Satellite Attitude Dynamics and Control
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Lecture - 58
Control Moment Gyro (Contd.)

Welcome to the 58th lecture. So, we have been discussing about the Control Movement Gyros. So, we have looked into the its working principle and that was a pretty simplified one.

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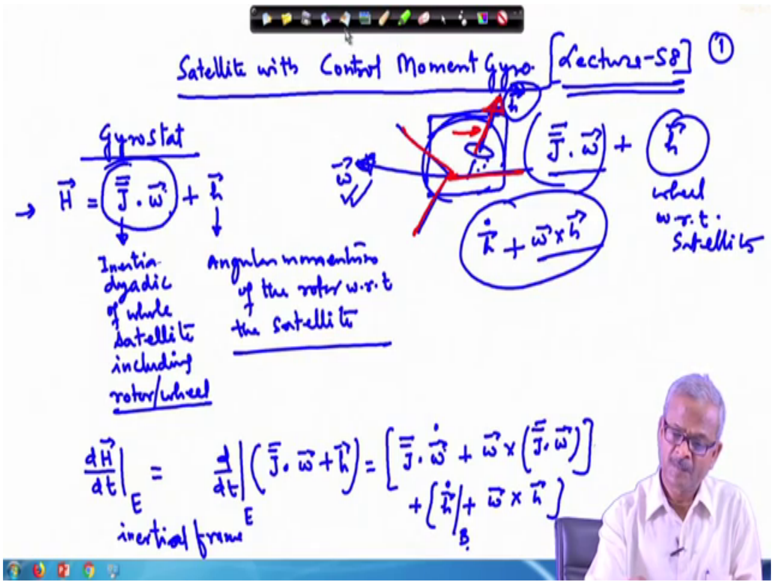

So, now we will do some mathematics and see the difference between the already I have through the figure; I have shown you difference between the control movement gyros and the basic gyros state.

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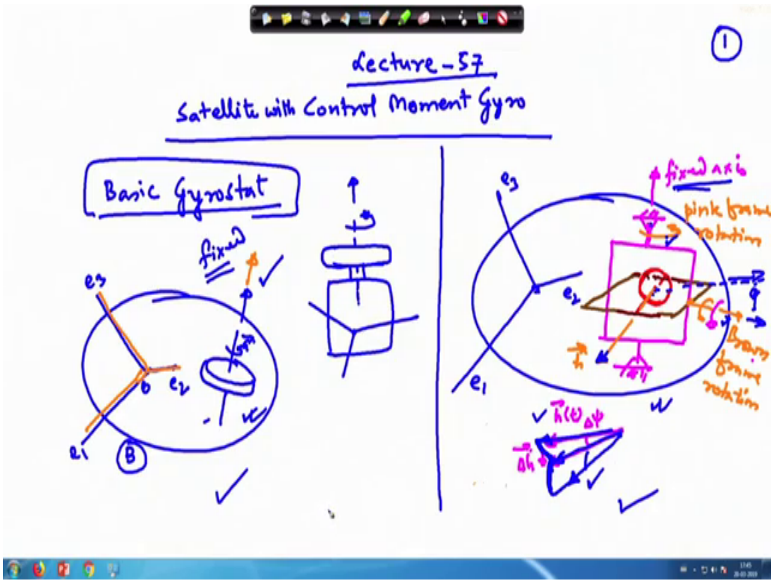

So, this is basic gyro state and this is the control movement gyros. So, how do they differ; and this part is very important which we have to keep in mind. So, if you remember for the gyro state, the angular momentum which we have written as J double bar dot omega and plus h; where this is the angular momentum. This moment of inertia, this part is moment of inertia; moment of inertia dyadic. So, this is inertia dyadic of the whole satellite, including the rotor wheel or simply the rotors wheel.

Reaction wheel and this rotor which are the momentum storage devices or momentum transfer devices, we use it in separate sense we will discuss that this we will leave it for the time being. So, inertia dyadic for the whole satellite this relationship we have derived and this is the angular momentum and therefore, this is the angular momentum of the whole satellite ok. But inside the there is a device which is rotating. So, this is the angular momentum; momentum of the rotor with respect to the satellite.

So, that gives you the total angular momentum. So, this is a satellite and there is a rotor here and suppose that this rotor is not rotating. So, what will be the moment of inertia of this which is we are writing as J double dot ok. And then if this body is rotating at omega angular velocity. So, this is your angular associated angular momentum. In this case we are assuming that this wheel is not rotating, if the wheel is rotating with respect to the satellite.

So, you need to add further one part. So, this is one part and another part this is for the wheel with respect to the satellite. So, the same equation is written here. And therefore, if I take the derivative of this dH dt, with respect to the E frame this is your inertial frame ok. So, we can write it as d by dt and breaking it this is with respect to the body axis ok. In the body at this moment of inertia of the whole satellite will not change and then omega cross, this is one part and for this part, then you have similarly h dot and plus omega cross h. So; that means, what we are doing that this is rotating with respect to the satellites.

So, this is suppose its angular momentum vector. So, as the satellite rotates at this omega ok. So, therefore, because of this omega cross h, the change will take place and another change we have to write, if the wheel itself is speeding on this axis. So, that you have to write this part ok. So, this is for the wheel speeding with respect to the body axis. Because here it is succeeds in the body axis; its direction is fixed ok. And thereafter what we did that we rearrange the terms and re-wrote the whole thing. So, go to the next page.

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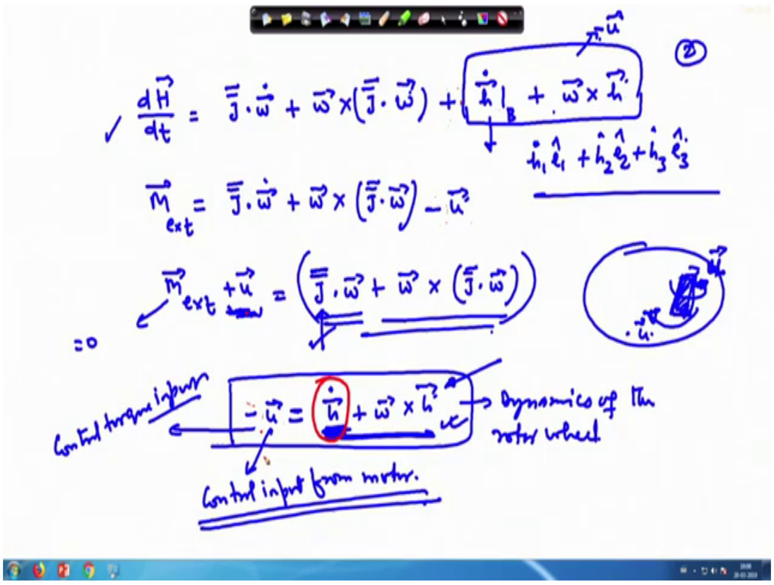

So, we have dH by dt this is equal to omega dot plus, omega cross h dot this is with respect to the body frame and plus omega cross h. So, this h dot implies h 1 dot e 1 cap, plus h 2 dot e 2 cap, plus h 3 dot e 3 cap, this is what it means. And we have written it in the other format also. So, if we try to write this site as the torque acting on the system M. So, this is M suppose we write this as a M external. So, M external this term then you will have J times omega dot plus, omega cross and this whole term from here to here; this term we can write as say u. So, or u or minus u whatever we can write both will be equally valid.

So, let us make it minus u here in this place. So, if we write here minus u. So, minus u will be present here in this place and we can bring it on the left hand side. So, in this contest how it is appearing, that this is your whole satellite J double dot this includes J double bar which is inertia dyadic, it includes also the wheel inertia. So, this as this whole satellite is rotating for this you have written this equation, but what is the modification if this quantity is 0 it is free from the external torque. So, your the internal system you are using and you are writing as minus u and you have taken here on the left hand side.

So, this minus u, from that places h dot with respect to the body axis which I will drop that notation and simply write this as omega cross h. So, this defines the dynamics of the rotor wheel and here this can be your control input from motor, control input from motor. So, this is your basically in control notation this is simply control torque or the control input and as a result you are getting the other side it is a changing ok. And the so if you apply a torque on some system ok; if you are applying this torque let us say this is the, this quantity we are writing as minus u so; obviously, the main body will also feel a torque in just in the opposite direction.

So, applying a torque or either you are writing this as the u here ok. So, just opposite of that any how if you change the sign if this is if you are writing this is minus u I will show this as u if you write this as plus minus u, so here the plus sign will come. So, by controlling your rotor you will be able to this is the control input or the torque applied to the rotor. So, if you apply this torque the rotor will change like this and the same torque will also be transferred to the in the opposite direction to the main body ok. Which also includes your the router ok, which is here in this place this also includes the rotor.

So, this way you will be able to control the attitude of the satellite. So, this is using that is just one wheel here shown and for the multiple wheels what we have done, that here in this equation we have just inserted a summation sign, here we inserted a summation sign we wrote it as h I and this as h I if you remember. So, this will be the summation with u i ok. Here this will be the summation with u i and then you can break it along the 3 axis then you can get the corresponding solution ok. So, you see the how the changes are applied.

So, this part I am deleting from this place not to confuse you, if you have multiple rotors along the different types. So, just put a summation there and used that to solve your problem ok. So, this is about the gyro state now we are going into and looking for the control moment gyro. So, how they are differing ok, in the case of the gyro state your this direction was fixed with respect to the body axis this direction we have kept it fixed because it is fixed inside the body axis. So, with respect to this axis this remains fixed this one will remain fixed; however, in the case of the control movement gyro satellite.

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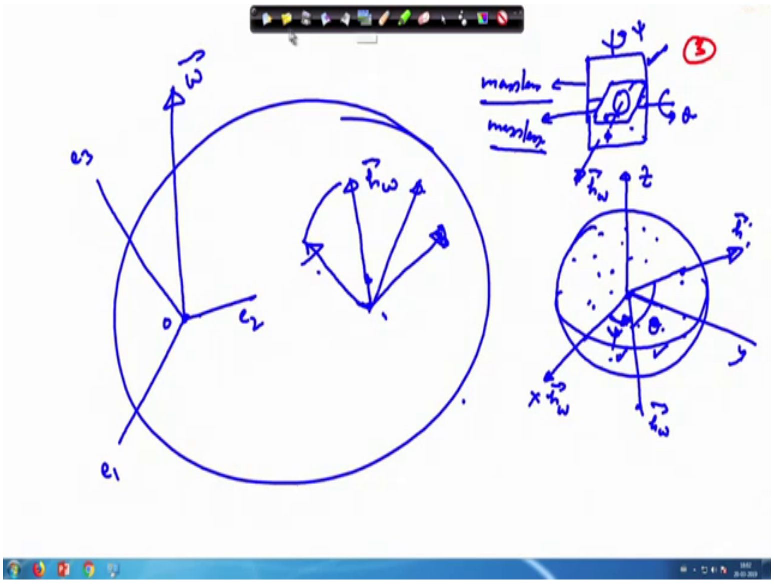

So, you have the satellite here ok. And as you have seen that your rotor is changing its direction. So, because of your the arrangement you are using the outer frame in the especially in the double gimbal, control moment gyros this is the outer frame and then we have the disk here in this place the outer frame inner frame and this disk. So, because this can rotate by psi here ok, and this can rotate about this by theta. So, psi and theta and this rotation about its own axis we have shown it by phi.

So, the angular momentum of this rotor disc which you are writing as hw; this wheel it can be oriented on the surface of a sphere anywhere how we are doing on the sphere, for suppose this is my reference axis here x axis. So, from here I measure any angle this is the y axis and vertically this is the z axis. So, starting from this place I show it by psi this angle ok. This we have looked while discussing into the orientation problem right in the beginning of this course ok. So, if each vector suppose say each wheel is directed along this line. So, this will come to this position once you have rotated from this position to this, and there after I can rotate it from this position and bring it on the surface of the sphere anywhere, is can go and this angle, let us say, this angle theta here in this case.

So, h vector comes here in this place. So, by a rotating wise psi and theta I can point this each vector on the surface of a sphere in any place ok. You can go to any place. So, the h victor, if I show this as the my body frame e 1, e 2 and e 3 ok. So, your h vector here, which is the pointed by the rotor and the frame we are assuming to be the mass less this frame and this frame is a massless these are not mass less they are having masses ok. So, for that the detailed calculations are done analytical and there after the it is a computed.

So, that way if your gyroscope suppose; it is a located here in this place itself or it will be located somewhere else. So, this h vector which is we have shown in the previous case as this one. This h vector, which is related to the wheel rotation ok; I will right as h w here. So, this can rotate with respect to this body anywhere this body itself is rotating at the rate omega, but this wheel also it can rotate, it can get oriented here in this direction it can go here in this direction it can keep changing its direction because of this psi and theta ok.

So, there is a difference between the gyro state and this control moment gyro satellites ok. And because of this the problem gets complicated and if you take the frame masses this is outer frame mass and the inner frame mass then the situation is much more complicated.

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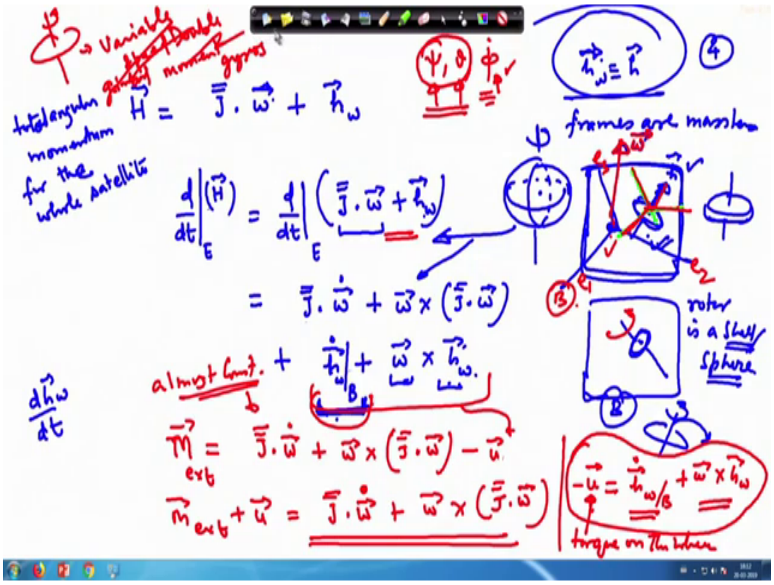

So, let us again write this is the total angular momentum, for the whole satellite. So, from the previous our discussion is the frame is mass less. So, we can write this in the terms of h wheel ok, h wheel or h they are same h wheel is identically h ok. Because there is the you are assuming frames to be mass less frames are mass less and therefore, only this one will count. So, either by write here h w or h it is the same.

Now, if I do the differentiation with respect to the E frame ok, that is with respect to the inertial frame, d by dt with respect to e frame. So, we will have on the right hand side d by dt, and following our earlier discussion for the main body you have J times w dot. Now remember this if this is the satellite and this is the rotor right now its oriented like this now after sometime the same rotor it gets oriented here in this direction.

So, until and unless this is an sphere this rotor is an sphere this J will also not remain constant. So, if I am writing like this. So, I am assuming here the rotor is a sphere rotor is a shell or a sphere a shell or a sphere and only then this is applicable otherwise; the moment of inertia of this will keep changing the satellite will keep changing as the rotor tilts here and there in other direction. So for simplicity it is assumed that this does not change ok. And it can be the case where it is a shell or either a sphere.

So, instead of wheel means instead of a disc you are using a sphere which looks like this. So, this is a sphere and its rotating on this axis. So, what about the way I tilts its moment of inertia it does not change ok. Along any axis bicot whichever the 3 orthogonal direction you choose for a sphere or shell the they remain the principal axis direction and therefore, it does not make to the difference to the moment of inertia of the whole body.

So, for this case this is applicable what we are discussing here. So, this when we can write as omega cross J double dot omega plus this part, so ds by dt. So, this the angular momentum of the this body this wheel with respect to this is the wheel; so, this angular momentum of this wheel with respect to the body ok.

So, if we write it in this way omega cross this is the h dot plus. So, this is dot is with respect to the body ok. This is the main body here, and then omega cross h wheel. So, the term h w it is appearing h is appearing here in this place also, but there is a difference between this h dot and the h dot which is present here in the previous case your h dot can be only by speeding of the wheel ok. While here in this case this is also tilting ok.

This omega; so what you are doing exactly that there is a vector inside this is a suppose h vector. So, as your body frame rotates with angular velocity omega. So, this h also changes. So, for that you have written this omega comes cross h w, but beside that. So, this h w it changes because of the rotation of the whole satellite, but in addition to this your this access itself is tilting suppose if I fix another axis here in this place I fix here another axis ok.

So, if I fix another axis here in this place. So, this axis is rotating with respect to this body axis this is your body axis, e 1, e 2 and e 3. And this body axis itself its rotating at the angular velocity omega ok. And inside this body axis you have the rotor here to which I have attaching another frame and this frame is also rotating with respect to this body axis means that double rotation is involved the body itself is rotating and with respect to the body also the wheel is or the your shell this is rotating. So, this case, this term here and this term here they are different they are not the same and this difference you should note and this is very important part ok.

So, if you we use this and then we write it here in this way the torque the M is the torque external torque acting on the system and of course, here we are assuming that this is not changing much and therefore, this is remaining for almost constant almost constant. So, we write it like this and then omega cross J double dot omega and this quantity we write as here as minus u. So, you can see that M external plus u omega cross this is a very simplified model so; that means, you have the other term minus u this equal to h dot wheel with respect to the body axis and plus omega cross hw.

So, this is the torque that you apply on the wheel. So, as a result the angular momentum of the wheel it will change its direction ok. In addition we can have the wheel also a speeding of this on its own axis. So, if that happens that we called as the variable, speed control moment gyros and if it is a double gimbal. So, variable speed double gimbal control moment gyros variable speed, this is double gimbal control moment gyros, we dont have a space here. So, we will come to that topic again. So, what I was stating that in addition if your wheel is also speeding on its own axis.

So; that means, you have 3 degree of freedom, one is coming from psi, another is coming from theta and another will be for phi. Already it is rotating at the rate phi dot ok, but phi dot you can speed up. So, that constitute another degree of freedom and this can be used to produce the 3 access control torque while here in this case which psi and theta only if they are varying you can produce only 2 axis control torque.

So, these are the satellites involved with the control moment gyros. So, this is. So, we are going to discuss afterwards for the time being this is the situation. So, using this u here then the control can be planned for the control moment gyros. Now, the same problem what we have done here we look through some other perspective this is omega dot.

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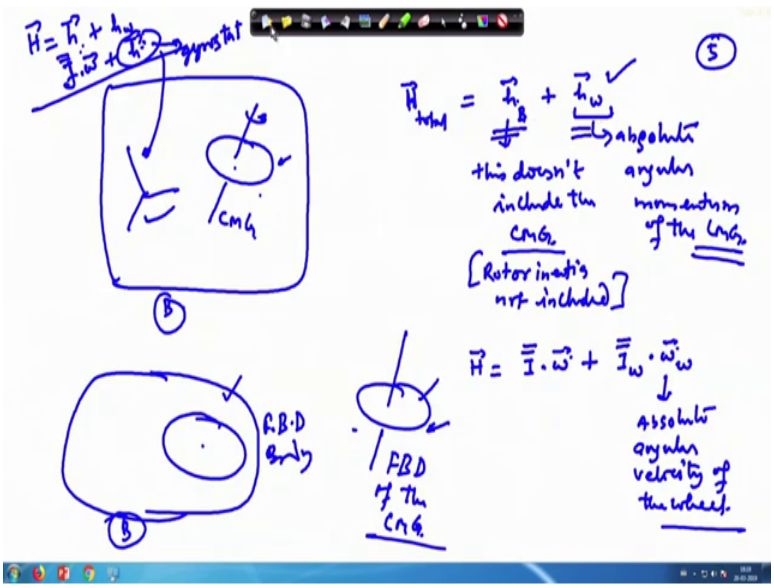

It is a very important to discuss this part without this it will be incomplete anyway let us start say this is my main body and here this is my control moment gyros whose axis can tilt. So, this is your CMG and this is the main body. So, the angular momentum h this is h total angular momentum I can write in the h angular momentum of the main body and plus angular momentum of the field how does what is the difference here I will put a notation here actually this is the absolute angular momentum. So, I need to make it sure that it different from the previous one.

Because here, I am not writing the whole body this is not only for this not include this does not include the CMG. So, this is CMG separately. So, the absolute angular momentum of the CMG, so you can see the difference in the case of the gyro state what we did that we took the angular momentum of the whole system as the angular momentum of the whole satellite including the gyro state or the rotor. So, there the rotor mass was also included here rotor mass is not included.

So, rotor inertia is not included rotor inertia not included. So, this is being considered separately here in this place; that means, we are taking this satellite the body and there is a hollow inside in which you have the CMG located here. So, this is free body diagram FBD of the body and here there is your FBD of the of this CMG ok. And this two angular momentum we are counting separately.

So, this is absolute angular momentum while in the previous case for the gyro state what we have done we have taken the angular momentum of the whole satellite including the CMG and plus with respect to the this satellite this is rotating on the axis. So, that we included separately and that we have written as h w. So, that part we wrote at J double bar omega M plus h ok. So, this is with respect to the body axis with respect to this axis in the case of the gyro state.

So, this is basic difference we have already discussed now here in this case you see that I have separated this part and this part here. So, your h total this total I will drop here. So, this can be written as I double bar dot omega this does not include the this wheel and plus the this is the wheel inertia of the wheel dot omega wheel. So, this is the absolute angular velocity of the wheel this is the absolute angular velocity of the wheel.

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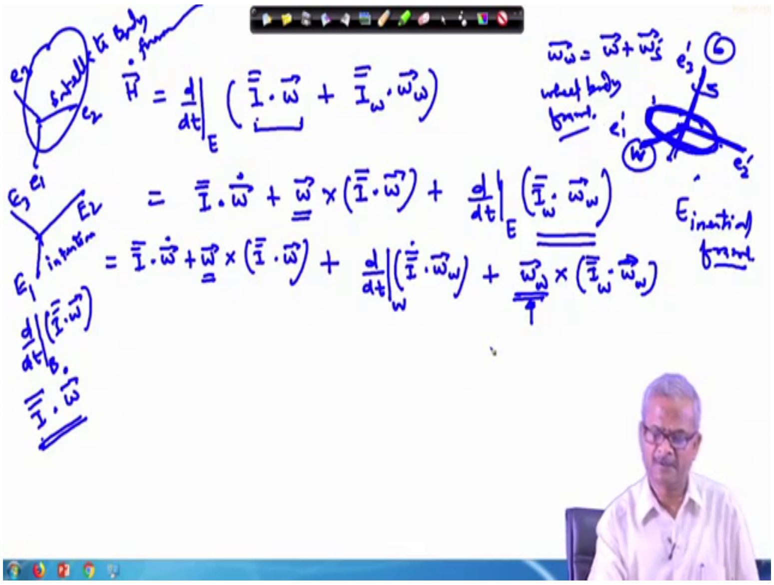

So, h dot means d by dt with respect to the E frame we have to work with here omega w this equal to omega plus omega s. So, once we differentiate this part as usual for this part and we are assuming that as a rotation of the wheel the I does not change ok. So, once the CMG is working. So, in this part there is no question of that because it a separately we are considering. So, there is no need of that assumption. So, just remove this part ok. So, I times w dot ok.

So, this with respect to the body part we can take this into d by dt with respect to the body axis. So, this I double bar comes out this inertia dyadic and we get here I times w dot. This is the dot product here and them omega cross and plus for this part. So, s by dt with respect to e now this part we can write here as d by d t with respect to the body axis here. In this case this body axis will because we are considering the this frame itself we are considering this wheel as a separate body ok. So, while writing this part this is with respect to the inertial frame e is the inertial frame inertial frame

So, this I have to consider it that we are taking this wheel as a separate entity. So, I fix a frame here which is a body frame here to this rotor which is rotating. So, while writing this equation then I have to write with respect to say this is the wheel axis. So, I have to write it like this is with respect to the wheel axis and then I and plus omega wheel cross I double bar this is wheel ok.

So, see the difference here this w w this is by w w here this is by omega because I am assuming that this body axis fixed to the body axis for this one which is this w I indicated by w I can write as e 1 prime e 2 prime e 3 prime. For the main body I have, for the main body I have taken axis as e 1 e 2 and e 3 and outside this I have taken capital E 1 capital E 2 and capital E 3, this is your inertial frame, this inertial. This is the satellite body frame, satellite body frame and this is wheel body frame. So, and because of this your omega w is appearing here this is very important to note; not omega this omega will not appear here if you do this then this will be a blunder.

So, we will continue in the next lecture, what we have discussed and we well derive this, with a shorter method ok.

Thank you very much for listening.