Satellite Attitude Dynamics and Control
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Lecture - 59
Satellite Dynamics with Control Moment Gyro

Welcome to the 59th lecture. We have been discussing about the Satellite with Control Moment Gyros. So, in that context, we were deriving some equation. So, we will continue with that equation.

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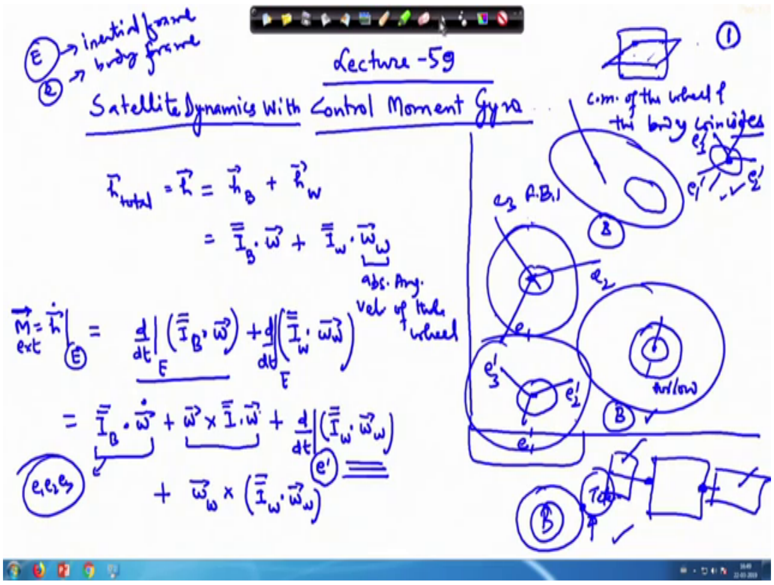

So, if you remember, we wrote h total or this h the total angular momentum of the satellite, we broke into two portion. One is this satellite which we have written by this b and inside, there is a hollow and in this hollow the control moment gyro which is located there and we are showing like this.

So, this part we have written as the hB and the other part we have written as the h cng or here in this case we have written it by hW. And we are assuming that the; this external frames are the internal and external frames they are not present. So, internal frame we showed like this ok.

So, both of these frames are not there, only this part is there or they are massless basically so and because of that we can ignore them. So, in that case, we are just dealing with the wheel. And then we wrote this part as, I body and in the matrix notation say if we write in terms of the this inertia dyadic so, we can write it like this where omega.

And the more over there is one more assumption that if this is the satellite so and suppose this is the hollow ok and in this h o. So, this is the hollow and in this only, you have the wheel located here ok. So, the centre of mass of the wheel and centre of mass of the, this body they are matching ok. So, the centre of mass of the wheel and the body coincide and this is a our mathematical processing. So, now this part we have written as I w wheel dot omega wheel ok. So, this is the absolute angular velocity of the wheel; absolute angular velocity of the wheel ok.

There after we have started with getting h dot. So, we know that if the system is free from the external torque ok. So, in that case we have to set m to 0. But here in this case if we assume that the external torque is there so, m will be equal to h dot and this we are doing with respect to the E frame where E frame throughout our this course, we have just use this for inertial frame. And small e we have used for the body frame; body frame.

So, here we have two bodies; the first one is the main satellite body, which you are showing it by hollow and here your e 1, e 2 and e 3 are located and simultaneously in the same place you can see it show that, you have another frame located which is e 1 prime, e 2 prime and e 3 prime. So, this is your main body and this is a subsidiary body which we have taken it out here and considering them as 2 different bodies.

So, basically we have taken this, the free body diagram FBD; this is a free body diagram. But on here, I am not showing the forces and the moments because they cancel each other and there is no need of going into this. If you so for this part one more problem I want to state here that we can have a body like this and to this another body is attached as in the case of a telescope over a main satellites.

So, this is the main body and here this may be the telescope ok. And this wheel rotate about the hinge or it can be a satellite to which the solar panels have been attached. So, in that case, this is the hinge point; this is the hinge point ok. So, this become the reference point for the secondary body which in this case it is a solar panel and here this is the telescope. So, this for this problem, I am going to upload supplementary material. So, you can go through that we do not have time to present all those mathematics here in this class.

So, taking up this problem here, so this becomes d by dt with respect to the E frame, d by dt with respect to the E frame and this part is pretty simple as we have done earlier. So, we break it into two portions and write like this where this part is this part we have written with respect to the e 1 e 2 e 3 frame. So in that frame the inertia, it is a remaining constant and this is the corresponding term which comes from the transport theorem in mechanics ok.

And the same way we can write for this part also. So, here this is I wheel. Now, but here there is a d frame; if we are writing so, here we have written with respect to the d frame. In this case we have to write it with respect to the e prime frame. Because e prime frame, it is embedded in this ok. Here is your e 1 prime, e 2 prime and e 3 prime. So, it is embedded in the wheel itself ok.

So, it is rotating along with the wheel. So, therefore, as usual we break it into two portion and write like this and plus the other part. So, other part here; this will be omega wheel cross. So, this is the equation we have got and we need to expand it and just work out the solution from here.

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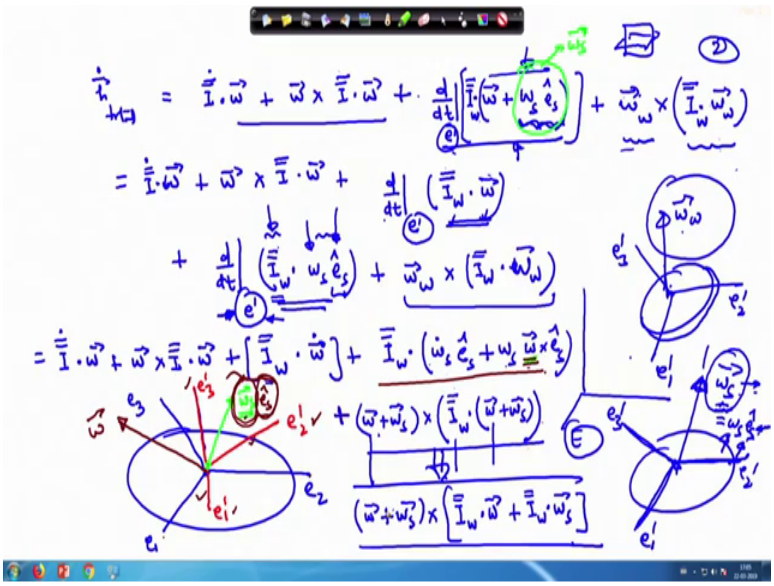

So, therefore, the h dot total this becomes I double dot and plus dot omega plus omega cross and the other term which we are writing here, d by dt and with respect to the this part is with respect to the e prime frame.

So, this is with respect to the e prime frame and omega wheel we can write as omega plus omega s times and the direction in which your momentum vector is located at any time. This angular velocity for spin vector basically, not momentum vector this is the spin vector; the omega w we are writing as omega plus omega spin ok.

So, this part we have broken up as omega s times e s cap. So, this is as per our usual notation we are doing this. Now here we have to process this caustically ok. And of course, the term missing is I W, here the term the outer term we have not written here. So, this should supplement. So, if we break it and this is with respect to e prime frame ok. So, if we differentiate it or rather we will do it later on, this not dot here and this part also we remove it we break into two portions first ok.

This is the first portion here, I double dot; this is the inertia dyadic times this omega and plus omega. Now we are considering this whole equation. So, we have to break into two portions as we are doing this here in this place. So, the third part we have not written here ok. Third part also we should write here in this place itself.

So, I will rub it out and write here this third part and the sorry instead of second part of this, so, following the same line of action as we have done for this part. So, we have omega wheel cross I double bar, this is W dot. So, this is your angular momentum vector and as usual what we have done that in this wheel, your e prime frame is embedded ok. Then here is your the inertial reference frame E. So, with respect to this is rotating at omega w. So, for that part because we are breaking here, we have already done the breaking part here in this place. So, this is fine.

So, this we are then this for this part, we have expanded. So, this part we have expanded here in this place and this part is there ok. So, we will take care of this part in the next step and right here in the next line. This plus d by dt e prime I double bar W dot. Now in the body frame as usual as you know that only this part will change. So, this part we are writing as omega dot. So, one thing we will do here that in this frame like while we are discussing this one so, this one is your e 1 prime, e 2 prime and e 3 prime. So, if we look here in this frame, so your because this is attached to this wheel itself ok. So, in this frame your inertia is not changing and therefore this has been taken out and rest this part we are writing as dot. Now we have to process this part.

So, for this part again because this is with respect to the e frame, but there is a difference that difference, I am going to tell you what does this mean here. This is written here with respect to the e prime. So, once we differentiate this part so, we have there are one term here, there are 3 terms here. This is the first term, this is the second term and this is the third term.

So, we need to take care of all these terms here. So, again in your e prime frame, I W is not changing. So, therefore, this will simply come up and then dot, you have to differentiate this ok. Now this differentiation this was written with respect to e prime frame and this is basically a vector ok. Your omega is a velocity vector and while we write it with respect to the e prime that comes because of the way we have written here, we have started from this place and we have written broken into this portion.

But actually omega is a vector which is changing with respect to your E frame ok. So, this difference you have to make. So this and therefore, this differentiation while we do it, so we simply will right here in this format. This is very important part and based on this your whole calculation is written. And then omega s and then differentiation of this part e s; so e s is a vector at any time. Suppose we are taking this body ok. In this body that anytime your omega is this is the in this frame you are defining ok.

So, this is the omega s vector, this is e 1 prime e 2 prime and e 3 prime. So, this is with respect to the body frame ok. But it so happened that we have the this thing we are writing as omega s times es cap, where es cap at any instant is the vector along this direction, which is the spin direction so, but there is a difference again.

This is the spin we have written if you remember we have written it for the gyros. Here in this case, this vector itself it is a changing with respect to the body axis, means this is no longer a spin vector. But because of the outer and the inner frame rotation ok, this vector is rotating it is a rotating and this vector will rotate.

So, one part of this is your scalar part, another part is the vector part, which is the unit vector ok. So, this unit vector it will rotate with respect to the body frame and therefore, you will write here, omega cross es cap because this es vector, it is a embedded in this frame. e 1, e 1, e 1, prime e 2 prime, e 3 prime frame which is we have return this part with respect to the body frame ok.

So, therefore, how your body frame is rotating, let me explain you in a separate figure. Let us see here, I have a main body and e 1, e 2, e 3; this is the body frame and then I am telling that e 1 prime, e 2 prime e 1 prime, e 2 prime and e 3 prime. This is another frame ok which rotates with respect to the body ok. And that rate we have indicated as omega s ok.

So, this omega s we have broken here in this portion, this part is your omega s ok. So, how your omega s, at what rate it is going to change? The rate of change of this vector is at the rate omega, because your the blue frame which is your shown by this e 1, e 2, e 3 cap ok this is rotating at the rate omega. And in this frame, there is a vector in which direction is e s cap ok, and which is embedded in this frame, your omega s is a vector. You can write in terms of the components of e 1 prime, e 2 prime and e 3 prime. And this frame itself is rotating at this rate ok.

So, at this frame the rate frame is rotating, at the omega s times es cap this is the angular velocity of this frame. And with respect that is with respect to the e 1, e 2 and e 3 frame. So, this is rotating with respect to this frame.

So, therefore, we have added omega plus, omega s and written this as the absolute angular velocity of the wheel. So, in therefore, for this region your es cap which is the unit vector along the omega s direction, this is the es cap this will rotate at the rate omega. So, this part is very important if you miss this part the whole thing will be wrong. And this concept, we are going to use for deriving complete equation for the control movement gyros.

So, this term we have expanded and then we have extra terms, which are due to this one ok. So, this also we break as omega this one is pretty simple we do not have to worry much about this. So, this is omega plus omega s cross ok. Now we need to expand this and expand and let us look into this what the result we are going to get.

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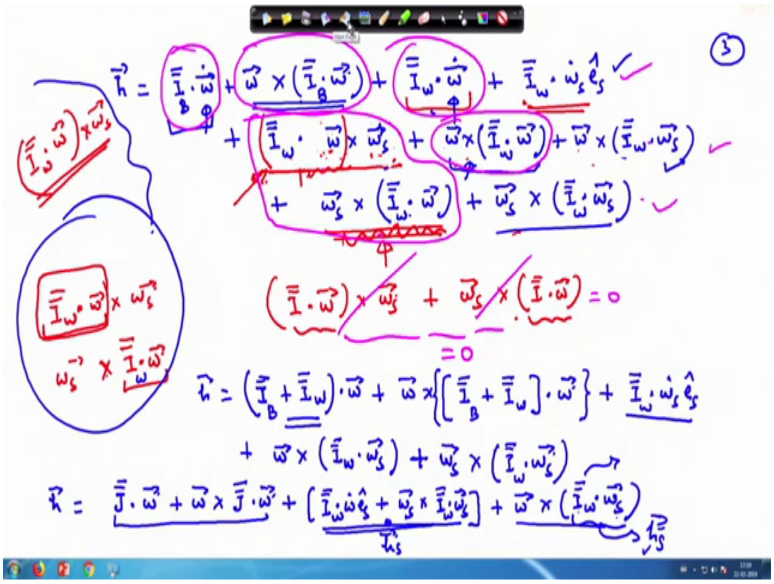

So, we have here h total this will be I double bar dot omega plus and plus times omega dot and then the other terms, omega s times omega cross e s. So, what we will do that this part we will take inside.

Here omega s will combine with es and write it together. So, this way we can write this as omega and then cross omega s times e s cap ok. So, this part we are going to replace and write simply as omega s. So, this part we will replace here, and write as omega s.

So, this part is over I W dot omega cross omega s and then this part we have to expand. So, if we expand that part. So, that becomes omega the first term is omega, then omega and we pick up this term and here this term and this will come together. So, if we multiply it, let us write this here in this place itself; I double bar W dot omega and then this is this part is plus I W double bar dot omega s and this we have to take cross product with omega cross omega s.

So, this part is this particular part is written here and this we have to expand we will get a total of four terms and this is plus ok. Now let us check the terms, I double bar, omega dot es we have taken I double bar omega s cross ok. This term we are here and the corresponding one more term we will get, this term and this term we will explore this 2 omega cross.

We have to be careful in writing a particular part here right. Once we are breaking this. So, basically we have to work little carefully. This part we have combined together and this we have written here, as one of the term. The first term we have got here this was the this term and there after this term we have written to the next, this is present here ok.

So, difficulty in copying from the previous page and then we have the next term. So, next term is I W dot omega s I W dot omega omega times omega s and then omega cross omega cross I w times omega s. So, I W times omega s. So, what the care we have to take care of that we have to choose the terms like this term is appearing here.

So, preference is given to this product first. So, we are not going to take the cross product first this, you are multiplying while we are writing it in a dyadic form and say while we write in the dyadic form I W, double dot and then we define this as a particular one term on that anything is best.

So, here this term times omega s cross it appears like this, not that this 2 will be combined together ok. So, now if you follow this procedure, so, this what we have written here as the bracket this should not be done rather we have to put it like this ok. So, once you do like this and then compare the terms, here this is I double dot w times omega and omega s omega s somewhere is here in the front and w dot. So, not this term. So, we compare now this 2 terms.

So, this term and this term if we compare omega and then cross omega s and from this place if you right here omega s cross. So, this is a vector this also a vector. So, you can see that there order has been reversed and therefore, they cancel each other and this makes it 0.

So, therefore, these 2 terms we are going to get rid of, and then rearrange the things. So, we are therefore, h becomes I double bar and you can see that omega dot is here and there is also omega dot here. So, we combine this terms and plus omega cross and then pick up a single term from this place, where the omega omega is involved.

So, this is present here in this place. So, we pick up that part. So, this becomes I w this is your body part. Somewhere we have not written body part only I we have written ok. Here we have written I B, so there after we have missed out that part. So, this is your I B ok. Therefore, this also this is I B this is I B this will kept under separate notation so that we can keep track of the things. So, this is I B I B and on the next page this is I B and this is also I B. So, this term and this term we have combined and this term and the another term which is omega this place, this two terms we are combining. So, if we combine, this is the I B here also this is I B ok.

So, this way we have covered this term we have taken care of this term is over this term is over and then these two terms are cancelling out therefore, then it not be considered ok. So, this quantity is already 0 and thereafter we have taken care of this term ok. What we are left with 1, 2 and 3 this one, this one and this one; these are the 3 extra terms that we have to take care of ok.

So, plus omega cross I w dot omega s plus; omega times, this term we have copied here this term we have to copy, cross ok. And then lastly this term which is present here this term together we can write as J; the moment of inertia of the whole body ok. Now we combined together I double dot e s cap and plus 1 more term will come here.

So, here we want to put it in a format, omega s cross I double bar dot omega s. Let us write it here and then I will explain you what I am trying to do. This I am trying to explain you 1 very fundamental aspect of the control moment gyros without this you will not be able to do ok. So, once we have written here in this format ok. And this part we have separated out. So, what does this mean, let us let me explain this.

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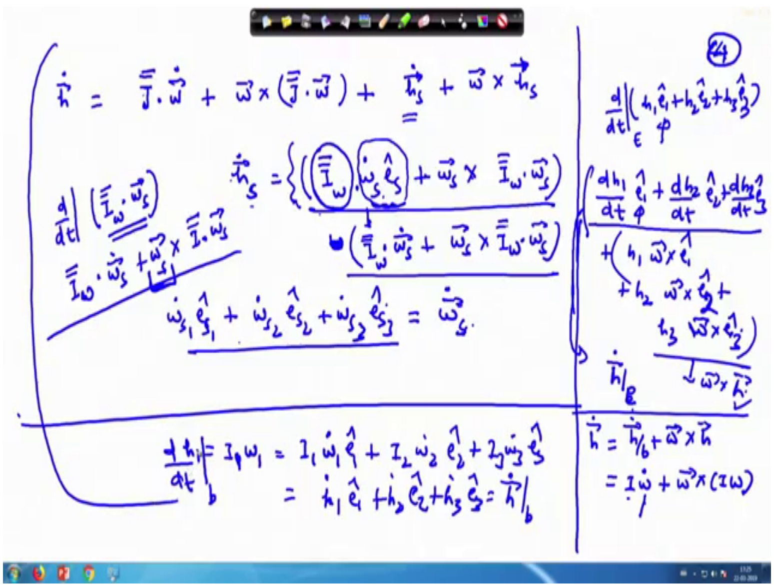

So, h dot this equal to the part which is in the bracket this part we can write this as h s, this we can write as hs dot ok. So, here we write this as hs dot and plus omega cross here look this part this is nothing but your hs. This is the angle angular this is the moment of inertia or moment of inertia dyadic of the wheel and this is the corresponding speed at which it is rotating with respect to the body.

So, with respect to the body then we are writing this hs, so, omega cross hs. Now let us look into this part hs dot which we have written as I double dot times omega I double bar wheel times, this is wheel these are all wheel, these are all wheel ok.

So, these are the wheel terms ok. So, this term I am going to explain you, I w double dot times omega s dot times, e s cap plus omega s cross omega s. Now look into this term what this is. The first term here this is I double bar w dot and this quantity if you remember that once we differentiated this h 1 e 1 cap, s 2 e 2 cap and s 3 e 3 cap this quantity with respect to the e frame.

So, in the e frame then what we are doing we are writing as dh 1 by dt times e 1 cap, dh 2 by dt times e 2 cap and dh 3 by dt times e 3 cap and plus h 1 times omega, cross e 1 cap plus h 2 times omega, cross e 3 cap, h 3 times omega cross e 2 cap and e 3 cap.

So, this part if you remember that this appears as h dot ok, but this is with respect to the in that case we have done with respect to the body frame, or the e frame ok. So, whenever we have done this exercise. So, h dot we have written as h dot with respect to the body and plus omega cross h and this part we have written as I times omega dot because in that frame I does not change and this part it remains I omega in the matrix notation this is what we have followed.

So, if you follow that notation. So, this part is your moment of inertia term ok. And this part is nothing but your omega dot term omega dot term in the sense that this is say if omega is 1 we write it like this, times e 1 cap, plus e s 1 cap and omega s 2 times dot times e s 2 cap and omega s 3 dot times e s 3 cap ok. And this we write as omega s dot ok. So, this is appearing in the same way as I am writing here ok.

So, for that place one more step, I will take you to explain this because this oe s 1 is I 1 times say the omega 1. And once you are differentiating this with respect to the body frame, so, what quantity you are going to reduce? This is I 1 times omega 1 dot and obviously, there after your e 1 term is there. So, that e 1 term here what we have written?

So, this e 1 term we are keeping it here and finally, the same kind of terms we are adding up e 2 cap, I 3 times omega 3 dot times e 3 cap and this we are writing as h 1 dot e 1 cap, plus h 2 dot e 2 cap, plus h 3 dot e 3 cap and this is nothing but your we have written as h dot. And that is with respect to the body frame. So, the same notation I am following here in this place. So, if we go back I hope that this is clear because, I am not available to interact with you at this stage, but this is a straightforward and this part gives rise to simply here the omega cross the h part ok.

So, this part is your omega dot s. So, this can be written as omega I can write it like this, and this part already we have written like this. So, what we have doing? This differentiation while we are trying to do we are trying to write it in this format ok. We have written here the hs date for this whole thing. So, what we have done? We have broken into two portions; d by dt and we are hs we are writing as I W dot omega s ok and then just differentiating it.

So, this part is written like this and omega s dot and plus omega cross omega s times, omega I dot times. So, every place we have to notice that where what we are using. Here we are using omega s. If you look back here we have while expanding in this place.

Let us say on the previous page it may be d by dt here say this is omega w we have written. So, where I am writing what? You should be very much clear, without this going into CMG it is a difficult ok. So, this whole part I will wind up on the next page.

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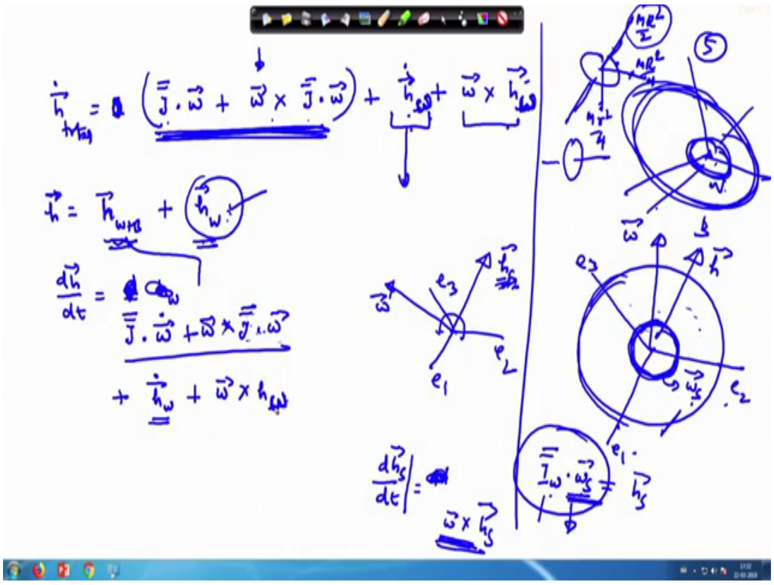

So, we have here h dot total this becomes equal to j double bar dot omega, plus omega cross j double bar dot omega and this constitutes your satellite part along with the CMG. CMG is also there and this is the main body is also there and this is the wheel. So, that is also present. So, those two things are combined together and the other part we are writing as h s dot plus omega this part we have combined here. So, other part omega times hs ok. So, this is omega times hs.

So, this part was important where now here also ponder over this point. While we write like this ok, so, what does this mean? This is the whole satellite I have taken ok. So, and here this omega s present ok; that means, this satellite is rotating this is say the, this is my satellite and there is the wheel which I will just showing by a circle. This is your body frame e 1, e 2 and e 3.

So, the combined moment of inertia ok, this has been taken care here in this place, but it so happens that your combined moment of inertia that gets affected because of the orientation of the this wheel. This wheel will be rotating like this sometimes it will become like this ok. So, you can see that the moment of inertia along the body axis then it will not remain the same, its moment inertia along this direction is different and while in this 2 direction it is a same ok.

So, if it is a wheel. So, you know that this is mr square divided by 4 mr square divided by 4 and this is mr square divided by 2. If it is a ring then this quantity becomes mr square ok. So, for that reason you have already taken care of the wheel along with the satellite and this whole system is rotating at the rate of omega.

Now, with respect to this system wheel is rotating at omega s ok. So, that I have to take care of ok. So, that omega s appears as. So, if you multiply this I w dot omega s, so, this shows the angular momentum of this wheel with respect to the body axis or in the body axis because this is with respect to the body axis which is e 1 e 2 and e 3. And therefore, if you write this as hs ok, so, rate of change of hs how we are going to write? So; obviously, we have the d one term will appear like this and another term we have to write is omega cross hs because your wheel is embedded in this frame.

So, consider that this whole wheel is replaced by a h vector. So, this is your e 1 e 2 and e 3 vector and this whole thing has been replaced by this h vector ok. So, this h vector rotates at the angular velocity omega this is this is rotating at angular velocity omega. So, this is omega cross h s. Beside this in its own frame h is also this hs is also changed which is taken care here in this place ok. And this we have already done. So, if you remember that we wrote for the h as h of the whole satellite including the wheel. So, this is w plus b first we have written and the one part we have written only for the wheel which is rotating with respect to the body and then we just differentiated it with respect to time.

So, this part we have written as d by dt. So, directly I can write here. If the total moment of inertia we write at J, so this part we were writing as J times omega dot and this appears as omega cross h which is h w plus b which is nothing, but here in this case J double J double bar J double bar dot omega. So, this part is taken care of here and the other part which you differentiate. So, that part you have to write separately.

So, that part we wrote as just as omega w dot ok. So, in the gyrostat this is what you have done. This is wheel this is h w wheel ok. So, this is the whole purpose of discussing this of course, little it is a looks haphazard, but we will do the this derivation completely in the next session. So, we will stop here and then continue with the control moment gyros deriving its complete equation in the next session.

Thank you very much.