will rotate only along with the satellite, but not with respect to the satellite, this frame.

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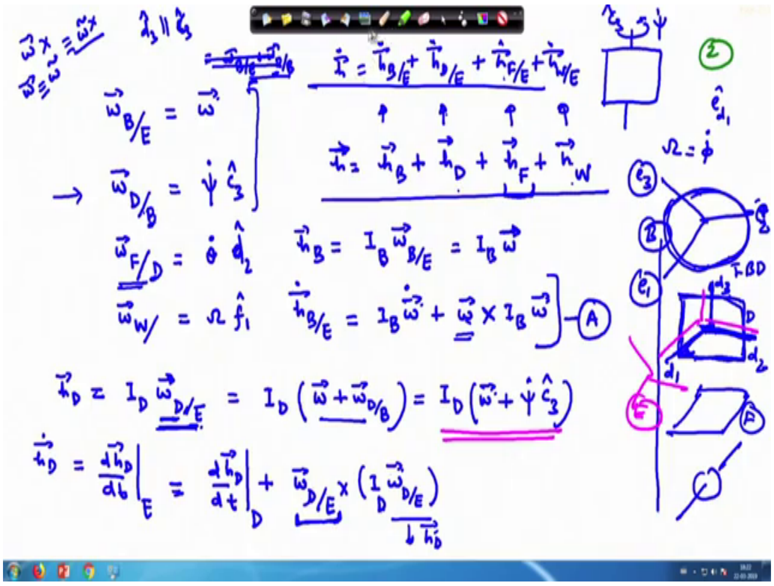

Next we can start composing our problem. So, we will write as omega angular velocity of the main body which is your B this part ok, angular velocity of the moment body with respect to the inertial frame and this we will write as simply as omega. Then angular velocity of the D frame with respect to the; with respect to the B frame because your this is the frame here and this frame is rotating here psi dot. So, and this direction is c 3 cap. So, only this part will appear here ok.

So, D with respect to B frame ok. Thereafter, the angular velocity of the F frame with respect to the D frame that will be theta dot times d2 cap which I have shown here ok; theta dot here is in this direction. This vector goes here in this direction multiplied by the corresponding unit vector. So, I am not using this notation e d cap and something like that because again we have to carry so many subscript another thing.

So, this is easier to comprehend here in this place and thereafter the lastly we have the angular velocity of the wheel with respect to the which frame? This is rotating with respect to the F frame ok, this is attached to the F frame; axle of the wheel it is attached to the F frame ok.

Therefore, this we can write as capital omega. So, capital omega we have written for phi dot which is a spin rate of the wheel. So, capital omega times f1 cap. Now f1 cap will change direction, d2 cap will change direction with respect to the satellite all of them are changing ok. So, with respect to the satellite f1 will change and also this changes with respect to the D frame and so on. So, this procedure we have to carry out ok. Now here the total angular momentum it can be written as the summation of the main body frame and the main body and then the outer frame which we are noted as D and then the outer frame we have written as D and then the F frame and then the angular momentum of the wheel ok.

So, this approach that we are taking here this is showing you these are the absolute angular momentum not a relative one. Just as in the previous lecture, we have done that and then we have combined to get a derivation which was in the sense relative sense with respect to the satellite. Here all these are the absolute angular momentum ok. So, we are making free body diagram of each and every one. This is your satellite this is your outer frame then this is your inner frame and to this attached is the wheel.

So, all of them are the free body diagram FBD. So, for all of them we are writing here like this. Now we have to compose each of the term here. So, the first term h B this will be equal to moment of inertia of the B frame. B frame means it is at the main satellite body which excludes the CMG ok. So, I B times omega B slash E; this is the angular momentum of this and this we are writing as I B times for convenience this will just write as I B times omega. Thereafter we need to also. So, your h dot then will be written as h dot equal to h B dot plus h D dot and all of them with respect to E frame h dot ok. So, you can see that we have written all of them with respect to means we are considering the free body diagram.

So, therefore, h dot B slash E this quantity will be written as I B times omega. So, there is a frame fixed into this which we are writing as the e1 e2 and e 3 this is your body frame attached to the main body this B ok. So, in this body frame the moment of inertia of this main body is not going to change and therefore, we can write it like this; times omega cross I B times omega. I am not putting here dot just 1 necessarily. This I am considering that this part will give you a vector and this is multiplied by a vector. So, it is very simple unnecessarily completing a complicating in terms of the dyadic it will not benefit here in this place.

So, it is a follow this omega cross here omega cross what we have written omega cross this is simply implies this is nothing but omega filled a cross which is a matrix and this omega here this omega this implies that this is nothing but identical to omega tilde equivalent to omega tilde ok. So, this way you equation then gets simplified. So, this is your equation number let us say A. Now we take the next one which is h D; h D now this is your D frame. So, D frame to this we have attached one frame which is d, d2 and d3 and this frame will be rotating with respect to the satellite.

So, in the D frame the moment of inertia because it is a fixed to the outer frame and its rotating along with the outer frame ok. Therefore, moment of inertia of this frame in d1, d2, d3 is not going to change and it makes us easier to write. So, I can write it like I D times omega D cross E. Here see the difference; this is D cross B D this is B. This is with respect to B ok. Here I am writing omega D square E; so this angular velocity of the frame D with respect to frame E which is an inertial frame. So, this can be written as I D then the angular velocity of the main body and this is because it is embedded in this main body. So, we can write this as psi dot. So, or we can write as omega times omega D with respect to the B frame ok.

So, this will be equal to this summation is equal to this part which will write as omega times psi dot c 3 cap because psi dot is lining along the c 3 cap direction ok; c 3 cap and d3 cap they are along the same direction all the time c 3 cap and d3 cap. And therefore, d3 cap can be replaced by this and it makes easier, some of the steps can be skipped. Now we can write here h dot. So, h D dot then it becomes dh D by dt with respect to E frame and this we can then expand as dh D by dt with respect to the D frame and plus omega D slash E cross remember the first derivation we have done.

Omega this is I D times omega D slash E this is nothing, but your the h D term ok. So, this term is your h D. So, this is the way we have written it. You have to particularly take care of the terms while I am writing here this is omega D slash E ok. If we look here in this place this was omega ok. So, this is the difference here which we have to take care of because this is now frame your considering this frame here this particular frame and this is the you are considering with respect to the E frame each of them you are make as a free body diagram and then you are considering ok. So, in that case this is the way you will write ok. Now, h D is given to be this quantity ok. So, we can expand it in the next step.

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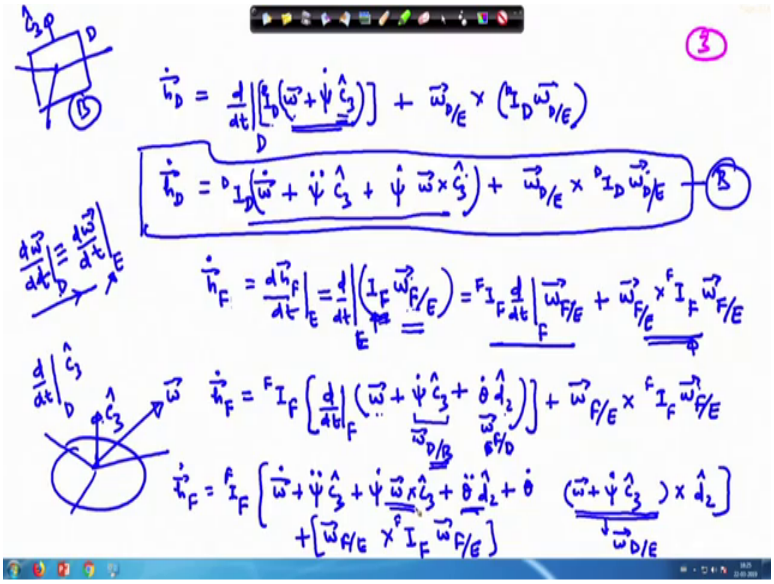

So, we have h dot D dh D by d t. So, this is the quantity h D we have to insert there with respect to the D frame and this is omega plus psi dot times c 3 cap and then I D is also there. So, I D we have to put that the I D is there and this plus omega D E cross I D times omega D slash E ok. Now this part we will expand.

So, in the D frame I D does not change. So, I D will simply come out, omega gets differentiated and this is with respect to. So, your d omega by dt while you are writing here this subscripted D, but this is identically equal to this is what we have done earlier also. This is with respect to basically E frame while we differentiate the angular velocity, so, this goes with the E frame.

So, this way if we write it here in this place. So, I D times omega dot and then plus psi double dot then c 3 cap and then psi dot and this c 3. W what this c 3 how it is going to change; d by dt with respect to the D frame and here is your c 3 cap. So, c 3 cap is a vector which is fixed in the body ok. This is your c 3 cap vector and this body itself this is rotating at the rate omega ok. Therefore, we will write here this is omega times c 3 cap. So, this is the expansion of this term and rest we can copy like this.

So, this is your equation number B and I D is common to all of them. So, I D we can keep it outside here ok. In the next step then we have to take the F frame. So, h dot F and this we have to do with respect to h dot F with respect to the E frame ok. So, h F now we have to pick up; h F is your this frame, the horizontal one which is the inner frame this is the F frame. So, this will be I F times the corresponding movement of inertia. So, omega F this with respect to E; moment of inertia I and then omega F slash E means the velocity angular absolute angular velocity of the frame F with which we are writing with respect to frame E.

Now, this F I F we are writing this is defined in the F frame itself ok. So, F frame we have shown previously this is your F frame. So, along the f1 f2 and f3 vector it will it is defined. So, as this F frame rotates. So, f1 f2 f3 will rotate and therefore, in the F frame the corresponding moment of inertia I F does not change ok. So, this you have to dou with respect to E frame and then we write this as I F times d by dt with respect to the F frame omega F slash E and plus omega F slash E cross ok.

This term we need to expand this is as usual. So, this becomes I F times now d by dt and remember while I am writing here I D, so, this is defined in the D frame itself this is defined in the F frame itself here this is defined in the F frame itself. Putting so many subscript it makes the things complicated. Understanding the control moment gyros complete function is not impossible or very difficult, but we have to take care of all the subscript and other things and rest we have to write for omega F E.

So, omega F E will be consisting of omega first omega is the angular velocity of the main body then the D frame is rotating with respect to the main body. So, therefore, for this we have to keep it here and thereafter your F frame is rotating with respect to the D frame ok. So, that is given by theta dot theta dot times and in the corresponding D frame the corresponding unit vector is the theta dot times d2 cap ok. So, theta dot times d2 cap ok.

So, what this quantity is this is your omega psi dot psi dot v. We have written as D cross B omega D slash B and then there after F with respect to D. So, this is omega D slash B this is omega D slash B and this part is omega this for the D frame with respect to the B frame or with respect this is ok. This is with respect to the B frame and the lastly we have used the notation theta dot D, here this is F frame with respect to the D frame.

So, this is the omega F frame with respect to the D frame. So, this 3 are added to get this omega F slash E. Now we expand this. This quantity will which simply omega dot this quantity will be psi double dot and then c 3. As earlier how this is changing that we have to write here. So, that becomes psi double dot c 3 cap and plus this will change because of the rotation of the D frame, sorry this is fixed in the c 3 frame, sorry what I am stating that this is your D frame and this is along the c 3 cap direction psi dot is along the c 3 cap direction.

So, this is because of the rotation of the main body. Your this quantities c 3 cap will change as we have done earlier. So, therefore, we write here psi dot times omega cross c 3 cap and then we go to the next term. So, this is theta double dot d2 cap and how the theta dot is going to this d2 vector is going to change. So, theta dot cross theta dot times. Now, this theta 2 theta 2 dot times d2 cap. This is a vector which is d2 cap is a vector which is fixed in the this pink frame fixed in the blue frame and it is a pointing here in this direction. So, the question is how we are going to change this part.

So, omega times c 3 cap and then theta double dot is coming here; h F we are taking here ok. Next part we have to write here the proper cross d2 cap and then there bracket will be closed and thereafter this part omega F slash E cross I this F F omega F slash E. So, this part we are keeping as it is and the other part we are expanding ok. So, the question is how your d2 vector is changing. So, d2 vector will change and already the psi vector we have taken care of. Psi vector is changing because of the omega only and d2 will vector will change because of omega and plus psi because d2 is here in this place. So, this body is rotating at omega and also together with this frame is rotating at the psi dot to which your d2 vector is attached ok; d2 the D frame and D frame is rotating at phi dot.

So, we have to take care of these 2 factors. So, this is omega plus psi dot times c 3 cap. This is the angular velocity of the frame, the outer frame ok. So, the outer frame angular velocity. So, this part is nothing but your here this part is ending this part is nothing but your omega frame D with respect to frame E. Here this is not a cross product. Here just we have written it like this. This is theta dot times omega. Omega is the main body angular rate and D frame is rotating, so that we are adding. So, D slash E this becomes. So, omega dot D slash E perhaps we have used for some other notation; D D slash B we have written here. What D slash E we have written ok?

So, if you add these two so your omega B slash E plus omega D slash B that gives you the total omega D with respect to the E frame; omega of D with respect to the E frame. So, this part I am not writing here this is obvious and we can expand it and write it. So, this constitutes your angular the equation for the f1 the frame F and thereafter we have to write the equation for the will. So, you considered how much complication is arising. If you take the exact equation for the CMG; Control Moment Gyros and that too we are assuming that the center of mass of the control moment gyros and rest of the satellite it is a conceding. If you do not assume that then the system will be equation will be further complicated.

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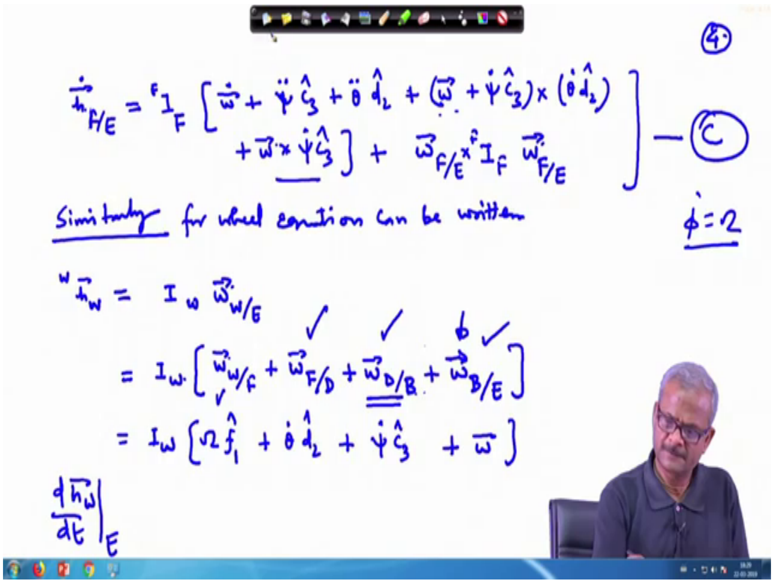

So, we can wind up here. We can write as h F slash E h dot F slash E this we have written as h dot F this part and then the other part. So, I F psi double dot c 3 cap; just rearranging the terms theta dot d2 cap this is this part. So, this part lastly I am writing omega cross this omega cross psi dot c 3 cap and plus I F F and times this cross. So, this is your equation number C. This we could have this term we could have written before this that is fine does not matter. Similarly, we write for similarly for the wheel equation can be written ok.

So, in that case h wheel we are writing that we are writing in the wheel frame itself ok. So, wheel is attached to the frame F which is a rotating frame ok. Omega wheel with respect to E this is the basic equation and omega wheel we can write as angular velocity of the wheel with respect to the F frame plus angular velocity of the F frame with respect to the D frame and then angular velocity of the D frame with respect to the B frame. We are not writing with respect to the C frame because C and C is also fixed in the body and B is also fixed in the body. So, it is a just in matter of orientation ok. It is not rotating and this can be expanded as I w omega D omega D slash B and one more term is there omega B slash E.

So, this is the rotation rate of the angular velocity of the body with respect to the E frame then with respect to the V frame, the outer frame angular velocity with respect to the outer frame and inner frame angular velocity and with respect to the inner frame wheel angular velocity. So, these are the four terms which are appearing here in this place. So, we can write this part is simply your omega dot as per our earlier notation. Omega D slash B this we have written as omega D slash B psi dot c 3 cap. So, this is psi dot c 3 cap and omega F slash D. This is theta dot times d2 cap and this capital omega this is nothing but capital omega along the 1 direction of the frame F.

So, this is capital omega times f1 cap ok. Now we need to differentiate this part again. This is not omega dot this is just omega, dot will come in the next step and here this capital omega which we have replaced phi dot by capital omega; you can keep phi dot as well it does not matter ok. So, the next step involves getting the dot of this means we have to differentiate this dh by dt. Write it in this format with respect to the E frame with respect to the inertial frame how it is changing. So, that we can do it on the next phase.

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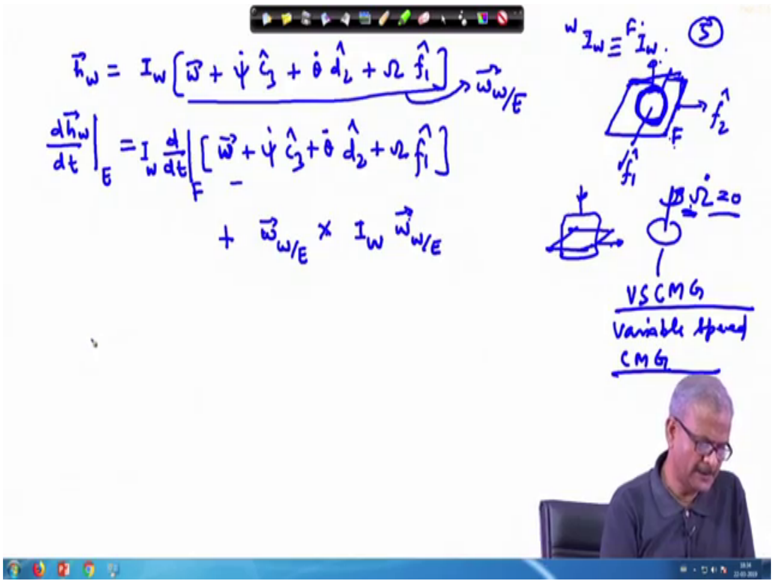

So, I will write here; h wheel this is the absolute angular velocity of the wheel equal to I w and I wheel this is defined in the F frame. So, instead of writing I w like this I can this is equivalent to writing I w in the F because in the F frame the direction of the axial of the wheel it is not changing or neither the movement of inertia of the wheel in changing in the F frame which is the inner frame. In the inner frame is like this ok, it is a coming like this ok.

So, along this direction we have kept as f1, here as f2 cap and f3 cap is going vertically out of the wheel. So, this is the rim of the wheel. So, therefore, in this D frame sorry the F frame which is the inner frame the wheel movement of inertia will not be changing. So, we will describe this I w you can write as I w or either you can write with respect to F, so, both are same.

So, we will put a tag letter on. Let us start with and then we have the other terms. We will just reverse all the terms from we will start with omega. So, this is omega and plus psi dot times c 3 cap and then theta dot times d2 cap we have got and capital omega times f1 cap we have got and we need to differentiate this in order to get the dh by dt of the wheel with respect to E frame.

So, this we will do with respect to first the F frame in which the wheel moment of inertia it is not changing and there after we can go up ok. So, here in this case I am discussing a case where your wheel is speed ok. This is the angular velocity of the wheel, this is also changing. And this particular part is called variable speed control moment gyros; this is variable speed CMG means you are changing also this and this provided 3 degree of control ok. Earlier with if your this wheel. If you cannot speed up this wheel say if omega dot is not present omega dot is 0, so, only 2 controls 2 access controls you are getting.

One along the this axis which is the outer frame and another you can get along this axis ok. If you restrain along this axis you get output here depending, if you are torqueing along this you will get output along this axis, if you torque along this you will get output along this axis.

So, you just have to look into where the resultant control moment will be. So, under this assumption, now we can if we differentiate this; so, we can write here I w outside and then with respect to the F frame and this is omega plus psi dot c 3 cap theta dot d2 cap and capital omega times f1 cap and plus; so, this quantity is your omega wheel with respect to the E frame. So, here this will come as omega wheel with respect to E frame and cross I wheel times omega wheel. This from we need to expand here. Thank you for listening, we will continue in the next lecture.

Thank you very much.